2022 19th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE). Mexico City, Mexico. November 9-11, 2022

New Results for Stability Analysis of Time-Delay Nonlinear Systems Represented by Exact Takagi-Sugeno Models

Rafael Duarte
Electrical and Electronics Engineering Department
Sonora Institute of Technology
Ciudad Obregón, Sonora, México
email: max_paguer30@hotmail.com

Raymundo Márquez
Electrical and Electronics Engineering Department
Sonora Institute of Technology
Ciudad Obregón, Sonora, México
email: raymundo.marquez@itson.edu.mx

Miguel Bernal
Electrical and Electronics Engineering Department
Sonora Institute of Technology
Ciudad Obregón, Sonora, México
email: miguel.bernal@itson.edu.mx

Adolfo Soto
Electrical and Electronics Engineering Department
Sonora Institute of Technology
Ciudad Obregón, Sonora, México
email: asoto@itson.edu.mx

***Abstract*—This paper addresses the problem of stability analysis of constant time-delay nonlinear systems rewritten as exact Takagi-Sugeno models. More relaxed sufficient conditions for stability analysis in terms of linear matrix inequalities have been obtained combining a novel non-quadratic Lyapunov-Krasovskii functional with an artificially extended system modelled in a Takagi-Sugeno. The Lyapunov functional analysis makes use of the very well-known Finsler's lemma. An academic example is provided to illustrate the effectiveness and advantages of the presented approach in comparison with other approaches.**

***Index Terms*—Lyapunov-Krasovskii Functional, Linear Matrix Inequality, Takagi-Sugeno, Time-Delay System.**

I. Introduction

The study of systems with time delays has become relevant due to their common presence in real systems and its effects on their stability. It is well known that small delays can cause poor system performance, oscillations and even lead to instability [1]. Delays are common in practice, they can appear during the measurement of a system variable, due to the physical nature of some component or in the transmission of some signal [2]; these facts highlight the importance of analyzing systems under the effect of delays. Time delays can be constant or variable in time and they generally appear in the states of the system as well as in the output and/or input signals [3]. Usually, stability analysis has been carried out from two approaches: (1) in the temporal domain where the extension of the second Lyapunov method enables finding results with the Lyapunov-Krasovskii functional [4] or the Lyapunov–Razumikhin function [5]; (2) in the frequency domain through analytical and graphical tests such as the generalization of the Hurwitz method for linear [6] or uncertain linear [7] delayed systems.

On the other hand, exact rewriting of nonlinear dynamics in the Takagi-Sugeno (TS) form (also known as convex model) [8] is widely used when addressing stability analysis and stabilization of nonlinear systems, whether under the effect of time-delays or not [9]. The TS representation is obtained by applying the sector nonlinearity approach [10]. The convex form allows a systematic and optimal stability analysis or controller/observer synthesis through the direct Lyapunov method that generally leads to design conditions expressed as linear matrix inequalities (LMIs), whose advantage is that they can be efficiently solved using convex optimization techniques [11] implemented in commercial software [12].

A wide variety of works in the literature are related to the stability and stabilization of time-delay nonlinear systems via TS models, for instance, [13], [14], [15], [16] based on a Lyapunov-Krasovskii functional (LKF) or in [3] where robust stabilization of a system with time-delay in the input and parametric uncertainties is analyzed. However, all the approaches provide only sufficient conditions, which implies a certain degree of conservativeness in the solutions, so there is room for improvement. Some strategies to deal with such conservativeness are: i) to use various integral inequalities, for instance, the Jensen inequality proposed in [17] or the Wirtinger inequality given in [18]; ii) to include slack variables in the conditions [19]; iii) to consider fuzzy [20], [21], line integral [19], [22], or non-quadratic terms in the Lyapunov-Krasovskii functional [23], [24],[25]; iv) to use an artificially extended system [26] which introduces slack variables. This work belongs to the latter two classes as a novel non-quadratic Lyapunov-Krasovskii functional (NQLKF) will be used in combination with an artificially extended system.

This paper is organized as follows: section II presents a) the methodology to rewrite a time-delay nonlinear system in an exact TS model; b) some useful properties and notation employed throughout this paper; c) the way to construct an artificially extended system. Section III develops the main results, relaxed sufficient LMI conditions for stability analysis

978-1-6654-5508-4/22/$31.00 ©2022 IEEE

of time-delay nonlinear systems that arise from a combination of an integral inequality with a novel non-quadratic Lyapunov-Krasovskii functional and the Finsler's lemma. Section IV illustrates the effectiveness and advantages of our proposal with a numerical example. Section V gives the conclusions and future work.

II. Preliminaries

A. Convex modelling

The following autonomous time-delay nonlinear system is considered:

$$\begin{aligned}\dot{x}(t) &= f_1(x(t))x(t) + f_2(x(t))x(t-\tau), \\ x(\theta) &= \phi(\theta),\ \ \theta \in [-\tau, 0]\,,\end{aligned} \tag{1}$$

where $f_i(\cdot)$, $i \in \{1,2\}$ are sufficiently smooth nonlinear matrix functions, $x(\cdot) \in \mathbb{R}^n$ is the state vector, $\tau \in \mathbb{R}^+$ is a constant time delay, $\phi \in \ell\big([-\tau,0],\mathbb{R}^n\big)$ is the initial function, $\ell\big([-\tau,0],\mathbb{R}^n\big)$ is the Banach space of real continuous functions on the interval $[-\tau, 0]$ with

$$||\phi||_\tau := \sup_{\theta\in[-\tau,0]} ||\phi(\theta)||,$$

where $||\cdot||$ is the Euclidean norm in $\mathbb{R}^n$. A unique solution of the system, $x(t;\phi) \in \mathbb{R}^n$ (state vector), is assumed for each initial condition $\phi \in \ell\big([-\tau,0],\mathbb{R}^n\big)$ and $t \geq 0$. A segment of solution $x(t;\phi)$ is represented by $x_t(\phi) := \{x(t+\theta;\phi) : \theta \in [-\tau,0]\} \subset \mathbb{R}^n$. For simplicity, $x(t)$ and x_t are employed instead of $x(t;\phi)$ and $x_t(\phi)$.

The sector nonlinearity approach presented in [10] can be employed to obtain an exact TS fuzzy model of a time-delay nonlinear system as in (1) using the following steps:

1) Identify the p non-constant terms in $f_i(x)$, $i \in \{1,2\}$, which are the elements of the premise vector function $z(x(t))$ considered smooth and bounded in a compact set $\mathcal{C}_x$ including the origin, i.e

$$z(x(t)) = \begin{bmatrix} z_1(x(t)) & z_2(x(t)) & \cdots & z_p(x(t))\end{bmatrix}^T,$$

with

$$z_j : \mathbb{R}^n \to [z_j^0, z_j^1] \subset \mathbb{R}, j \in \{1,2,\ldots,p\},$$

where $z_j^0 = \min\limits_{x(t)\in\mathcal{C}_x} z_j(x)$, $z_j^1 = \max\limits_{x(t)\in\mathcal{C}_x} z_j(x)$.

2) Construct the following pairs of convex functions for each nonlinear term in the premise vector function:

$$w_0^j(x) = \frac{z_j^1 - z_j(x)}{z_j^1 - z_j^0},\ w_1^j(x) = 1 - w_0^j(x),$$

which satisfy the convex sum property in $\mathcal{C}_x$, i.e., $\sum_{i_j=0}^{1} w_{i_j}^j(x) = 1$, $w_{i_j}^j(x) \geq 0$ for $j \in \{1,2,\ldots,p\}$. Note that each non-constant term in the premise vector can be rewritten using the convex function as

$$z_j(x(t)) = w_0^j z_j^0 + w_1^j z_j^1 = \sum_{i_j=0}^{1} w_{i_j}^j(x) z_j^{i_j},$$

$j \in \{1,2,\ldots,p\}$. Convexity in the functions on step 2 leads to LMI conditions under the direct Lyapunov-Krasovskii method for stability analysis.

3) Finally, rewrite the time-delay nonlinear system in (1) as its exact TS convex representation in $\mathcal{C}_x$ defined by:

$$\begin{aligned}\dot{x}(t) &= \sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(x(t))(\mathbf{A_i}x(t) + \mathbf{A_{di}}x(t-\tau)), \\ x(\theta) &= \phi(\theta),\ \theta \in [-\tau, 0]\,,\end{aligned} \tag{2}$$

where $\mathbf{i} = (i_1, i_2, \ldots, i_p)$, $\mathbf{w_i}(x) = w_{i_1}^1 w_{i_2}^2 \cdots w_{i_p}^p$, $\mathbf{w_i}(x) \in [0,1]$, $\forall x \in \mathcal{C}_x$, $\mathbb{B} = \{0,1\}$, $\sum_{\mathbf{i}\in\mathbb{B}^p} \mathbf{w_i}(x) = 1$, $\forall x \in \mathbb{R}^n$, $\mathbf{A_i} = f_1(x(t))|_{\mathbf{w_i}=1}$ and $\mathbf{A_{di}} = f_2(x(t))|_{\mathbf{w_i}=1}$ are matrices of proper size.

B. Notations and Properties

In this paper, for any matrix $\mathcal{M}$, $\mathcal{H}_e(\mathcal{M})$ will denote $\mathcal{H}_e(\mathcal{M}) = \mathcal{M} + \mathcal{M}^T$; for a symmetric matrix $\mathcal{N}$, $\mathcal{N} > 0$ ($\mathcal{N} < 0$) means that $\mathcal{N}$ is positive definite (negative definite, respectively). When convenient, arguments will be omitted.

The following lemmas and properties will be applied to obtain the main results of this work:

Property 1. (Jensen inequality) [27]: For a given definite positive matrix $\mathcal{M} \in \mathbb{R}^{n\times n}$ and for all continuous functions $w(s)$ in $[a,b] \to \mathbb{R}^n$, the following inequality holds:

$$\begin{aligned}(b-a)\int_a^b w^T(s)\mathcal{M}w(s)ds \geq \\ \left(\int_a^b w(s)ds\right)^T \mathcal{M}\left(\int_a^b w(s)ds\right).\end{aligned} \tag{3}$$

Property 2. (Finsler's lemma) [28]: Let $x \in \mathbb{R}^n$, and matrices $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{n\times n}$, and $\mathcal{B} \in \mathbb{R}^{m\times n}$ such that $rank(\mathcal{B}) < n$, the following statements are equivalent:

$$i)\ x^T\mathcal{Q}x < 0, \quad \forall x \in \{x \in \mathbb{R}^n : \mathcal{B}x = 0, x \neq 0\}. \tag{4}$$

$$ii)\ \left(\mathcal{B}^\perp\right)^T \mathcal{Q}\mathcal{B}^\perp < 0. \tag{5}$$

where $\mathcal{B}^\perp$ denotes a basis for the null-space of $\mathcal{B}$.

C. Extended system

Consider the TS model (2) wich can be rewritten for any β such that $0 \leq \beta \leq \tau$ as:

$$\begin{aligned}\dot{x}(t+\beta) &= \sum_{\mathbf{i}\in\mathbb{B}^p}\mathbf{w_i}(x(t+\beta))(\tilde{\mathbf{A}}_\mathbf{i}x(t+\beta) + \tilde{\mathbf{A}}_\mathbf{di}x(t+\beta-\tau)), \\ x(\theta+\beta) &= \phi(\theta+\beta),\ \theta \in [-\tau, 0]\,.\end{aligned} \tag{6}$$

Now, if $\beta = \frac{\tau}{2}$ the state equation in (6) yields:

$$\begin{aligned}\dot{x}(t+\frac{\tau}{2}) = \sum_{\mathbf{i}\in\mathbb{B}^p}\mathbf{w_i}\left(x\left(t+\frac{\tau}{2}\right)\right)\left(\tilde{\mathbf{A}}_\mathbf{i}x\left(t+\frac{\tau}{2}\right)\right. \\ \left. + \tilde{\mathbf{A}}_\mathbf{di}x\left(t-\frac{\tau}{2}\right)\right).\end{aligned} \tag{7}$$

Using an auxiliar vector $\bar{x}^T(t) = \begin{bmatrix} x^T(t+\frac{\tau}{2}) & x^T(t)\end{bmatrix}$, then, an artificially extended TS model of (7) and (2) is defined as:

$$\dot{\bar{x}}(t) = \sum_{\mathbf{i}\in\mathbb{B}^{2p}} \mathbf{w_i}(\bar{x}(t))(\bar{\mathbf{A}}_\mathbf{i}\bar{x}(t) + \bar{\mathbf{A}}_\mathbf{di}\bar{x}(t-\tau)), \tag{8}$$

where $\bar{\mathbf{A}}_{\mathbf{i}} = \begin{bmatrix} \tilde{\mathbf{A}}_{\mathbf{i}} & 0_n \\ 0_n & \mathbf{A}_{\mathbf{i}} \end{bmatrix}$ and $\bar{\mathbf{A}}_{\mathbf{di}} = \begin{bmatrix} \tilde{\mathbf{A}}_{\mathbf{di}} & 0_n \\ 0_n & \mathbf{A}_{\mathbf{di}} \end{bmatrix}$.

Applying a shorthand notation, (8) can be rewritten as follows:

$$\dot{\bar{x}}(t) = \bar{\mathbf{A}}_{\mathbf{w}}\bar{x}(t) + \bar{\mathbf{A}}_{\mathbf{dw}}\bar{x}(t-\tau)) \tag{9}$$

with $\bar{\mathbf{A}}_{\mathbf{w}} = \sum_{\mathbf{i}\in\mathbb{B}^{2p}} \mathbf{w}_{\mathbf{i}}(\bar{x}(t))\bar{\mathbf{A}}_{\mathbf{i}}$, $\bar{\mathbf{A}}_{\mathbf{dw}} = \sum_{\mathbf{i}\in\mathbb{B}^{2p}} \mathbf{w}_{\mathbf{i}}(\bar{x}(t))\bar{\mathbf{A}}_{\mathbf{di}}$.

Remark 1. Note that in the case of the artificially extended system (9), the number of non-constant terms is twice in regard to the original system (2), i.e., $\mathbf{i} \in \mathbb{B}^{2p}$ instead of $\mathbf{i} \in \mathbb{B}^{p}$.

III. Main Results

In this section, conditions in a LMI form have been obtained for stability analysis of time-delay nonlinear systems based on the NQLKF in (11) altogether with the Jensen integral inequality given in (3) and the Finsler's lemma in property 2.

Theorem 1. The origin of the TS model (2) with a constant time-delay, $\tau > 0$, is asymptotically stable if there exist symmetric and definite positive matrices $P_{\mathbf{j}}^i \in \mathbb{R}^{2n\times 2n}$, $Q_{\mathbf{j}}^i \in \mathbb{R}^{2n\times 2n}$, $R^i \in \mathbb{R}^{2n\times 2n}$, $i \in \{1,2\}$ $\mathbf{j} \in \mathbb{B}^{2p}$, such that the next LMI conditions hold:

$$\left(\mathcal{B}_{\mathbf{i}}^{\perp}\right)^T \mathbf{\Gamma}_{\mathbf{ijkl}}\mathcal{B}_{\mathbf{i}}^{\perp} < 0,\ \{\mathbf{i},\mathbf{j},\mathbf{k},\mathbf{l}\} \in \mathbb{B}^{2p} \tag{10}$$

where

$$\mathbf{\Gamma}_{\mathbf{ijkl}} = \begin{bmatrix} \Gamma^{11} & \Gamma^{12} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} \\ \Gamma^{21} & \Gamma^{22} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & -Q_{\mathbf{j}}^1 & 0_{2n} & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & -Q_{\mathbf{k}}^2 & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & -\frac{2}{\tau}R^1 & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & -\frac{1}{\tau}R^2 \end{bmatrix}$$

with $\Gamma^{11} = \sum_{i=1}^2 \frac{i\tau}{2}R^i$, $\Gamma^{12} = \Gamma^{21} = \sum_{i=1}^2 P_{\mathbf{l}}^i$, $\Gamma^{22} = \sum_{i=1}^2 Q_{\mathbf{i}}^i + \frac{2}{\tau}\left(P_{\mathbf{i}}^1 - P_{\mathbf{j}}^1\right) + \frac{1}{\tau}\left(P_{\mathbf{i}}^2 - P_{\mathbf{k}}^2\right)$, $\mathcal{Z}_1 = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}$, $\mathcal{Z}_3 = \begin{bmatrix} 0_n & 0_n \\ -I_n & 0_n \end{bmatrix}$, $\mathcal{Z}_4 = \begin{bmatrix} I_n & -I_n \\ 0_n & I_n \end{bmatrix}$, and $\left(\mathcal{B}_{\mathbf{i}}^{\perp}\right)^T = \begin{bmatrix} \bar{\mathbf{A}}_{\mathbf{i}}^T & I_{2n} & \mathcal{Z}_1^T & 0_{2n} & \mathcal{Z}_4^T & I_{2n} \\ \bar{\mathbf{A}}_{\mathbf{di}}^T & 0_{2n} & \mathcal{Z}_1 & I_{2n} & \mathcal{Z}_3^T & -I_{2n} \end{bmatrix}$.

Proof. A NQLKF with the next structure is considered :

$$V(\bar{x}) = V_1(\bar{x}) + V_2(\bar{x}) + V_3(\bar{x}), \tag{11}$$

where

$$V_1(\bar{x}) = \bar{x}^T(t)\left(\sum_{i=1}^2 P_{\mathbf{v}}^i\right)\bar{x}(t),$$
$$V_2(\bar{x}) = \sum_{i=1}^2 \int_{t-\frac{i\tau}{2}}^t \bar{x}^T(s)Q_{\mathbf{w}^s}^i\bar{x}(s)ds,$$
$$V_3(\bar{x}) = \sum_{i=1}^2 \int_{t-\frac{i\tau}{2}}^t \int_\theta^t \dot{\bar{x}}^T(\theta)R^i\dot{\bar{x}}(\theta)d\theta ds,\ R^i = \left(R^i\right)^T > 0$$

with

$$P_{\mathbf{v}}^i = \sum_{\mathbf{j}\in\mathbb{B}^{2p}} \frac{2}{i\tau}\int_{t-\frac{i\tau}{2}}^t \mathbf{w}_{\mathbf{j}}(\bar{x}(s))P_{\mathbf{j}}^i ds,\ P_{\mathbf{j}}^i = \left(P_{\mathbf{j}}^i\right)^T > 0$$
$$Q_{\mathbf{w}^s}^i = \sum_{j\in\mathbb{B}^{2p}} \mathbf{w}_{\mathbf{j}}(\bar{x}(s))Q_{\mathbf{j}}^i,\ Q_{\mathbf{j}}^i = \left(Q_{\mathbf{j}}^i\right)^T > 0.$$

The time-derivative of $V_1(\bar{x})$ in (11) is:

$$\dot{V}_1(\bar{x}) = \bar{x}^T(t)\left(\sum_{i=1}^2 P_{\mathbf{v}}^i\right)\dot{\bar{x}}(t) + \dot{\bar{x}}^T(t)\left(\sum_{i=1}^2 P_{\mathbf{v}}^i\right)\bar{x}(t) + \bar{x}^T(t)\left(\sum_{i=1}^2 \frac{d}{dt}P_{\mathbf{v}}^i\right)\bar{x}(t),$$

where $\frac{d}{dt}P_{\mathbf{v}}^i = \frac{2}{i\tau}\sum_{j\in\mathbb{B}^{2p}}\left(\mathbf{w}_{\mathbf{j}}(\bar{x}(t)) - \mathbf{w}_{\mathbf{j}}(\bar{x}(t-\frac{i\tau}{2}))\right)P_{\mathbf{j}}^i$.

The time-derivative of the next term, $V_2(\bar{x})$, in (11) is:

$$\dot{V}_2(\bar{x}) = \sum_{i=1}^2\left(\bar{x}^T Q_{\mathbf{w}}^i\bar{x} - \bar{x}_{\frac{i\tau}{2}}^T Q_{\mathbf{w}_{\frac{i\tau}{2}}}^i \bar{x}_{\frac{i\tau}{2}}\right),$$

where $\bar{x}_{\frac{i\tau}{2}} = \bar{x}(t-\frac{i\tau}{2})$, $Q_{\mathbf{w}}^i = \sum_{j\in\mathbb{B}^p}\mathbf{w}_{\mathbf{j}}(\bar{x}(t))Q_{\mathbf{j}}^i$ and $Q_{\mathbf{w}_{\frac{i\tau}{2}}}^i = \sum_{j\in\mathbb{B}^{2p}}\mathbf{w}_{\mathbf{j}}(\bar{x}(t-\frac{i\tau}{2}))Q_{\mathbf{j}}^i$.

Finally, the time-derivative of $V_3(\bar{x})$ in (11) is:

$$\dot{V}_3(\bar{x}) = \sum_{i=1}^2\left(\frac{i\tau}{2}\dot{\bar{x}}^T(t)R^i\dot{\bar{x}}(t) - \int_{t-\frac{i\tau}{2}}^t \dot{\bar{x}}^T(s)R^i\dot{\bar{x}}(s)ds\right).$$

Then, $\dot{V}(\bar{x})$ yields

$$\dot{V}(\bar{x}) = \sum_{i=1}^2\Bigg(\bar{x}^T P_{\mathbf{v}}^i\dot{\bar{x}} + \dot{\bar{x}}^T P_{\mathbf{v}}^i\bar{x} + \bar{x}^T\left(Q_{\mathbf{w}}^i + \frac{d}{dt}P_{\mathbf{v}}^i\right)\bar{x} - \bar{x}_{\frac{i\tau}{2}}^T Q_{\mathbf{w}_{\frac{i\tau}{2}}}^i\bar{x}_{\frac{i\tau}{2}} + \frac{i\tau}{2}\dot{\bar{x}}^T R^i\dot{\bar{x}} - \int_{t-\frac{i\tau}{2}}^t \dot{\bar{x}}^T(s)R^i\dot{\bar{x}}(s)ds\Bigg).$$

Applying Jensen's inequality in property 1 to the previous expression, the condition $\dot{V}(\bar{x}) < 0$ is satisfied if:

$$\sum_{i=1}^2\Bigg(\mathcal{H}_e\left(\bar{x}^T P_{\mathbf{v}}^i\dot{\bar{x}}\right) + \bar{x}^T\left(Q_{\mathbf{w}}^i + \frac{d}{dt}P_{\mathbf{v}}^i\right)\bar{x} - \bar{x}_{\frac{i\tau}{2}}^T Q_{\mathbf{w}_{\frac{i\tau}{2}}}^i\bar{x}_{\frac{i\tau}{2}} + \frac{i\tau}{2}\dot{\bar{x}}^T R^i\dot{\bar{x}} - \frac{2}{i\tau}\left(\int_{t-\frac{i\tau}{2}}^t \dot{\bar{x}}(s)ds\right)^T R^i\left(\int_{t-\frac{i\tau}{2}}^t \dot{\bar{x}}(s)ds\right)\Bigg) < 0. \tag{12}$$

Condition (12) can be rewritten as

$$\zeta^T \underbrace{\begin{bmatrix} \Gamma^{11} & \Gamma^{12} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} \\ \Gamma^{21} & \Gamma^{22} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & -Q_{\mathbf{w}_{\frac{\tau}{2}}}^1 & 0_{2n} & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & -Q_{\mathbf{w}_\tau}^2 & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & -\frac{2}{\tau}R^1 & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & -\frac{1}{\tau}R^2 \end{bmatrix}}_{\Gamma_{\mathbf{w}\mathbf{w}_{\frac{\tau}{2}}\mathbf{w}_\tau\mathbf{v}}} \zeta < 0 \tag{13}$$

where $\Gamma^{11} = \sum_{i=1}^{2} \frac{i\tau}{2} R^i$, $\Gamma^{12} = \Gamma^{21} = \sum_{i=1}^{2} P_{\mathbf{v}}^i$, $\Gamma^{22} = \sum_{i=1}^{2} \left(Q_{\mathbf{w}}^i + \frac{d}{dt} P_{\mathbf{v}}^i\right)$, with

$$\zeta = \begin{bmatrix} \dot{\bar{x}}(t) \\ \bar{x}(t) \\ \bar{x}(t-\frac{\tau}{2}) \\ \bar{x}(t-\tau) \\ \int_{t-\frac{\tau}{2}}^{t} \dot{\bar{x}}(s)ds \\ \int_{t-\tau}^{t} \dot{\bar{x}}(s)ds \end{bmatrix} = \begin{bmatrix} \dot{\bar{x}}(t) \\ \bar{x}(t) \\ \bar{x}(t-\frac{\tau}{2}) \\ \bar{x}(t-\tau) \\ \bar{x}(t) - \bar{x}(t-\frac{\tau}{2}) \\ \bar{x}(t) - \bar{x}(t-\tau) \end{bmatrix}.$$

From the extended TS model in (9) and the auxiliary vector ζ, the following expression has been obtained:

$$\underbrace{\begin{bmatrix} -I_{2n} & \bar{\mathbf{A}}_{\mathbf{w}} & 0_{2n} & \bar{\mathbf{A}}_{\mathbf{dw}} & 0_{2n} & 0_{2n} \\ 0_{2n} & -I_{2n} & I_{2n} & 0_{2n} & I_{2n} & 0_{2n} \\ 0_{2n} & -I_{2n} & 0_{2n} & I_{2n} & 0_{2n} & I_{2n} \\ 0_{2n} & \mathcal{Z}_1 & \mathcal{Z}_2 & \mathcal{Z}_3 & 0_{2n} & 0_{2n} \end{bmatrix}}_{\bar{\mathcal{B}}_{\mathbf{w}}} \zeta = 0 \qquad (14)$$

with $\mathcal{Z}_1 = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}$, $\mathcal{Z}_2 = \begin{bmatrix} -I_n & 0_n \\ 0_n & I_n \end{bmatrix}$, $\mathcal{Z}_3 = \begin{bmatrix} 0_n & 0_n \\ -I_n & 0_n \end{bmatrix}$.

The inequality (13) altogether the restriction (14) can be combined using the Finsler's lemma in property 2 which leads to the following equivalent condition:

$$\left(\bar{\mathcal{B}}_{\mathbf{w}}^{\perp}\right)^T \mathbf{\Gamma}_{\mathbf{w}\mathbf{w}_{\frac{\tau}{2}}\mathbf{w}_{\tau}\mathbf{v}} \bar{\mathcal{B}}_{\mathbf{w}}^{\perp} < 0. \qquad (15)$$

where $\bar{\mathcal{B}}_{\mathbf{w}}^{\perp}$ is a basis for the null-space of $\bar{\mathcal{B}}_{\mathbf{w}}$, it has been defined as follows:

$$\bar{\mathcal{B}}_{\mathbf{w}}^{\perp} = \begin{bmatrix} \bar{\mathbf{A}}_{\mathbf{w}} & \bar{\mathbf{A}}_{\mathbf{dw}} \\ I_{2n} & 0_{2n} \\ \mathcal{Z}_1 & \mathcal{Z}_1^T \\ 0_{2n} & I_{2n} \\ \mathcal{Z}_4 & \mathcal{Z}_3 \\ I_{2n} & -I_{2n} \end{bmatrix}$$

with $\mathcal{Z}_4 = \begin{bmatrix} I_n & -I_n \\ 0_n & I_n \end{bmatrix}$.

Considering that functions $\mathbf{w}$,$\mathbf{w}_{\frac{\tau}{2}}$, $\mathbf{w}_{\tau}$, $\mathbf{v}$, hold the convex sum property, it is possible to eliminate them from conditions (15) which leads to LMI conditions in Theorem 1. Thus, the proof is concluded. □

Remark 2. The structure of the NQLKF (11) includes the information of the artificially extended system taking advantage of it in order to relax the LMI conditions for stability analysis. On the other hand, the term $V_1(\bar{x})$ avoids the problem of handling the time-derivatives of the so-called fuzzy Lyapunov function [29] as well as the limitations of the special structure for Lyapunov matrices in line-integral Lyapunov functions [30].

Remark 3. If the terms in the NQLKF (11) are considered as $P_{\mathbf{v}}^1 + P_{\mathbf{v}}^2 = P = P^T > 0$, $Q_{\mathbf{w}^s}^i = Q^i = \left(Q^i\right)^T > 0$ and following a similar development as in theorem 1, the LMI conditions that guarantee asymptotic stability yields:

$$\left(\bar{\mathcal{B}}_{\mathbf{i}}^{\perp}\right)^T \mathbf{\Gamma} \bar{\mathcal{B}}_{\mathbf{i}}^{\perp} < 0,\ \mathbf{i} \in \mathbb{B}^{2p} \qquad (16)$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma^{11} & \Gamma^{12} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} \\ \Gamma^{21} & \Gamma^{22} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & -Q^1 & 0_{2n} & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & -Q^2 & 0_{2n} & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & -\frac{2}{\tau}R^1 & 0_{2n} \\ 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & 0_{2n} & -\frac{1}{\tau}R^2 \end{bmatrix}$$

with $\Gamma^{11} = \sum_{i=1}^{2} \frac{i\tau}{2} R^i$, $\Gamma^{12} = \Gamma^{21} = P$, $\Gamma^{22} = \sum_{i=1}^{2} Q^i$, $\mathcal{Z}_1 = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}$, $\mathcal{Z}_3 = \begin{bmatrix} 0_n & 0_n \\ -I_n & 0_n \end{bmatrix}$, $\mathcal{Z}_4 = \begin{bmatrix} I_n & -I_n \\ 0_n & I_n \end{bmatrix}$, and $\left(\bar{\mathcal{B}}_{\mathbf{i}}^{\perp}\right)^T = \begin{bmatrix} \bar{\mathbf{A}}_{\mathbf{i}}^T & I_{2n} & \mathcal{Z}_1^T & 0_{2n} & \mathcal{Z}_4^T & I_{2n} \\ \bar{\mathbf{A}}_{\mathbf{di}}^T & 0_{2n} & \mathcal{Z}_1 & I_{2n} & \mathcal{Z}_3^T & -I_{2n} \end{bmatrix}$.

It is possible to observe that LMI conditions (16) are included as a particular case of condition (10) in theorem 1; these conditions reduce the number of decision variables but decrease the space of solutions, it will be shown in example section.

IV. Example

The effectiveness of the proposed approach is illustrated in the following example; it has been programmed using the LMI toolbox [12] within a MATLAB R2019a platform.

Example 1. Consider the following time-delay TS system:

$$\begin{aligned} \dot{x}(t) &= \sum_{\mathbf{i} \in \mathbb{B}^p} \mathbf{w}_{\mathbf{i}}(x(t))(\mathbf{A}_{\mathbf{i}} x(t) + \mathbf{A}_{\mathbf{di}} x(t-\tau)), \qquad (17) \\ x(\theta) &= \phi(\theta),\ \theta \in [-\tau, 0], \end{aligned}$$

where $p = 1$, $\mathbf{w}_0(x(t)) = \frac{1}{1+\exp(-2x_1(t)x_2(t))}$, $\mathbf{w}_1(x(t)) = 1 - \mathbf{w}_0(x(t))$, $\mathbf{A_0} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}$, $\mathbf{A_1} = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}$, $\mathbf{A_{d0}} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$, $\mathbf{A_{d1}} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}$.

Now, the artificially extended system of (17) yields,

$$\dot{\bar{x}}(t) = \sum_{\mathbf{i} \in \mathbb{B}^{2p}} \mathbf{w}_{\mathbf{i}}(\bar{x}(t))(\bar{\mathbf{A}}_{\mathbf{i}} \bar{x}(t) + \bar{\mathbf{A}}_{\mathbf{di}} \bar{x}(t-\tau)), \qquad (18)$$

with

$$\begin{aligned} \mathbf{w}_{00}(\bar{x}(t)) &= \mathbf{w}_0(x(t))\mathbf{w}_0(x(t+\frac{\tau}{2})), \\ \mathbf{w}_{01}(\bar{x}(t)) &= \mathbf{w}_0(x(t))\mathbf{w}_1(x(t+\frac{\tau}{2})), \\ \mathbf{w}_{10}(\bar{x}(t)) &= \mathbf{w}_1(x(t))\mathbf{w}_0(x(t+\frac{\tau}{2})), \\ \mathbf{w}_{11}(\bar{x}(t)) &= \mathbf{w}_1(x(t))\mathbf{w}_1(x(t+\frac{\tau}{2})), \end{aligned}$$

and

$$\bar{\mathbf{A}}_{\mathbf{00}} = \begin{bmatrix} \mathbf{A_0} & 0 \\ 0 & \mathbf{A_0} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -0.9 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -0.9 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{01}} = \begin{bmatrix} \mathbf{A_1} & 0 \\ 0 & \mathbf{A_0} \end{bmatrix} = \begin{bmatrix} -1 & 0.5 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -0.9 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{10}} = \begin{bmatrix} \mathbf{A_0} & 0 \\ 0 & \mathbf{A_1} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -0.9 & 0 & 0 \\ 0 & 0 & -1 & 0.5 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{11}} = \begin{bmatrix} \mathbf{A_1} & 0 \\ 0 & \mathbf{A_1} \end{bmatrix} = \begin{bmatrix} -1 & 0.5 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0.5 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{d00}} = \begin{bmatrix} \mathbf{A_{d0}} & 0 \\ 0 & \mathbf{A_{d0}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{d01}} = \begin{bmatrix} \mathbf{A_{d1}} & 0 \\ 0 & \mathbf{A_{d0}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0.1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{d10}} = \begin{bmatrix} \mathbf{A_{d0}} & 0 \\ 0 & \mathbf{A_{d1}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0.1 & -1 \end{bmatrix},$$

$$\bar{\mathbf{A}}_{\mathbf{d11}} = \begin{bmatrix} \mathbf{A_{d1}} & 0 \\ 0 & \mathbf{A_{d1}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0.1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0.1 & -1 \end{bmatrix}.$$

In Table I, a comparison of the maximum delay bound for which the nonlinear system (17) is asymptotically stable under different approaches is presented between results in [23], [31], [32], [25], and conditions in Theorem 1. LMI conditions (10) in Theorem 1 gives the best result with a maximum delay bound $\tau = 2.2350s$. Also, it is possible to observe that the maximum delay bound with conditions (16) in Remark 3 is lower than results in theorem 1 because they are a particular case, it means that the use of non-quadratic terms in the Lyapunov-Krasovskii functional provides less conservative conditions.

TABLE I
MAXIMUM DELAY WITH DIFFERENT APPROACHES: EXAMPLE 1

METHOD	τ (s)
[23]	1.6010
Remark 3, conditions (16)	1.9745
[31]	2.002
[32]	2.0689
[25]	2.1602
Theorem 1	2.2350

The following matrices involved in the NQLKF have been obtained using conditions in theorem 1 with $\tau = 2.2350s$; some of them are given for illustration purposes due to space limitation:

$$P^1_{\mathbf{00}} = \begin{bmatrix} 0.1730 & 0.0626 & 0.0032 & 0.0276 \\ 0.0626 & 0.1144 & 0.0170 & -0.0336 \\ 0.0032 & 0.0170 & 0.2746 & 0.0741 \\ 0.0276 & -0.0336 & 0.0741 & 0.1966 \end{bmatrix} \times 10^{-6}$$

$$P^2_{\mathbf{11}} = \begin{bmatrix} 0.1897 & 0.0584 & -0.0146 & 0.0029 \\ 10.0584 & 0.1318 & 0.0223 & 0.0112 \\ -0.0146 & 0.0223 & 0.1562 & -0.0371 \\ 0.0029 & 0.0112 & -0.0371 & 0.1243 \end{bmatrix} \times 10^{-6}$$

$$Q^1_{\mathbf{00}} = \begin{bmatrix} 0.3288 & 0.0163 & 0.0713 & -0.0683 \\ 0.0163 & 0.1033 & 0.0576 & -0.0357 \\ 0.0713 & 0.0576 & 0.7216 & 0.1265 \\ -0.0683 & -0.0357 & 0.1265 & 0.1940 \end{bmatrix} \times 10^{-6}$$

$$Q^2_{\mathbf{11}} = \begin{bmatrix} 0.3127 & 0.0624 & -0.1838 & -0.0222 \\ 0.0624 & 0.1090 & -0.0481 & -0.0886 \\ -0.1838 & -0.0481 & 0.6852 & 0.1111 \\ -0.0222 & -0.0886 & 0.1111 & 0.2876 \end{bmatrix} \times 10^{-6}$$

$$R^1 = \begin{bmatrix} 0.1745 & 0.0870 & -0.1177 & -0.0169 \\ 0.0870 & 0.1656 & -0.0305 & -0.0135 \\ -0.1177 & -0.0305 & 0.1377 & 0.0627 \\ -0.0169 & -0.0135 & 0.0627 & 0.0914 \end{bmatrix} \times 10^{-6}$$

$$R^2 = \begin{bmatrix} 0.2413 & -0.0778 & 0.1061 & -0.0991 \\ -0.0778 & 0.1047 & 0.0387 & -0.0252 \\ 0.1061 & 0.0387 & 0.2895 & -0.1841 \\ -0.0991 & -0.0252 & -0.1841 & 0.1798 \end{bmatrix} \times 10^{-11}$$

The time response of the states is shown in Figure 1. As expected, the states are driven asymptotically towards the

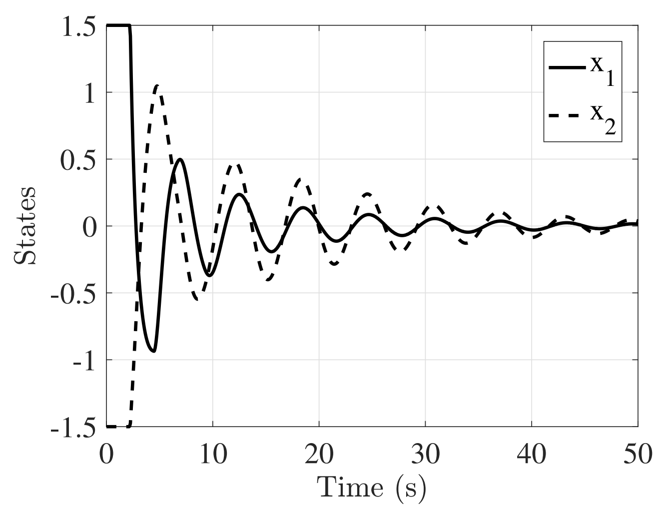

Fig. 1. Time response of the states for nonlinear system (17) with $\tau = 2.2350s$.

origin. Simulations have been performed for initial functions $x(\theta) = \begin{bmatrix} 1.5 & -1.5 \end{bmatrix}^T$, $\theta \in [-2.2350, 0]$.

V. Conclusion

In this paper, more relaxed sufficient conditions in a LMI way have been presented such as the solution space of the problem of stability analysis for time-delay nonlinear systems modelled by an exact TS representation outperforms previous approaches. The LMI conditions have been obtained using a novel non-quadratic Lyapunov-Krasovskii functional altogether with an artificially extended system and the Finsler's lemma. The effectiveness of the proposal has been illustrated via a numerical example. In this work, the Jensen inequality has been applied; future work is on course to apply recent approaches with different integral inequalities such as the Wirtinger one.

Acknowledgment

This work has been supported by the Mexican Council of Science and Technology (CONACYT) through scholarship No. 769169 and the ECOS Nord Mexico-France Project 291309/M17M08. The authors gratefully acknowledge the support of these institutions.

References

[1] S.-I. Niculescu, E. I. Verriest, L. Dugard, and J.-M. Dion, "Stability and robust stability of time-delay systems: A guided tour," in *Stability and control of time-delay systems*. Springer, 1998, pp. 1–71.

[2] E. Fridman, *Introduction to time-delay systems: Analysis and control*. Springer, 2014.

[3] D. Yue, "Robust stabilization of uncertain systems with unknown input delay," *Automatica*, vol. 40, no. 2, pp. 331–336, 2004.

[4] N. Krasovskii, "On the application of the second method of Lyapunov for equations with time delays," *Prikl. Mat. Mekh*, vol. 20, no. 3, pp. 315–327, 1956.

[5] B. Razumikhin, "On the stability of systems with a delay," *Prikl. Mat. Mekh*, vol. 20, no. 1, pp. 500–512, 1956.

[6] V. L. Kharitonov and A. P. Zhabko, "Robust stability of time-delay systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 12, pp. 2388–2397, 1994.

[7] S. Angulo, R. Márquez, and M. Bernal, "Quasi-polynomial-based robust stability of time-delay systems can be less conservative than lyapunov–krasovskii approaches," *IEEE Transactions on Automatic Control*, vol. 65, no. 7, pp. 3164–3169, 2020.

[8] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[9] M. Bernal, D. A. Sala, Z. Lendek, and T. M. Guerra, *Analysis and Synthesis of Nonlinear Control Systems: A Convex Optimisation Approach*. Springer Nature, 2022, vol. 408.

[10] T. Taniguchi, K. Tanaka, and H. Wang, "Model construction, rule reduction and robust compensation for generalized form of Takagi-Sugeno fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 2, pp. 525–537, 2001.

[11] S. Boyd, L. Ghaoui, E. Feron, and V. Belakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, USA: SIAM: Studies In Applied Mathematics, 1994, vol. 15.

[12] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*. Natick, USA: MathWorks, 1995.

[13] B. Chen and X. Liu, "Delay-dependent robust H_∞ control for T-S fuzzy systems with time delay," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 4, pp. 544–556, 2005.

[14] H.-N. Wu and H.-X. Li, "New approach to delay-dependent stability analysis and stabilization for continuous-time fuzzy systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 3, pp. 482–493, 2007.

[15] C. Peng, D. Yue, T.-C. Yang, and E. G. Tian, "On delay-dependent approach for robust stability and stabilization of T-S fuzzy systems with constant delay and uncertainties," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 5, pp. 1143–1156, 2009.

[16] S.-H. Tsai, Y.-A. Chen, and J.-C. Lo, "A novel stabilization condition for a class of t–s fuzzy time-delay systems," *Neurocomputing*, vol. 175, pp. 223–232, 2016.

[17] K. Gu, "An integral inequality in the stability problem of time-delay systems," in *Decision and Control, 2000. Proceedings of the 39th IEEE Conference on*, vol. 3. IEEE, 2000, pp. 2805–2810.

[18] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, 2013.

[19] Z. Zhang, C. Lin, and B. Chen, "New stability and stabilization conditions for T-S fuzzy systems with time delay," *Fuzzy Sets and Systems*, vol. 263, pp. 82–91, 2015.

[20] C. Lin, Q.-G. Wang, T. H. Lee, and Y. He, "Fuzzy weighting-dependent approach to H∞ filter design for time-delay fuzzy systems," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2746–2751, 2007.

[21] F. O. Souza, L. A. Mozelli, and R. M. Palhares, "On stability and stabilization of T-S fuzzy time-delayed systems," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 6, pp. 1450–1455, 2009.

[22] O. Kwon, M. Park, J. H. Park, and S. Lee, "Stability and stabilization of TS fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals," *Information Sciences*, vol. 372, pp. 1–15, 2016.

[23] S. Angulo, D. Vazquez, R. Márquez, and M. Bernal, "Stability of Takagi-Sugeno fuzzy systems with time-delay: A non-quadratic functional approach," in *2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, July 2017, pp. 1–6.

[24] S. Angulo, D. Vázquez, R. Márquez, and M. Bernal, "Nonquadratic stabilization conditions for time-delay nonlinear systems in a Takagi-Sugeno form," in *2018 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, July 2018, pp. 1–7.

[25] S. Angulo, R. Márquez, and M. Bernal, "Relaxed conditions for stability and stabilization of takagi-sugeno fuzzy systems with time-delay via non-quadratic functional," in *2019 16th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)*. IEEE, 2019, pp. 1–6.

[26] F. Gouaisbaut and D. Peaucelle, "Delay-dependent robust stability of time delay systems," *IFAC Proceedings Volumes*, vol. 39, no. 9, pp. 453–458, 2006.

[27] K. Gu, J. Chen, and V. L. Kharitonov, *Stability of time-delay systems*. Springer, 2003.

[28] M. Oliveira and R. Skelton, "Stability tests for constrained linear systems," in *Perspectives in robust control*, ser. Lecture Notes in Control and Information Sciences. Berlin: Springer-Verlag, 2001, vol. 268, pp. 241–257.

[29] T. Guerra and M. Bernal, "Strategies to exploit non-quadratic local stability analysis," *International Journal of Fuzzy Systems*, vol. 14, no. 3, pp. 372–379, 2012.

[30] R. Marquez, T. Guerra, A. Kruszewski, and M. Bernal, "Improvements on non-PDC controller design for Takagi-Sugeno models," in *Proceedings of the 2013 IEEE International Conference on Fuzzy Systems*, Hyderbad, India, 2013, pp. 1–7.

[31] H.-B. Zeng, J. H. Park, J.-W. Xia, and S.-P. Xiao, "Improved delay-dependent stability criteria for t–s fuzzy systems with time-varying delay," *Applied Mathematics and Computation*, vol. 235, pp. 492–501, 2014.

[32] F. Yang, S. Guan, and D. Wang, "Quadratically convex combination approach to stability of T-S fuzzy systems with time-varying delay," *Journal of the Franklin Institute*, vol. 351, no. 7, pp. 3752–3765, 2014.