Robust Adaptive Control of Servo Systems

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***Abstract*—This paper presents the evaluation of a novel Robust Adaptive Control algorithm on a servo system. The main feature of the proposed approach is the use of an updated law based on the e-modification, endowed with a saturation function that maintains the parameter estimates within known lower and upper bounds without resorting on parameter projection techniques. The performance and advantages of the proposed update law compared to the use of a standard updated law based on the e-modification are evaluated. The experimental results highlight the advantages of the proposed robust adaptive controller. In addition, the experiments also show that the use of the proposed update law produces a smooth control signal without large peaks.**

***Index Terms*—Adaptive Control, DC servomotor, parameter identification, update law, robustness, e-modification.**

I. INTRODUCTION

Adaptive Control is a classical tool for the design of control algorithms for systems with parametric uncertainties. In general terms, an adaptive controller is an algorithm that can modify its behavior in the presence of changes in a system parameters and when the system is affected by disturbances. There exist two methods that adjust the controller when the system parameters are unknown, these methods are called direct and indirect. In the case of Direct Adaptive Control the controller parameters are estimated directly whereas in Indirect Adaptive Control the parameters of the system under control are estimated, subsequently, these estimates allow computing the controller [1].

An adaptive controller may be divided into two loops, the first is referred as the parameter identification loop and the second as the control algorithm. The parameter identification process is performed on-line, and the controller may be designed for tracking the output of a reference model, a constant or a time-varying reference signal [1], [2], [3], [4], [5], [6]. In this article, we will emphasize the Direct Adaptive Control (DAC), whose purpose, is to track the output of a reference signal [7], [8], [9]. The update mechanism, which performs the parameter identification, may be obtained by the application of stability theory [10], [11].

There are two problems when using the classic DAC. First, it exhibits parametric drift if the plant under control suffers the effect of disturbances. A way to deal with this problem is to modify the update law by means of the e-modification [12]. The second problem is the fact that the DAC may produce large control signals. This behaviour may be alleviated by adding standard projection mechanisms to the update law to limit the parameter estimates to known bounds [13].

The objective of this work is to present a novel robust Adaptive Control algorithm. The key feature of the proposed approach is the use of an update law, which is based on the standard e-modification, but adds a smooth saturating function to limit the values of the parameter estimates without relying on standard parameter projection techniques. In this way, the proposed controller is robust against disturbances and at the same time the control signal does not exhibit large peaks. It should be noted that the proposed update law has not been described in any reference so far.

The article is divided as follows: Section II presents the mathematical model of a DC servo system. Section III describes the proposed direct adaptive control algorithms applied to the servo system. Section IV gives details of the experimental platform used for the experiments, and presents the experimental results. Finally, the conclusions of the study are presented.

II. MATHEMATICAL MODEL OF A SERVO SYSTEM

Assume that the DC motor is driven by a power amplifier working in current mode. Moreover, a position sensor allows measuring the motor angular position. The above elements form a servo system whose model is described as follows:

$$J\ddot{y} = -f\dot{y} + ku + \eta \tag{1}$$

where y, $\dot{y}$ and $\ddot{y}$ are respectively the DC motor angular position, velocity and acceleration, u is the control voltage, J corresponds to the DC motor and load inertias, f to the viscous friction, k the input gain, which depends on the power amplifier gain and the motor torque constant, and and η corresponds to bounded external disturbances.

III. DIRECT ADAPTIVE CONTROL

Multiplying (1) by $\dfrac{1}{k}$ gives:

$$\frac{J}{k}\ddot{y} = -\frac{f}{k}\dot{y} + u + \frac{\eta}{k} \tag{2}$$

which has the next alternative writing:

$$\theta_1^*\ddot{y} + \theta_2^*\dot{y} = u + d \tag{3}$$

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with $\theta_1^* = \frac{J}{k}$, $\theta_2^* = \frac{f}{k}$ and $d = \frac{\eta}{k}$, where $|d| \leq D$. It is important to note that D is not known, and for the design of the proposed adaptive controller this knowlege is not necessary.

Consider the next control law assuming that θ_1^* and θ_2^* are known [14]:

$$u = \theta_1^* \dot{v} + \theta_2^* v - K_d r \tag{4}$$

where

$$\begin{aligned} v &= \dot{y}_m - \lambda(y - y_m) \\ e &= y - y_m \\ r &= \dot{y} - v = \dot{e} + \lambda e. \end{aligned} \tag{5}$$

and y_m is a reference signal with continuous first and second time derivatives.

From the above, we obtain:

$$\begin{aligned} \dot{y} &= r + v \\ \ddot{y} &= \dot{r} + \dot{v} \end{aligned} \tag{6}$$

Substituting (4) into (3) and using (5) yields the next closed-loop system:

$$\begin{aligned} \theta_1^* \ddot{y} + \theta_2^* \dot{y} - d_b &= \theta_1^* \dot{v} + \theta_2^* v - K_d r \\ \theta_1^* (\dot{r} + \dot{v}) + \theta_2^* (r + v) - d_b &= \theta_1^* \dot{v} + \theta_2^* v - K_d r \\ \theta_1^* \dot{r} + \theta_2^* r - d_b + K_d r &= 0 \end{aligned} \tag{7}$$

Now, suppose a control law computed using parameter estimates:

$$u = \hat{\theta}_1 \dot{v} + \hat{\theta}_2 v - K_d r \tag{8}$$

Performing the same procedure as in (7), but substituting (8) into (3) and using (5) gives the next error dynamics:

$$\theta_1^* \dot{r} + \theta_2^* r - d + K_d r = \tilde{\theta}_1 \dot{v} + \tilde{\theta}_2 v \tag{9}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*,\ i = 1, 2$ are the parametric errors.

A. Update law with standard e-modification

The parameters estimates $\hat{\theta}_i,\ i = 1, 2$ are obtained from the following adaptation law with an additional term $\gamma_1 \kappa |r| \hat{\theta}_i$ which is called the e-modification to eliminate the parametric drift in the parameter estimates due to external disturbances in the system to be controlled [12]:

$$\begin{aligned} \dot{\hat{\theta}}_1 &= -\gamma_1 \dot{v} r - \gamma_1 \kappa |r| \hat{\theta}_1 \\ \dot{\hat{\theta}}_2 &= -\gamma_2 v r - \gamma_2 \kappa |r| \hat{\theta}_2 \end{aligned} \tag{10}$$

B. Proposed update law

Assume that θ_i^* belongs to the set $\Omega = [\theta_{\min i}, \theta_{\max i}],\ i = 1, 2$. Then, the next transformation:

$$\theta_1^* \ = \ \lambda_1 [1 + \tanh(\eta_1^*)] + \theta_{\min 1} \tag{11}$$

$$\theta_2^* \ = \ \lambda_2 [1 + \tanh(\eta_2^*)] + \theta_{\min 2} \tag{12}$$

guaranties that for any value of η_i^* the parameter θ_i^* remains in the set Ω. Function $\tanh(\cdot)$ corresponds to the hyperbolic tangent.

Bearing in mind the above transformation, the estimation of the parameters estimates $\hat{\theta}_i,\ i = 1, 2$ are obtained as follows. Define:

$$\hat{\theta}_i = \frac{1}{2}[\theta_{\max i} - \theta_{\min i}][1 + \tanh(\hat{\eta}_i)] + \theta_{\min i}, i = 1, 2 \tag{13}$$

The next adaptation law produces the estimates $\hat{\eta}_i,\ i = 1, 2$:

$$\dot{\hat{\eta}}_i = -\gamma_i \dot{v} r - \gamma_i \kappa |r| \hat{\theta}_i \tag{14}$$

For the sake of space, the stability proof, which uses the error dynamics (9) and the update law (13)-(14), is omitted. However, some comments regarding the update laws are in order. The main difference between the update law (10) and (13)-(14) is the fact that the parameter estimates produced by the latter are bounded to prescribed limits whereas in the case of (10) the parameter estimates are bounded only when it is used in closed-loop with an adaptive controller, but the bound is unknown. Moreover, in both cases the e-modification provides closed-loop robustness against disturbances, and the use of the $\tanh(\cdot)$ function improves smoothness of the parameter estimates and consequently of the control signal.

IV. Experiments

A. Experimental platform

The experimental platform used to evaluate the adaptive control algorithms, depicted in Fig. 1, consists of a personal computer equipped with a data acquisition card. The control algorithms are coded using the MATLAB/SIMULINK programming platform under the real-time software WINCON environment *QUARC* from *Quanser Consulting*, with a sampling time of 1 ms, and the Euler01 integration method is used because it is simple to implement and its use opens the possibility of a simpler implementation in low cost processors. The control signal output produced by the data acquisition card feeds a power amplifier Copley Controls 413 working in current mode through a box that galvanically isolates the data acquisition card from the power amplifier with a saturation voltage of $\pm 10V$.

B. Parametric Identification of a DC Servomotor

Consider the matematical model of a DC servomotor (1) without disturbance. To translate it into an alternative writing multiply by $\frac{1}{J}$ to obtain:

$$\ddot{y} = -a\dot{y} + bu \tag{15}$$

with $a = \frac{f}{J}$, $b = \frac{k}{J}$ positives parameters. This system is marginally stable, so a proportional derivative controller is implemented to stabilize it in closed loop and carry out the parameter identification.

Applying the Least-Squares method to the servomotor model (15) requires measurements of u, y control signal u, angular velocity $\dot{y}$, and angular acceleration $\ddot{y}$ at different time instants to form an over-determined system. These variables are estimated through linear filters to generate an expression that contains the unknown parameters [10].

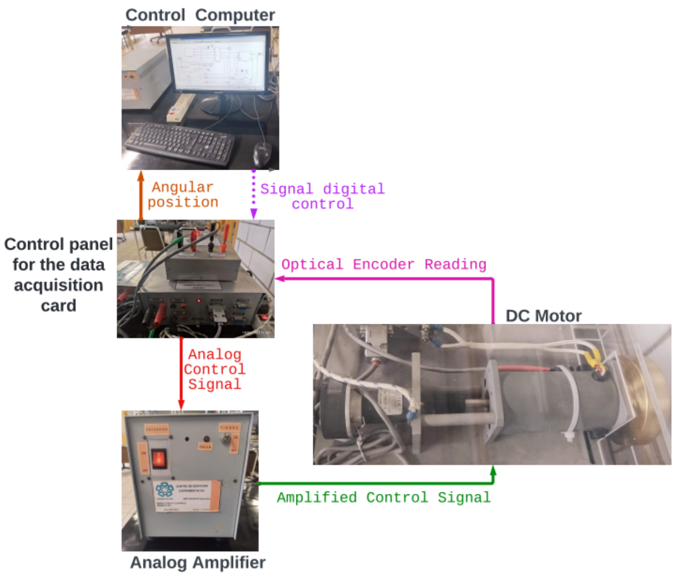

Fig. 1: Experimental platform.

The filter is defined:

$$F(s) = \frac{f_2}{s^2 + f_1 s + f_2} \tag{16}$$

with f_1 y f_2 positive real numbers.

Applying the filter to the model (15) produces:

$$\ddot{y}_f + a\dot{y}_f = bu_f \tag{17}$$

The filters used to generate the variables u_f, $\ddot{y}_f$ y $\dot{y}_f$ are the following:

$$\begin{aligned} y &\rightarrow \frac{-f_2 s}{s^2 + f_1 s + f_2} \rightarrow -\dot{y}_f \\ y &\rightarrow \frac{f_2 s^2}{s^2 + f_1 s + f_2} \rightarrow \ddot{y}_f \\ u &\rightarrow \frac{f_2}{s^2 + f_1 s + f_2} \rightarrow u_f \end{aligned} \tag{18}$$

The filtered model given by (17) allows a linear regression based on available signals, that is:

$$z(t) = \phi^T(t)\theta_{LS} \tag{19}$$

where:

$$z(t) = \ddot{y}_f$$

$$\phi(t) = \begin{bmatrix} -\dot{y}_f \\ u_f \end{bmatrix}, \quad \theta_{LS} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \theta_{LS1} \\ \theta_{LS2} \end{bmatrix}$$

vector $\phi(t)$ is a vector of known functions and $\theta_{LS} \in \Re^2$ is a vector of unknown constant parameters.

The equation (19) is valid for any time t, therefore also it is valid for the time moments $T, 2T..., (k-1)T, kT$, where T it is the sampling period. The previous observation allows writing it as:

$$z(k) = \phi^T(k)\theta_{LS} \tag{20}$$

where the sampling period T has been eliminated to keep the notation simple.

Measurements y and u are made at the sampling periods $1.2, ..., (k-1), k$, and the corresponding values of $\dot{y}_f$, $\ddot{y}_f$ and u_f are computed at these times. The above allows forming the following overdetermined system:

$$\begin{aligned} -\dot{y}_f(1)\theta_1 + u_f(1)\theta_2 &= \ddot{y}_f(1) \\ -\dot{y}_f(2)\theta_1 + u_f(2)\theta_2 &= \ddot{y}_f(2) \\ &\cdot \\ &\cdot \\ -\dot{y}_f(k)\theta_1 + u_f(k)\theta_2 &= \ddot{y}_f(k) \end{aligned}$$

which can be expressed as:

$$\begin{bmatrix} \phi^T(1) \\ \phi^T(2) \\ \cdot \\ \cdot \\ \phi^T(k) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} z(1) \\ z(2) \\ \cdot \\ \cdot \\ z(k) \end{bmatrix}$$

Finally, the solution in the Least-Squares sense for system overdetermined corresponds to [3], [15]:

$$\hat{\theta}_{LS} = (A^T A)^{-1} A^T Y \tag{21}$$

where

$$A = \begin{bmatrix} \phi^T(1) \\ \phi^T(2) \\ \cdot \\ \cdot \\ \phi^T(k) \end{bmatrix} \in \Re^{n\times 2} \;\; ; \;\; Y = \begin{bmatrix} z(1) \\ z(2) \\ \cdot \\ \cdot \\ z(k) \end{bmatrix} \in \Re^{n\times 1}$$

For more information on the development of the parametric identification of a DC servomotor model, see [16].

By applying the Least-Squares method (21) to servomotor model (15), we obtain the estimates:

$$\hat{\theta}_{LS} = \begin{bmatrix} \hat{\theta}_{LS1} \\ \hat{\theta}_{LS2} \end{bmatrix} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} 2 \\ 50 \end{bmatrix}$$

.

To obtain the $\theta_{\min i}$ and $\theta_{\max i}$ values used in the evaluation of the proposed adaptive controller, the following is used:

$$\begin{aligned} \theta_{\min 1} &= \frac{1}{2}\hat{\theta}_I \\ \theta_{\max 1} &= 75\hat{\theta}_I \\ \theta_{\min 2} &= \frac{1}{4}\hat{\theta}_{2I} \\ \theta_{\max 2} &= 37.5\hat{\theta}_{2I} \end{aligned} \tag{22}$$

where $\hat{\theta}_I = \dfrac{1}{\hat{\theta}_{LS2}}$ and $\hat{\theta}_{2I} = \dfrac{\hat{\theta}_{LS1}}{\hat{\theta}_{LS2}}$. The bounds in (22) were chosen in order to allow the parameters estimated in the proposed adaptation law to evolve and to consider possible increases in the inertia J of the servo system.

C. Experimental results

The variables presented in Table I are used to implement the adaptive control algorithms. They are tuned by trial and error so that the tracking error converges to zero. The input signal y_m applied is a sum of sinusoidal signals:

$$y_m = 0.7\sin(0.2t) + 0.5\sin(0.3t) + 0.3\sin(0.5t)$$

processed by a low-pass filter with a cut-off frequency of 5 rad/s. The reference model uses a damping factor $\zeta = 1$ and a natural frequency $\omega_n = 5$. The servomotor angular velocity $\dot{y}$ is estimated from position measurements through the cascade connection of a high-pass filter and a low-pass filter:

$$G_f(s) = \frac{300s}{s+300}\,\frac{300}{s+300} \tag{23}$$

where the high-pass filter estimates the angular velocity, while the low-pass filter attenuates the high frequencies.

Fig. 2 shows the tracking of the Reference output using the update law (10) while Fig. 3 presents the the tracking results using the update law (13)-(14). In both Figures it can be seen that the differences in the tracking of the reference are minimal, indicating that in both cases the controller produce essentially the same performance. The corresponding control signals are displayed in Fig. 4 and Fig. 5 respectively. It can be observed that the signal produced by the controller whose parameter estimates are generated by the update law (10) generates large peaks. However, if the parameter estimates are produced by the update law (13)-(14) then the control signal do not longer exhibits these peaks.

The tracking errors are presented in Fig. 6, 7 corresponding to each of the update laws reported above. Fig. 8 shows the time evolution of the parameters estimated generated by the update laws (10). In this case, the variations of the estimates at the time instants $10s$ and $20s$ coincides with the peaks observed in the control signal in Fig. 4. Then, then the variations in the parameter estimates seem to be responsible of the large peaks observed in the control signal. On the other hand, the parameter estimates produced by the update law (13)-(14) exhibits very small variations after an initial transient.

V. Analysis of Results

To assess the performance of the adaptive controller under the two update laws and applied to the servo system, the following performance criteria were used: the integral squared error (ISE), the integral of the absolute value of the control

TABLE I: Variables used in the implementation of the Model Reference Adaptive Control law.

Control algorithm variables	
$K_d = 2$	$\lambda = 5$
Adaptation law variables	
$\gamma_1 = \gamma_2 = 20$	$\kappa = 0.1$
Bounds on the parameter estimates	
$\theta_{\min 1} = 0.01$	$\theta_{\max 1} = 1.5$
$\theta_{\min 2} = 0.01$	$\theta_{\max 2} = 1.5$

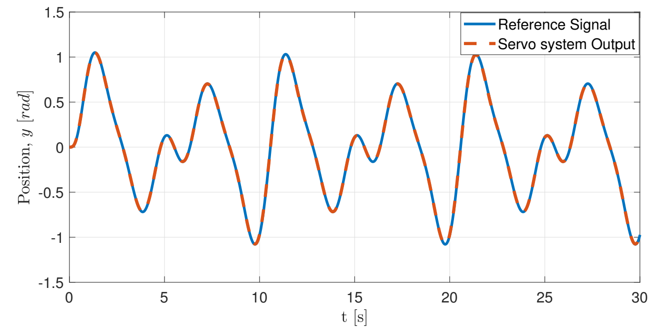

Fig. 2: Signal y_m and servo system output y using update law (10)

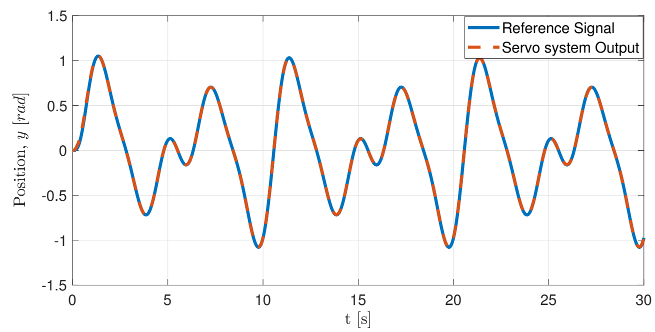

Fig. 3: Signal y_m and servo system output y using update law (13)-(14).

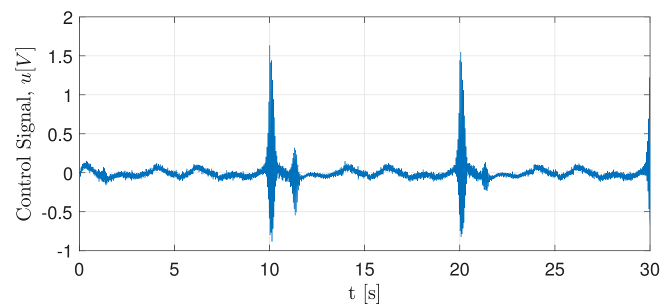

Fig. 4: Signal produced by the controller (8) and the update law (10).

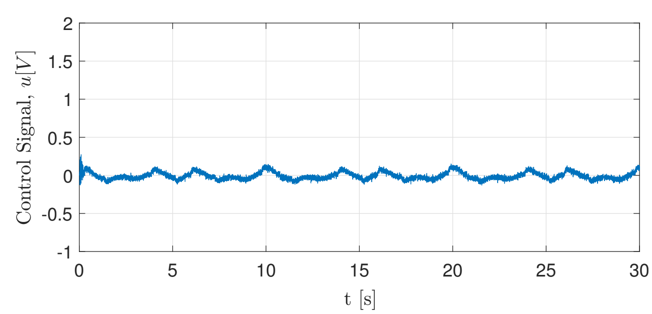

Fig. 5: Signal produced by the controller (8) and the update law (13)-(14).

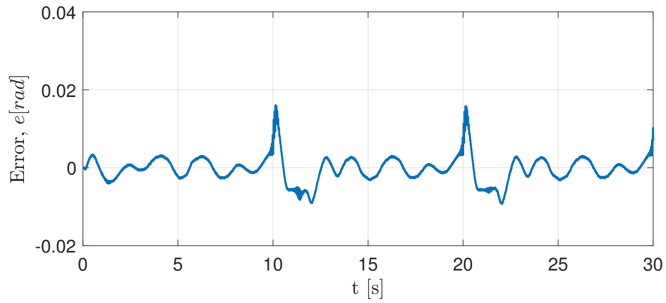

Fig. 6: Tracking error produced by the controller (8) and the update law (10) .

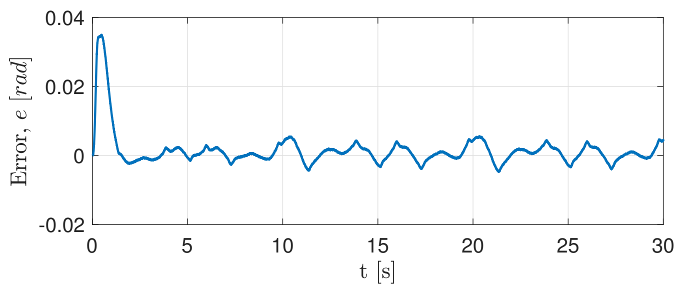

Fig. 7: Tracking error produced by the controller (8) and the update law (13)-(14).

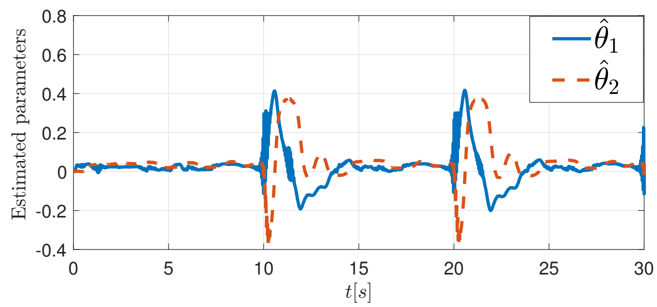

Fig. 8: Parameters estimated $\hat{\theta}_1$ and $\hat{\theta}_2$ produced by the update law (10).

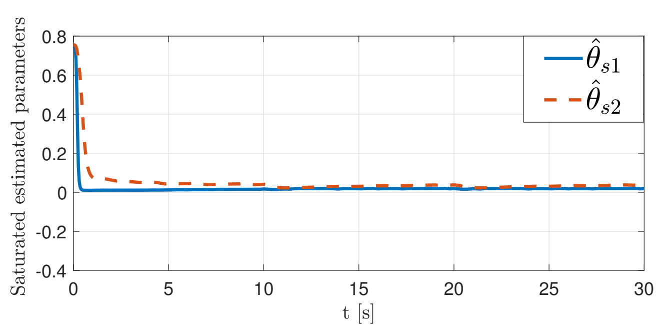

Fig. 9: Parameter estimated $\hat{\theta}_{s1}$ and $\hat{\theta}_{s2}$ obtained from the update law (13)-(14).

TABLE II: Performance of the adaptive controller using the update laws (10) and (13)-(14).

Update laws	ISE	IAC	IACV
Update law (10)	0.7627	0.3714	84.5816
Update law (13)-(14)	**0.2012**	**0.1875**	**56.9439**

signal (IAC) and the integral of the absolute value of the control signal variation (IACV). These indices are mathematically expressed as follows:

$$ISE = \int_{T_1}^{T_2} k[e(t)]^2 dt \tag{24}$$

$$IAC = \int_{T_1}^{T_2} |u(t)| dt \tag{25}$$

$$IACV = \int_{T_1}^{T_2} \left| \frac{du(t)}{dt} \right| dt \tag{26}$$

where k represents a scaling factor and $\{T_1, T_2\}$ defines a time interval during which performance indexes are calculated. For the comparative study a value of $k = 100$ is used with $T_1 = 10s$ and $T_2 = 15s$. As it can be observed in Table II the values of the ISE, IAC and IACV indices for the proposed update law (13)-(14) are lower compared with those corresponding to the update law (10). Note that the higher value of the IACV index is in agreement with the large variations observed in the control signal in Fig. 4 and in the tracking error signal in Fig. 6.

Note that the peaks observed in the control signal generated by the adaptive controller using the standard e-modification shown in Fig. 4 reach a value close to 2V whereas the corresponding peaks of the adaptive controller endowed with the proposed update law depicted in Fig. 5 are significantly smaller. It is worth noticing that even if larger peaks do not affect the tracking performance, they may damage the DC motor and the mechanical components attached to it. None of the control signals reaches the maximum value of ± 10V imposed by the power amplifier.

VI. Conclusions

Based on the experimental results presented in previous sections, it can be concluded that the proposed Direct Adaptive Control algorithm, whose parameter estimates are produced by a standard and a proposed update laws, is able to track the output of a reference signal. However, the standard update law produces large variations in the parameter estimates, which seems to be responsible of large peaks in the control signal. On the other hand, the proposed update law avoids these peaks, produces a smoother control signal and a reduced tracking error. The above is due to the use of a saturation function in the update law that maintains the parameter estimates inside known bounds.

Future work includes comparing the proposed algorithm with other recently proposed controllers and to extend the study to the case of trajectory tacking control.

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