Model Reference Adaptive Control for an unmanned aerial vehicle with variable-mass payloads

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Abstract—This paper deals with a model reference adaptive based control for an unmanned aircraft under payload variations. This situation is motivated by some tasks performed by autonomous aircrafts in precision agriculture, for example seed dispersal or spraying pesticides, where the vehicle payload is released during flight. This payload variation involves a change of the vehicle mass as well as the inertia moments. Thus, it is necessary to apply a feedback controller to counteract this phenomena while a hovering flight is performed. In this study, a backstepping-like adaptive control approach based on model reference adaptive procedure is designed. Finally, some numerical simulations to test the performance of the designed controller using MATLAB Simulink are developed.

Index Terms—payload variation, unmanned aerial vehicle, adaptive backstepping control, hovering, numerical simulation

I. INTRODUCTION

Quadrotors are very popular unmanned aerial vehicles (UAVs) due their multiples advantages like vertical take-off and landing (VTOL) [1], lower manufacturing and operating costs, greater efficiency and portability (they can be easily transported), also good agility and maneuverability. Moreover, as they do not need a human pilot on board, they can be used in dangerous places or environments [2], [3]. This features combined with the technological progress in the development of micro controllers, electronic devices, and control laws, help to explain why the quadrotors are more and more used, even for demanding tasks like complex trajectory tracking [4]. However, designing a high-performance controller for a quadrotor that operates in the presence of parametric uncertainties stills as an interesting challenge [5]. Actually, UAVs are used in real situations where mass parameter variation occurs, for example in tasks related to transportation of objects where payload release and drop is required. This applications are in the frame of construction tasks, delivery services, military operations, search and rescue missions, and others [6]–[8]. On the other hand, precision agriculture represents another important and becoming demand application that requires transport and payload release in different tasks, like irrigation, fumigation or sowing [9]. In [10] a quadrotor drone applied in the rice seed sowing process of the wet seeded rice farming is addressed.

Due to the multiple applications and scenarios where the payload transportation and release tasks are involved, it currently represents a promising and a great interest area [11]. However, it is complicated to guarantee a good performance in an autonomous flight process where payload variation occurs. For instance, payload motion generates changes on the aircraft’s aerodynamic like the gravity center and moments of inertia [12]; also when it is necessary lift or release some payload, the aircraft’s mass becomes an uncertain parameter, this issue generates a degradation of flight performance if the current controller is not able to compensate this parameter uncertainty [13], [14].

In order to provide the aircraft systems with the capacity to perform in pick up/drop payload missions without degradation, a reliable control algorithms should be developed, like non-linear control schemes including adaptive robust controllers and sliding mode controllers [15]. In fact, adaptive control represents an interesting option to control systems with the particularity of their dynamics models present uncertain parameters or that can change their values in time without a predictable pattern. Generally, adaptive control can be seen as a combination of an online parameter estimator mechanism with a control law that allows the system output behaves as a desired reference signal even with the presence of uncertain parameters [16]. Nowadays, this assignment has been dealt in several research, for example, in [17] an altitude adaptive robust control for a quadrotor carrying an unknown payload is proposed.

An interesting approach to design adaptive controllers is the Model Reference Adaptive Control (MRAC). This control scheme involves a plant with uncertain parameters, a model reference that specifies the desired output, an adaptation law to estimate the parameters and a control law that use these estimated parameters [18]. The purpose of the controller is to make the plant output behave according to that of the reference model, as plant parameters are not precisely known, the adaptation rule works to estimate them and the controller works to achieve an acceptable convergence to desired output tracking. In [19], in order to stabilize a quadrotor in hovering flight, an adaptive robust control scheme was designed, however its adaptive control scheme does not guarantee the exact estimation of UAV parameters.

The aim of this work is the design of a model reference adaptive control in order to control the position of a quadrotor under payload variation during hover. As a consequence, mass
and moments of inertia are considered both as uncertain parameters. The reference model is proposed considering the whole dynamics of the vehicle and the control law is obtained by the Lyapunov formalism. The proposed controller can be seen as an adaptive backstepping control since it applies a recursive procedure from a Lyapunov function to obtain both a feedback control to stabilize the system and an adaptation rule that adjust the control parameters on-line. Therefore, to test the adaptive control performance, some numerical simulations on MATLAB Simulink were carried out.

It is important to mention that parametric variation problems due to changes in the payload drone during the flight have been dealt in different ways in the literature. For example, in [20] a robust adaptive control scheme as a solution for mass changes was presented. This control scheme involves a sliding mode observer in combination with a least-squares method (LSM) to estimate the system parameters. Then for the identified system a feedback controller via pole placement is proposed. On the other hand, the structure proposed in our work, we addressed the same problem by using a simple MRAC scheme without the use of an observer structure in the closed loop system. It uses only an adaptation rule based on MRAC to estimate the plant parameters in order to design a feedback control law to command the whole state vector to stabilize the Quadrotor position. On the other hand, the structure proposed in our work in comparison with the study presented in [20], represents an advantage for future real-time implementations. This is due to it only needs a control law based on a MRAC algorithm to be programmed, it represents a less demanding algorithm in the sense of computational cost in comparison with the complex structure presented in [20].

Paper outline is as follows. Section 2 presents the dynamic model of quadrotor. Section 3 shows the design procedure of the model reference adaptive control. Section 4 gives the results of numerical simulations. Finally, conclusions are located in Section 5.

II. DYNAMIC MODEL

In this section, the quadrotor model used for the design of our control law is addressed.

In Fig. 1, a quadrotor rigid body located in a three-dimensional space is depicted. The generalized coordinates \( \xi = (x, y, z) \in \mathbb{R}^3 \) that denotes the position relative to the inertial fixed frame \( I \) in East-North-Up coordinates, and \( \eta = (\psi, \theta, \phi) \in \mathbb{R}^3 \) represents vehicle’s orientation expressed by \( \{\psi, \theta, \phi\} \) [21]. In addition, quadrotor is subject to the control inputs corresponding to the main thrust \( u = f_1 + f_2 + f_3 + f_4 \), where \( f_i \) is the individual thrusts of each motor, and the roll, pitch and yaw moments, i.e., \( \tau_\phi = f_2 - f_4 \), \( \tau_\theta = f_1 - f_3 \) and \( \tau_\psi = \tau_{M1} + \tau_{M2} + \tau_{M3} + \tau_{M4} \) where \( \tau_{M1} \) is the reaction torque of each motor [22].

In order to obtain the dynamic model of quadrotor by the Euler-Lagrange method is applied. Then, as described in [23], it is possible to obtain a simplified quadrotor defined by the following set of dynamical equations:

\[
\begin{align*}
\dot{x} &= \frac{1}{m} (\cos \phi \sin \theta) u \\
\dot{y} &= \frac{1}{m} (-\sin \phi) u \\
\dot{z} &= \frac{1}{m} (\cos \theta \cos \phi) u - g \\
\dot{\phi} &= \frac{1}{I_{xx}} \tau_\phi \\
\dot{\theta} &= \frac{1}{I_{yy}} \tau_\theta \\
\dot{\psi} &= \frac{1}{I_{zz}} \tau_\psi
\end{align*}
\]

where, \( m \) is the vehicle’s mass that changes when payload is released, \( I_{xx}, I_{yy}, I_{zz} \) are the moments of inertia, which are also variable parameters, and \( g \) is the gravitational acceleration.

III. ADAPTIVE BACKSTEPPING CONTROL

In this section, the proposed adaptive control law based on the system (1) is developed.

Considering that a parametric variation can appear in problems related to payload transportation or other applications, a MRAC for the quadrotor is designed. First, a feedback control law \( u \) is proposed without taking the mass parameter as follows

\[
u = \frac{v}{\cos \theta \cos \phi}
\]

then, substituting (2) in (1), yields

\[
\begin{align*}
\dot{m} \ddot{x} &= \tan \theta v \\
\dot{m} \ddot{y} &= -\frac{\tan \phi}{\cos \theta} v \\
\dot{m} \ddot{z} &= v - mg \\
\end{align*}
\]

The equations in (3) can be rewritten into the following matrix structure:

\[
P\ddot{Q} = U + PG
\]

where matrix \( P = \text{diag}(m, m, I_{xx}, I_{yy}, I_{zz}) \) contains the parameters, \( Q = [x \ y \ z \ \phi \ \theta \ \psi]^T \) the state of the system, \( G = [0 \ 0 \ -g \ 0 \ 0 \ 0]^T \) gravitational acceleration, and control inputs \( U \) are defined as

\[
U = \begin{bmatrix}
\tan \theta \dot{v} \\
-\frac{\tan \phi}{\cos \theta} \dot{v} \\
\dot{v} \\
\tau_\phi \\
\tau_\theta \\
\tau_\psi
\end{bmatrix}
\]
where \( u_x \) and \( u_y \) are virtual control inputs defined via backstepping.

The dynamics of the reference model is proposed as follows:

\[
\dot{Q}_m = K_b(\dot{Q}_d - \dot{Q}_m) + K_a(Q_d - Q_m)
\]

where \( K_a = \text{diag}(a_1, a_2, a_3, a_4, a_5, a_6) \), \( K_b = \text{diag}(b_1, b_2, b_3, b_4, b_5, b_6) \) and \( \in \mathbb{R}^{6 \times 6} \) are positive definite matrices. By the way, \( Q_d \) is a vector containing the desired positions, while \( Q_m \) vector contains the positions of the reference model, that is, \( Q_d = [x_d \ y_d \ z_d \ \phi_d \ \theta_d \ \psi_d]^T \) and \( Q_m = [x_m \ y_m \ z_m \ \phi_m \ \theta_m \ \psi_m]^T \). According to the adaptive control scheme there exist a tracking error between the real plant and the reference model

\[
e = Q - Q_m = [e_x \ e_y \ e_z \ e_{\phi} \ e_{\theta} \ e_{\psi}]^T
\]

As mentioned above, the parameters of the dynamic model that are considered not precisely known are the vehicle’s mass \( m \) and therefore the moments of inertia \( I_{xx}, I_{yy}, I_{zz} \). So, the vector containing the values of the uncertain parameters is defined as \( \gamma = [m \ I_{xx} \ I_{yy} \ I_{zz}]^T \); otherwise, the vector containing the corresponding estimated quantities is represented by \( \hat{\gamma} = [\hat{m} \ \hat{I}_{xx} \ \hat{I}_{yy} \ \hat{I}_{zz}]^T \). Thus, the estimation error is given by

\[
\dot{\hat{\gamma}} = \gamma - \hat{\gamma}
\]

In order to obtain the control law, consider the following candidate Lyapunov function

\[
V_1 = w_1 + w_2 + w_3
\]

where \( w_1 = \frac{1}{2} e^T e, \ w_2 = \frac{1}{2} (\dot{e} + K_a e)^T P (\dot{e} + K_a e) \) and \( w_3 = \frac{1}{2} R^T \hat{\gamma} \), while \( P, K_a \in \mathbb{R}^{6 \times 6} \) and \( R, \in \mathbb{R}^{4 \times 4} \) are a real positive-definite matrices. The time derivative of (9) along the trajectories of the system is given by

\[
\dot{V} = e^T \ddot{e} + (\dot{\dot{e}} + K_a e)^T P (\dot{e} + K_a e) - \hat{\gamma}^T R \dot{\hat{\gamma}}
\]

Then, adding and subtracting the term \( 2 w_1 \) on the right hand of (10), yields to

\[
\dot{V} = -e^T K_a e - (\dot{\dot{e}} + K_a e)^T (e + P \ddot{e} + PK_a \dot{e}) - \hat{\gamma}^T R \dot{\hat{\gamma}}
\]

Now, according to (4) and (7), the next relation can be obtained

\[
P(\ddot{Q}_m + K_a \dot{e} + G) = U + P(\ddot{Q}_m + K_a \dot{e} + G)
\]

Here it is possible to establish a parameterization connecting a regression matrix \( \phi \) with vector \( \gamma \) as follows

\[
\begin{bmatrix}
\varphi_{11} & 0 & 0 & 0 \\
\varphi_{21} & 0 & 0 & 0 \\
\varphi_{31} & 0 & 0 & 0 \\
\varphi_{42} & 0 & 0 & 0 \\
\varphi_{53} & 0 & 0 & 0 \\
\varphi_{64} & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
m \\
I_{xx} \\
I_{yy} \\
I_{zz}
\end{bmatrix}
\begin{bmatrix}
\gamma
\end{bmatrix}
\]

\[
(13)
\]

where

\[
\begin{align*}
\varphi_{11} & = a_1 e_x - \ddot{x}_m \\
\varphi_{21} & = a_2 e_y - \ddot{y}_m \\
\varphi_{31} & = a_3 e_z - \ddot{z}_m - g \\
\varphi_{42} & = a_4 e_{\phi} - \ddot{\phi}_m \\
\varphi_{53} & = a_5 e_{\theta} - \ddot{\theta}_m \\
\varphi_{64} & = a_6 e_{\psi} - \ddot{\psi}_m
\end{align*}
\]

In this way, considering (12) and (13), the derivative (11) is now expressed as

\[
\dot{V} = -e^T K_a e + (\dot{\dot{e}} + K_a e)^T (e + \varphi^T \gamma + U) - \hat{\gamma}^T R \dot{\hat{\gamma}}
\]

Considering (8), equation (15) is rewritten as

\[
\dot{V} = -e^T K_a e + (\dot{\dot{e}} + K_a e)^T e + \varphi^T \hat{\gamma} + \varphi^T \dot{\hat{\gamma}} + U
\]

If we choose the control input with the following form

\[
U = -e - \varphi^T \dot{\hat{\gamma}} - K_b (\dot{e} + K_a e)
\]

then (16), results in

\[
\dot{V} = -e^T K_a e - (\dot{\dot{e}} + K_a e)^T K_b (\dot{e} + K_a e) + \dot{\hat{\gamma}}^T (\varphi (\dot{e} + K_a e) - R \hat{\gamma})
\]

In order to guarantee that \( \dot{V} \leq 0 \), the adaption rule takes the following form

\[
\dot{\hat{\gamma}} = R^{-1} \varphi (\dot{e} + K_a e)
\]

and the derivative of (9) satisfies the following condition

\[
\dot{V} = -e^T K_a e - (\dot{\dot{e}} + K_a e)^T K_b (\dot{e} + K_a e) \leq 0
\]

Thus, the candidate function (9) is a Lyapunov function and according to the Lyapunov’s stability theorem, the functions \( e, \dot{e} \) are bounded. Finally, according to Barbalat’s Lemma (Corollary) [24], is ensured that \( e_1 \) and \( \dot{e}_1 \) goes to zero as \( t \to \infty \).

Since the quadrotor is an underactuated system, with only four control inputs \((u, \tau_{\psi}, \tau_{\theta}, \tau_{\phi})\) and six degrees of freedom to control; \( x \) and \( y \) positions cannot be directly controlled. Therefore, the control design must provide the virtual inputs for the desired angles \( \phi_d \) and \( \theta_d \). In this way, using (2) and (5), the following expressions are established:

\[
u = \frac{u_x}{\cos \phi \cos \theta}
\]

\[
\theta_d = \arctan \left( \frac{u_y}{u_z} \right)
\]

\[
\phi_d = \arctan \left( -\frac{u_y \cos \theta}{u_z} \right)
\]

Additionally, in order to guarantee the correct implementation of the control law (21), it is considered that the angles \( \phi, \theta \in (-\frac{\pi}{2} - \delta, \frac{\pi}{2} + \delta) \), for a small \( \delta > 0 \). The previous condition is a common assumption in related works [25], [26].
IV. RESULTS

In order to test the performance of the designed controller, numerical simulations were developed using MATLAB Simulink R2020b. The numerical method settings consider a Runge-Kutta with a fixed step size of 0.001 seconds and a simulation time of 60 seconds. The parameters of the dynamic system (1) are shown in Table I. Also, the initial conditions, \( \xi_i = [0.2 \ -0.12 \ 0] \) and \( \eta_i = [0 \ 0 \ 0.5] \), were considered. Additionally, Table II shows the control gains.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>1.25</td>
<td>kg</td>
</tr>
<tr>
<td>( g )</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
<tr>
<td>( I_{xx} )</td>
<td>0.01</td>
<td>kgm²</td>
</tr>
<tr>
<td>( I_{yy} )</td>
<td>0.011</td>
<td>kgm²</td>
</tr>
<tr>
<td>( I_{zz} )</td>
<td>0.0206</td>
<td>kgm²</td>
</tr>
</tbody>
</table>

The reference inputs were considered constants for \( x_d = 1.8 \) and \( y_d = 1.6 \), while desired altitude is set as a class of Heaviside signal as follows

\[
z_d = \begin{cases} 
1.8 & \text{if } t \leq 20 \\
3.5 & \text{if } 20 < t \leq 40 \\
2.4 & \text{if } t > 40
\end{cases}
\]  
(24)

Taking into consideration the problem of payload variation, which directly affects the value of the mass, it is proposed that the mass behaves according to the following function

\[
m = \begin{cases} 
2.2 & \text{if } t \leq 30 \\
1.5 & \text{if } 30 < t \leq 50 \\
1.25 & \text{if } t > 50
\end{cases}
\]  
(25)

Fig. 2 shows the results generated by the simulation for the translational positions. Besides there are some oscillations, specially for \( x \) and \( y \), it can be seen that controller have a good performance since the convergence time is relatively short (less of 7 seconds for all positions) and a satisfactory response for altitude changes is observed with no significant overshoot (seconds 20 and 40). Also, the control shows a good performance in order to compensate the payload variation (seconds 30 and 50) keeping the aircraft in the desired reference without notable alterations, this allows a properly flight process. Also, Fig. 3 shows an excellent performance of the controller for the vehicle orientation.

Adaptive control signals are shown in Fig. 4; it is easy to see how the main thrust \( u \) works to adjust the change on altitude reference and the variation on the mass parameter.

Finally, Fig. 5 shows the adjust of the parameters during the experiment, the estimated mass \( \hat{m} \) coincides with the real value of \( m \), in the same way, the moments of inertia matches for \( I_{zz} \) and \( I_{yy} \), strictly in the case of \( I_{xx} \), the estimation is slightly different of the true value, but still close to it.
V. CONCLUSION

In this work, a model reference adaptive control has been presented for an unmanned aerial vehicle under payload variations during hovering. The control law has been obtained proposing a reference model and using Lyapunov formalism considering the whole dynamics of an unmanned aerial vehicle. Numerical simulations showed a good performance for the designed controller.

The developed controller represents an interesting option for different applications where the transport and drop of cargo is required, as in agricultural tasks like irrigation, spraying pesticides, seeding, and other applications. This constitutes a low cost alternative due the capability to be implemented in various open source platforms, such as Pixhawk autopilot and even others electronic boards with 32-bits processors. Moreover, despite simulations in this paper only consider the problem of sudden payload drop during hovering, this control law also has the possibility to be applied for processes of gradual payload release, or involving more complex trajectory tracking.

REFERENCES


