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On the evaluation of free-decay and impulse-response modal testing techniques

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***Abstract*—This paper addresses the evaluation of modal parameter identification techniques, based on transient response on lightly damped vibrating systems. This work focuses on the evaluation of the impulse-response and free-decay modal analysis techniques in vibrating systems for the construction of modal models, by applying frequency domain post-processing modal parameters extraction techniques to experimental data through the application of the classic peak picking and circle fit techniques for the extraction of modal parameters.**

***Keywords*—Modal testing, modal analysis, modal parameters identification, modal modeling.**

I. INTRODUCTION

In the field of mathematical modelling of dynamic systems there are plenty of powerful tools used to perform a deep analysis of the performance of engineering designs in operating conditions and under harmonic excitations. One of those technological tools, is the well-known modal analysis [1], defined as the process of determining the dynamic characteristics of a system in terms of natural frequencies, damping factors and mode shapes, this process leads to the use of those parameters in the construction of a mathematical model to describe the systems dynamic behavior when they undergo harmonic excitation. The final mathematical model is called modal model of the system and the information for this characteristics are known as modal data or modal parameters [1-3]. Modal models are used in vibration absorber designs, vibration control schemes and, in general engineering for design purposes, since good models of structures allow better understanding of dynamic properties.

Modal testing techniques are used to build the mathematical models since they are used in conjunction with specialized numerical methods that include peak picking, with some variants of it, as circle fitting and a curve fitting methods, usually in frequency domain, this implies the necessity of storing and processing reasonable and good quality data by means of power spectral calculations usually based on the FFT algorithm in order to extract the modal information. These numerical methods are valid when the system under analysis has a dominantly linear dynamic behavior.

In this context, the area of parametric identification in electrical and electronic systems involves a vast variety of research interests and applications, for example: power factor correctors, surge suppressors, harmonic distortion monitoring, filter systems and high precision medical equipment.

It is well known that the harmonic oscillations that describe the vibratory phenomenon can be characterized and described by parameters such as: amplitude, frequency and phase, in the context of modal analysis it is possible to make an analogous approach to the modal parameters natural frequency, damping factor and phase are implicit in the mode shapes or patterns of deformation [4] in electric or electronic systems.

The representation in the modal space that implies the use of a modal model, decomposes the complex response of a given vibratory system in a finite sum of simple responses, that are much easier to understand and manipulate, that is, the modal model allows to avoid the high complexity of the coupled mathematical models [1,3]. In this way, it is possible to use all the tools of linear algebra to make a deep analysis of the system and give a physical meaning to concepts as the values and eigenvectors of the system, even more: it is possible to make a change of coordinates that allows a decoupling of the differential equations that model the system to achieve a formal approach of the decomposition of the total response of the system as a linear combination of the different modes of vibration, described by the modal parameters; natural frequencies, modal amplitudes and damping ratios.

This work focuses on the evaluation of modal testing techniques, based on the impulse-response and free-decay excitation techniques, by analyzing the dynamic behavior of an electric system, the mathematical fundamentals of two one degrees of freedom modal parameters extraction techniques; peak picking and circle fitting are presented and then experimentally evaluated in a sixth order electric system.

II. TRANSIENT RSPONSE ANALYSIS TECHNIQUES

Consider a general signal $y(t)$ that comes from the transient response of a vibratory system (mechanical or electric), it is possible to determine the modal representation of such a signal, that is, an uncoupled version of the output obtained using an analytical model of the system. The schematic representation of the modal analysis of a transient signal is presented as a linear combination of single harmonic content is depicted in Fig. 1.

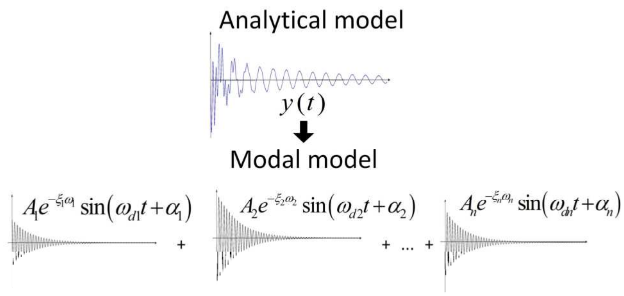

Fig. 1. Modal analysis of an electric signal

The signal $y(t)$ is considered to be quantifiable, or measurable in a time interval $[t_0, t_1]$ and can be represented as [4].

$$y(t) = \sum_{i=1}^{n} A_i e^{-\xi_i \omega_i} \sin(\omega_{di} t + \alpha_i) \quad (1)$$

with

$$\omega_{di} = \sqrt{1-\xi_i^2}\,\omega_i \quad (2)$$

where A_i is the modal amplitude, ω_i is the natural frequency, ξ_i is modal damping ratio $0 < \xi_i < 1$, ω_i is the damped natural frequency and α_i is the phase angle, all of them, associated to the i-th vibrating mode. The signal $y(t)$ can be described by the expressions:

$$A_i = \sqrt{y_{i,0}^2 + \left(\frac{\xi_i \omega_i y_{i,0} + \dot{y}_{i,0}}{\omega_{di}}\right)^2}, \qquad \alpha_i = \tan^{-1}\frac{\omega_{di}}{\xi_i \omega_i y_{i,0} + \dot{y}_{i,0}} \quad (3)$$

where $y_{i,0}$ and $\dot{y}_{i,0}$ are the initial conditions of each harmonic components of the signal. Notice that, the modal parameters come from the i-th characteristic polynomial:

$$P_i = s^2 + 2\xi_i \omega_i s + \omega_i^2 \quad (4)$$

The modal testing techniques are used to determine, experimentally the modal parameters A_i , ω_i , ξ_i and α_i to construct the modal model of the system from its transient response. In the following section, two experimental techniques for the extraction of modal parameters of a vibrating system are described.

A. Peak Picking

The Peak Picking method is also known as the mean power method [1]. This method is very portable and computationally cost effective to extract modal parameters [2,3]. The modal parameters are obtained from the frequency response function as:

$$|\mathrm{A}_{\max}| = \frac{A_r}{2\omega_r^2 \xi} \quad (5)$$

where $\omega_r = \omega_{peak}$, is the frequency at the peak of the FRF with magnitude A_r , the damping ratio ξ is then estimated as

$$\xi = \frac{\omega_b - \omega_a}{2\omega_r} \quad (6)$$

with $\omega_a < \omega_b$, the frequencies where the amplitude reaches a value of $|A_{a,b}| \approx 0.7071|A_r|$ .The modal constant A_i is directly related with the amplitude of the peak:

$$A_i = 2\alpha_r \omega_r^2 \xi_r \quad (7)$$

Finally, phase α_i is determined from the imaginary part of the FRF.

B. Circle fit

Circle fit method is more complex than peak picking, since it is based on the assumption of circularity of FRF near to resonance, in this case, modal parameters are given by:

$$\xi = \frac{\omega_a^2 - \omega_b^2}{2\omega_r\left(\tan\left(\theta_a/2\right) + \tan\left(\theta_b/2\right)\right)} \quad (8)$$

where ω_r is the undamped natural frequency, that is determined graphically by the rate at which the locus sweeps around the circular arc and takes the maximum value at resonance ω_r , the angles θ_a and θ_b are shown in Figure 2. The modal constant A_i is necessary to fit a circle using a set of points near the resonance ω_r , the diameter of that circle is directly related to the modal constant as:

$$A_r = D_r \omega_r^2 2\xi_r \quad (9)$$

where D_r is the diameter of the circle obtained by a fitting method as least squares.

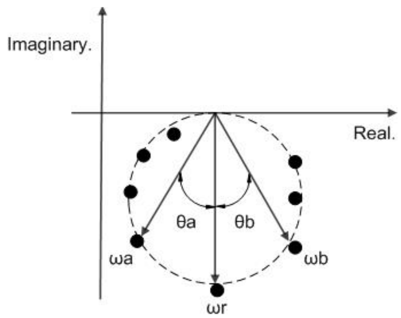

Fig. 2. Circle fit modal parameters extraction method

III. VIBRATING ELECTRIC SYSTEM

Consider the electric system shown in Fig. 3. Where the coupled system dynamics is modelled by the set of coupled differential equations:

$$\begin{aligned} L_1\ddot{i}_1 + R_1\dot{i}_1 + \frac{1}{C_1}(i_1 - i_2) &= \dot{V}_s \\ L_2\ddot{i}_2 + R_2\dot{i}_2 + \frac{1}{C_2}(i_2 - i_3) - \frac{1}{C_1}(i_1 - i_2) &= 0 \\ L_3\ddot{i}_3 + R_3\dot{i}_3 + \frac{1}{C_3}i_3 - \frac{1}{C_2}(i_2 - i_3) &= 0 \end{aligned} \quad (10)$$

where L_k , R_k and C_k , $k = 1,2,3$ are the inductance, resistance and capacitance respectively, i_k are the mesh currents and V_s is the voltage source, in addition the voltage of capacitor is given by:

$$v_{ck} = \frac{1}{C_k}\int i_{C_k} dt, \ k = 1,2,3 \quad (11)$$

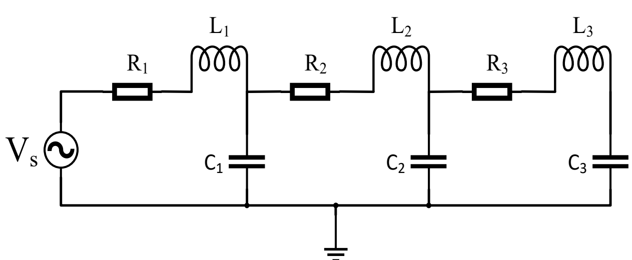

Fig. 3. Electric vibrating system.

The matrix representation of the system of equations (10) is: where $\mathbf{L}$, $\mathbf{R}$ and $\mathbf{C}$ are the inductance, resistance and

$$\mathbf{L}\ddot{\mathbf{i}} + \mathbf{R}\dot{\mathbf{i}} + \mathbf{C}\mathbf{i} = \dot{\mathbf{V}}_s \quad (12)$$

capacitance matrices respectively, with $\mathbf{L}, \mathbf{R}, \mathbf{C} \in R^{3\times3}$, the vectors $\mathbf{i} = [i_1, i_2, i_3]^T$ and $\dot{\mathbf{V}}_s = [V_s, 0, 0]^T$ are the current and voltage source vectors respectively. The inductance, resistance and capacitance matrices are given by

$$\mathbf{L} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix}, \quad (13)$$

$$\mathbf{C} = \begin{bmatrix} \frac{1}{C_1} & -\frac{1}{C_1} & 0 \\ -\frac{1}{C_1} & \frac{1}{C_1} + \frac{1}{C_2} & -\frac{1}{C_2} \\ 0 & -\frac{1}{C_2} & \frac{1}{C_2} + \frac{1}{C_3} \end{bmatrix}$$

The modal analysis representation of the system (12) leads to the expression in principal coordinates

$$\begin{aligned} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 &= \psi_{1i}\dot{V}_s \\ \mathbf{i}(t) &= \mathbf{\Psi}\mathbf{q}(t) \end{aligned} \quad (14)$$

where ξ_i and ω_i are damping ratio and natural frequency as defined in (4) and $\mathbf{q} = [q_1, q_2, q_3]^T$ .The matrix $\mathbf{\Psi} \in R^{3\times3}$ contains the eigenvector column space or mode shapes and is known as the modal matrix. In frequency domain eq. (14) takes the form:

$$\left(s^2 + 2\xi_i\omega_i s + \omega_i^2\right)Q_i(s) = \psi_{1i}\dot{V}_s(s) \quad (15)$$

Notice that , for an impulsive excitation voltage source $V(s) = \begin{cases} 1 \text{ if } t = 0 \\ 0 \text{ if } t \neq 0 \end{cases}$ a unit impulse function, by using Laplace transform operator $\mathrm{L}\{.\}$ we have.

$$\mathrm{L}\left\{\dot{V}_s(t)\right\} = \dot{V}_s(s) = s \quad (16)$$

On the other hand, for a free decay condition, that is, a change on the initial conditions, eq. (15) takes the form:

$$\left(s^2 + 2\xi_i\omega_i s + \omega_i^2\right)Q_i(s) = p_{0,1} + p_{1.i}s \quad (17)$$

where $p_{j,i}, i = 1,2,3, j = 0,1$ are constants that depend on the initial conditions of current and voltage of each capacitor.

IV. Experimental results

For the evaluation of the modal parameters extraction of the circuit shown in Fig. 3, some experiments were carried out. The system parameters are reported in Table I. Two different excitations were used for the experimental modal analysis: impulse-response and free-decay. The theoretical modal parameters of the system are reported in Table II, those parameters were obtained by solving, numerically, the eigen-problem of the system defined in (12).

TABLE I. System Parameters

Param. \ i	1	2	3
L	100 mH	400 mH	2 H
R	8Ω	8Ω	8Ω
C	0.1 µF	0.1 µF	0.1µF

A 32 bit and ARM® architecture microcontroller was used to effectively generate pulse with an amplitude of 1 volt and a duration of 100 micro seconds to emulate a pulse excitation for the circuit.

A. Impluse-response test

The system response was characterized by taking measurements of the capacitors voltage at a sample rate of 1×10^5 samples per second (100 KSPS). The system response and the excitation signal in time domain are shown in Fig. 4.

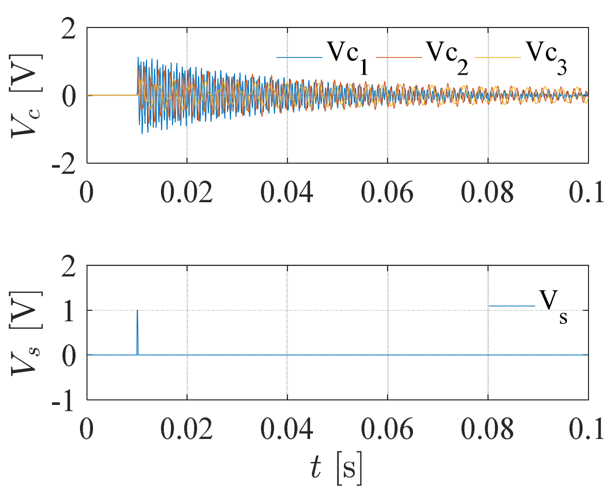

Fig. 4. Impulse response of the system.

TABLE II. Theoretical Modal Parameters

Param. \ Mode	1	2	3
ω_{di} $[Hz]$	310.7	795.8	1829
ξ_i $[\%]$	2.8	2.9	3.1
A_i $[V]$	1.3	1.98	1.7

The frequency response functions (FRF) are shown in Fig. 5, whereas the modal parameters estimated with peak picking and circle fit techniques are shown in Table 2. In Fig. 6, the Nyquist plot shows the phases with the implicit mode shapes of the system.

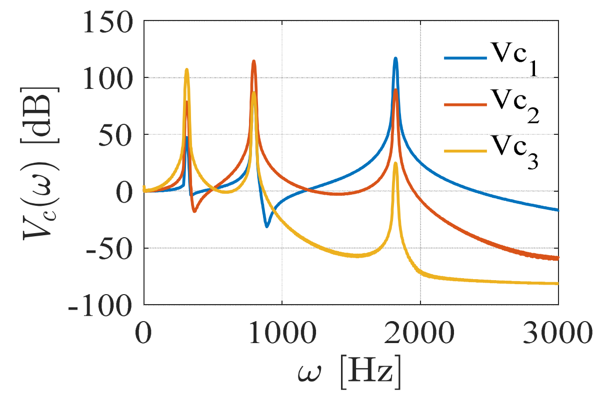

Fig. 5. FRF of the system.

TABLE III. Modal Parameters, Impluse-response test.

Param. \ Mode	Peak Picking 1	Peak Picking 2	Peak Picking 3	Circle Fit 1	Circle Fit 2	Circle Fit 3
ω_{di} $[Hz]$	311.27	794.98	1820	312.08	795.02	1816
ξ_i $[\%]$	2.73	2.8	2.98	2.69	2.82	3.0
A_i $[V]$	1.23	1.88	1.65	1.36	1.87	1.35

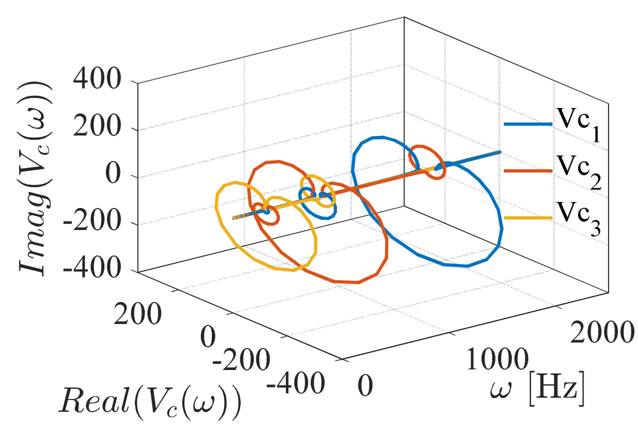

Fig. 6. Implicit mode shapes of the system.

The phases of the individual capacitor voltages are equivalent to the modal shapes of the electric system and are graphically represented in Fg.6. The 3D diagram shows the argand or Nyquist plot as a function of frequency instead of a trajectory curve.

B. Free-decay test

The free decay test was performed by applying a constant voltage of 1 [V], digitally generated by the 10 bits digital to analog converter (DAC) of the ARM® processor. The constant voltage was applied to the circuit during one second and then, the DAC was turned off to simulate a change in initial conditions in the electric system. The parameters of the system are reported in Table 1, the free response of the system y shown in Fig. 7.

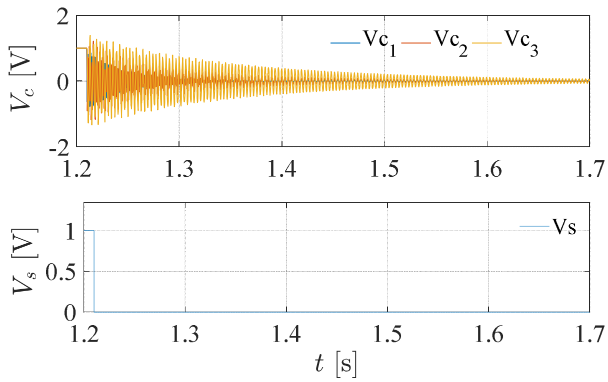

Fig. 7. Free-deaay test, system response.

The frequency response functions (FRF) of the free-decay test are shown in Fig. 8, whereas the estimated modal parameters are reported in Table 3.

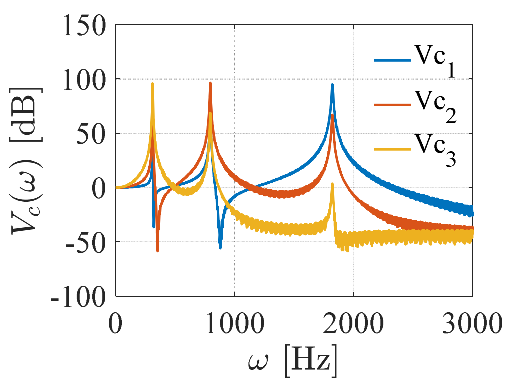

Fig. 8 Frequency response function of the system.

TABLE IV. MODAL PARAMETERS, FREE-DECAY TEST.

	Peak Picking			Circle Fit		
Param. / Mode	1	2	3	1	2	3
ω_i $[Hz]$	310.89	790.22	1818.5	308.98	790.40	1816.6
ξ_i $[\%]$	2.35	2.12	2.98	2.3	2.38	3.2
A_i $[V]$	1.43	1.38	1.45	1.33	1.5	1.65

A comparison of results is presented in Table V where the mean value of the both techniques is compared to the analytical values reported in Table II for each kind of excitation input.

TABLE V. RESULTS COMPARISON.

	Impulse-test			Free-decay		
Param. / Diff %	1	2	3	1	2	3
ω_i $[Hz]$	0.31	0.1	0.6	0.24	0.68	0.62
ξ_i $[\%]$	3.21	3.1	3.54	3.21	3.1	4.0
A_i $[V]$	0.38	5.3	2.94	3.07	3.53	3.82

In Fig. 9, the Nyquist plot shows the phase whit the implicit second mode shape of the system. The circle fit method for the modal parameters extraction of mode 2 is also depicted.

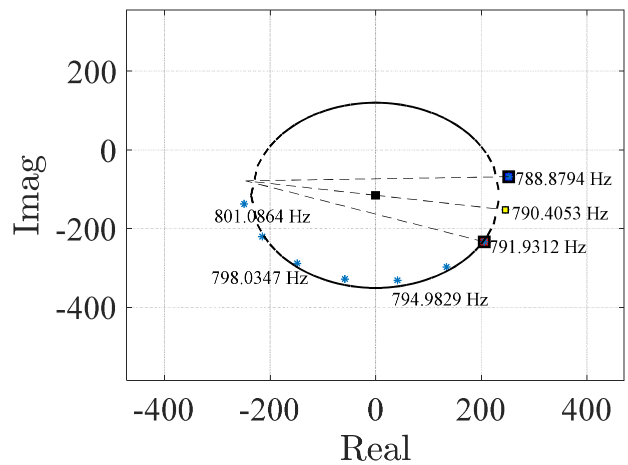

Fig. 9. Circle fit of the second resonance.

V. CONCLUSIONS

An evaluation of two transient response methods was performed in an electric system. The generation of high accuracy impulse signal tests is possible due to the versatility and high performance of the peripherals of the ARM® processors, in conjunction with the strict time restrictions and real time operation of the digital systems. The transient response methods are easy to implement compared to their mechanical version, due to the exact reproducibility of the input signal. Based on the results of the test, the impulse-response test is more stable and adequate for the extraction of modal parameters.

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