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A New Positioning Algorithm Robust to Measured Distances Errors for Non-Overdetermined Systems.

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Abstract— **In the position estimation topic, it is well-known that a minimum of three reference stations or anchors are required to solve the positioning problem by classical methods in a 2D space, otherwise the obtained errors are large. Thus, more information provided by a greater number of anchors will pay off in a more accurate estimation. Nevertheless, there are situations where this cannot be always achieved. In this work an algorithm able to estimate the position of an unmanned aerial vehicle with only the three anchors required for position estimation is presented and evaluated in terms of accuracy in a multipath environment. The proposed algorithm combines two positioning methods found in the literature in order to reduce the errors generated by each one implemented separately.**

Keywords— Autonomous navigation, Gauss-Newton method, Positioning systems, radical axis, UAV, UWB.

I. INTRODUCTION

Positioning systems are widely used in navigation applications such as guiding unmanned aircrafts which are equipped with a signal emitter or a receiver device. The position can be estimated by triangulation (based on angle measurements) or trilateration (using distance measurements) of the signals sent from base stations (also called *anchors*) whose positions are known. These signals can be of different nature such as sound waves, visible and non-visible light or radio waves [1]. For example, the GPS (Global Positioning System) is a trilateration system which uses radio waves and determines the position of a point in a three-dimensional space of the Earth from reference signals taken from at least 4 satellites [2].

In Wireless Sensor Networks (WSNs) applications the position estimation problem has also been addressed by different authors (e.g. [3], [4]) and so some approaches can be taken for aerial vehicles using nodes with limited computational and power resources. In these networks where the position of the sensor nodes is unknown, it is possible to calculate their position using localization algorithms. These algorithms can be divided into two groups: range-based and range-free localization algorithms. The range-based localization algorithms use the distance between sensor nodes which is estimated using physical properties of communication signals, e.g., Received Signal Strength Indicator (RSSI), Time of Arrival (ToA), Time Difference of Arrival (TDoA) and Angle of Arrival (AoA) [5], [6]. On the other hand, the range-free localization algorithms estimate the coordinates of the sensor nodes using connectivity information between sensors without ranging.

Now, there are also some strategies to estimate the position in indoor scenarios where multipath effects and non-line-of-sight (NLOS) conditions could be presented most of the time and reference signals coming from satellite navigation systems are lost. In these environments, ultra wideband (UWB) positioning systems using multiple anchors have demonstrated to estimate the position of a UWB tag with a high accuracy [7]. Another issue is the signal delay introduced by radio propagation through crowded multiple obstacles scenarios, which can lead to errors in ToA measurements. This problem has been also addressed by incorporating UWB technology in [8] where the measurement errors of distance based on the ToA parameter has shown to be around only 35-50 cm in scenarios with multiple obstacles.

All the above examples of UWB technology used for position estimation make use of overdetermined systems and 4 anchors or more. In contrast, for cases where there is just the three anchors required, the accuracy of classical positioning systems decreases to the point that this configuration has not been recommended for autonomous navigation. Nevertheless, we have found an alternative for positioning systems with non-overdetermined systems and using three anchors only. Then, we propose here a new algorithm based on UWB technology for position estimation of a UAV flying in outdoors. The essence of this algorithm lies in the Gauss-Newton method initialized with the radical axis scheme, both discussed and compared in [9] for overdetermined systems.

We show how our proposal allows improving the accuracy of estimation for cases where it is not possible to have many anchors and the errors in measured distances are large due to environments with multiple obstacles. The above is simulated in a two-dimensional (2D) scenario assuming that for a three-dimensional case, a UAV is able to determine its flight height through different sensors like accelerometers, barometer or Light Detection and Ranging (LIDAR) devices.

This work is organized as follows: Section II presents the proposed positioning method and the concepts of two positioning methods which are based on. In Section III the

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simulation settings are stated. In Section IV the results are presented and analyzed. Finally, the conclusions are discussed in Section V.

II. Proposed Positioning Algorithm

A. Trilateration Method: Principles of Classical Positioning for UWB Systems

UWB positioning systems usually use the well-known trilateration method to determine the position of an object or a mobile robot based on the measured distances between the object in question and anchors or references points, whose positions are known. This method is not only used in mobile robot positioning including UAVs, but also in kinematics, aeronautic, crystallography and computer graphics [10].

The trilateration method is able to estimate the position of a UAV in a 2D plane with a minimum of 3 anchors by solving a system of equations in the following form [11]

$$\begin{aligned}(x-x_1)^2+(y-y_1)^2&=d_1^2\\(x-x_2)^2+(y-y_2)^2&=d_2^2\\(x-x_3)^2+(y-y_3)^2&=d_3^2\end{aligned}\qquad(1)$$

where (x, y) is the position of the UAV to be estimated, (x_i, y_i) is the position of the i-th anchor and d_i the distances measured between the i-th anchor and the UAV with $i = 1,2,3$. Each equation in (1) represents a circle centered at (x_i, y_i) with a radius of d_i, in such way that the solution of the equations system is equivalent to the intersection of the three circles. If the distances d_1, d_2 and d_3 are precise, the position of the UAV could be determined by the intersection of the three circumferences. The need of having 3 equations is because the measured distances often have errors. Then, these errors produce two possible solutions, p or p' as is shown in Fig. 1a for a two equation system, whereas with a third equation it is possible to determine the true position as is illustrated in Fig. 1b.

On the other hand, if these errors are too large, the circles do not touch each other at all and there is no solution. There exist two different cases associated to these errors: i) when the distances between anchors are larger than the sum of the measured distances, then, circumferences do not touch each other as can be seen in the Fig. 2a; ii) when the distances between two anchors are smaller than the subtraction of the measured distances, in such a way that a circumference is inside the other one as is shown in the Fig. 2b. As can be expected, the larger the measured distance errors, the more likely these situations occur.

B. Basis of the Proposal

By inspecting the properties of different positioning techniques, we have devised a positioning algorithm robust to large errors of the measured distances and which make use of just three anchors. This is achieved by combining two positioning methods analyzed and compared in [9], the radical axis (RA) method and the Gauss-Newton (GN) method.

The radical axis method has the advantage over other methods of being able to generate a solution even if the circumferences do not touch each other. On the other hand, the iterative Gauss-Newton method has the best performance in terms of accuracy for overdetermined systems, although its performance is dependent on the initial values. Thus, the underlying idea of our proposed RA-GN algorithm is to take the solution generated by the radical axis method as a first approximation of the position to be introduced as initial value to the Gauss-Newton method. In this way, we take the advantages of both techniques; on the one hand, the radical axis method is able to obtain the point that have the equal power[1] respect to the three circles despite the errors introduced in the measured distances. This point provides a possible position where the errors respect to the three anchors are equivalent. On the other hand, the Gauss-Newton method is able to find the position where the errors of estimation are minimized. For more details of these methods, the interested reader can consult [9] and references therein.

C. Cost Function

Let us consider that the errors of the measured distances are given by

$$e_i=\sqrt{(\hat{x}-x_i)^2+(\hat{y}-y_i)^2}-\hat{d_i}\qquad(2)$$

where $(\hat{x}, \hat{y})$ is the estimated position and $\hat{d_i}$ are the measured distances between the i-th anchor and the UAV. When the Gauss-Newton method is used without redundant information with large errors in the measured distances, the existence and unicity of solution cannot be guaranteed, hence the initial estimation takes great importance. For example, if the initial

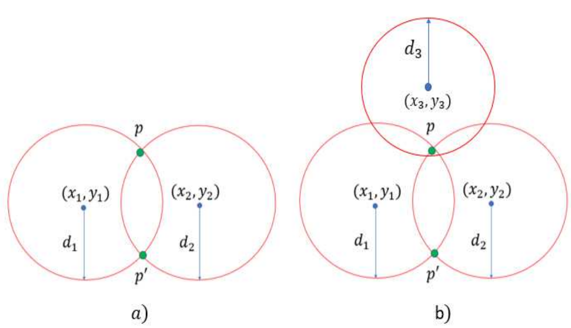

Fig. 1 Comparison of intersection points between circles a) ambiguity with two circles, b) solution by trilateration.

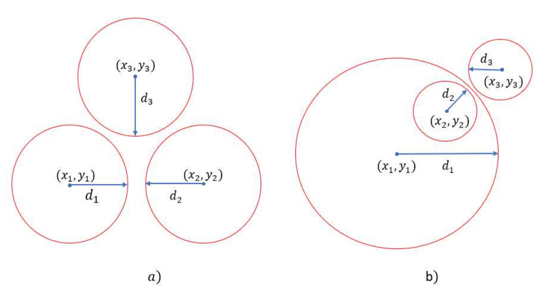

Fig. 2 Cases where there is no solution due to circles do not touch each other a) circles are in the exterior of each other b) one or two circles are inside the other one.

[1]In geometry the power of a point P with respect to a circle with center O and radius R is defined by $OP^2 - R^2$ [12].

position (0,0) is used as it is widely recommended in the literature (see [13] for instance), there exists a possibility that the estimation converges to an ambiguous solution based by to the initial position regardless of it is or not the correct solution. Furthermore, in cases of no solution, it is probable that the estimation results in an undefined value. Here is where arises the central contribution of this work of making use of the radical axis method as a first approach to estimate the position respect to the anchors. Despite possible positioning imprecisions introduced by the RA method, this scheme provides a rough position estimation but good enough to initiate the Gauss-Newton method instead of taking any arbitrary initial position like (0,0).

The objective of the Gauss-Newton method is to find the position where the errors respect to the three anchors are minimum. The above is carried out by evaluating a cost function at each iteration until a certain error tolerance ϵ is reached. In this paper, we use as cost function the root mean squared error (RMSE) of the distances between the UAV respect to the three anchors given by the following equation:

$$F(p) = \sqrt{\frac{1}{3}\sum_{i=1}^{N=3} {e_i}^2} \qquad (3)$$

III. Simulation Settings

Let us now present the simulation settings used in a program developed in Matlab to assess the accuracy of the proposed RA-GN algorithm. In this simulation the traditional radical axis method, the Gauss-Newton method and the proposed RA-GN algorithm are compared. Three anchors and one UAV are deployed in random positions uniformly distributed within an area of 30×30 m as is shown in Fig. 3. It is worth mentioning that the aim of this work is to evaluate and compare the performance of each of these methods for a non-overdetermined system. Thus, despite the lack of information that is characterized in this type of systems, each position is estimated regardless of the UAV model and the previous points of the trajectory.

In order to simulate the effects of crowded multipath environments, we introduce a random variation on the measured distances d_i which are taken as the mean value of a Gaussian distributed random variable with a standard deviation of $\sigma = 0.5$ m which represents the error. This value is taken from [8] for multipath environments where the ToA is affected by the obstacles in the environment.

Provided that we are interested in comparing our RA-GN algorithm with the original Gauss- Newton and the radical axis methods as they perform independently, it is necessary to state the initial values for the Gauss-Newton method. Thus, for this method we use two values, the origin (0,0) and the center of the area (15,15). The error tolerance value used in the Gauss-Newton method is $\epsilon = 0.001$ m and a maximum number of 20 iterations when the tolerance is not reached.

IV. Results

This section presents the simulation results obtained after 100,000 runs. The obtained results were analyzed using the cumulative distribution function (CDF) both of the cost function expressed in (3) and the absolute errors respect to the actual position given by

$$e_r = \left|\sqrt{(x-\hat{x})^2 + (y-\hat{y})^2}\right| \qquad (4)$$

where (x,y) is the actual position of the UAV and $(\hat{x},\hat{y})$ the estimated position. In the set of plots shown below and discussed through the text, RA represents the radical axis method, GN0 and GN15 correspond to the Gauss-Newton method using as initial value (0,0) and (15,15), respectively, and RA-GN is our proposal.

Let us first present and analyzed the results of the obtained CDF of the RMSE depicted in Fig. 4. As can been see, the RA-GN algorithm has a probability of 99.7% that the RMSE is less than 1 m which is an improvement compared with the corresponding probabilities of 86.8%, 84.7% and 93% for the radial axis, and the Gauss-Newton method using as initial value (0,0) and (15,15), respectively. More specifically, the RA-GN algorithm has a probability of 90% that the RMSE is less than 10 cm, whereas the RA, GN0, and GN15 present, respectively, 150 cm, 220 cm, and 40 cm for the same value of RMSE. Let us now analyze the results shown in Fig. 5 for the CDF of the error generated respect to the actual position. It is possible to observe that the combination of the two methods (i.e. RA-GN) is able to reduce the error with the best precision resulting in a 96.03% of probability to obtain an error less than 1 m. In contrast, for the Gauss-Newton method initializing it with (0,0) and (15,15) the probability to obtain an error less than 1 m is

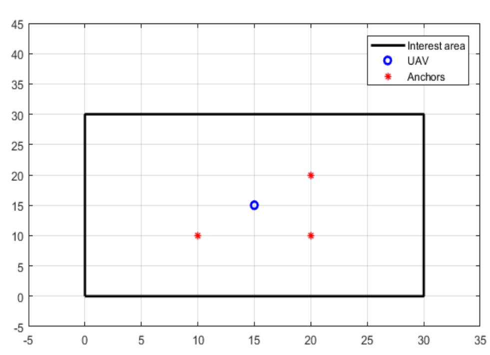

Fig. 3 Area of simulation where anchors and UAVs are deployed.

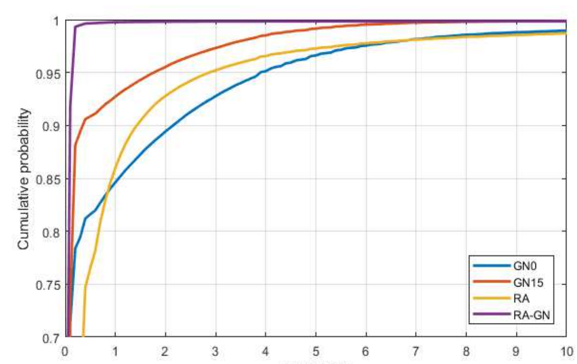

Fig. 4 Performance comparison of positioning methods in terms of CDF of the RMSE.

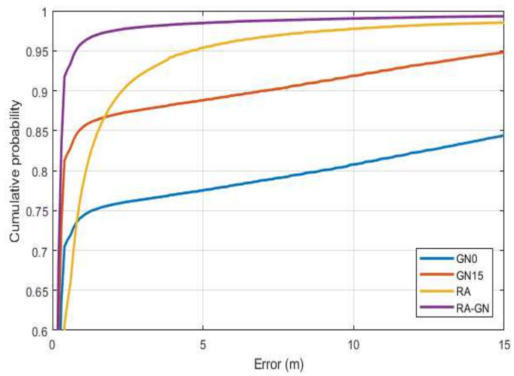

Fig. 5 Performance comparison of positioning methods in terms of CDF of the error.

74.28% and 85.41%, respectively. These results demonstrate the importance of the initial value in the Gauss-Newton method when there is not an overdetermined system. It is also evident that initializing from the center of the area improves the results more than 10% respect to one of the corners, but it is still far away from our RA-GN proposal. On the other hand, the radical axis method has a lower performance than the RA-GN and the GN15 methods with 78.04% of probability that the error is less than 1 m.

Now, the accuracy of the algorithm RA-GN has proven to have 90% of probability to obtain an error less than 40 cm which is a great improvement compared with the other methods analyzed, which are, for the same probability, 2.4 m for RA, 7.4 m for GN15, and 22.7 m for GN0.

From both error perspectives, it is worth pointing out that our results demonstrate the sensibility of the initial value used in the Gauss-Newton method and how it is a relevant factor for the reduction of the errors.

V. Conclusion

This work presents an algorithm able to estimate the position in the plane XY where overdetermined systems are not available. The central point of our algorithm lies in using the position estimation generated by the radical axis method as initial value for the Gauss-Newton method. The performance of the proposed algorithm demonstrates it can compensate the position errors by combining the characteristics of these two methods. Firstly, by making use of the radical axis concept it is possible to obtain an approach of the position robust enough to the measured distance errors without regard if the equations system which represents the problem has or not a unique solution, and subsequently, the errors are minimized using the Gauss-Newton method. According to our results, when there are three anchors and the errors of measured distances between each anchor and the UAV increase due to the obstacles present in the environment, the accuracy of the estimation using traditional methods is lower than the accuracy of our proposal which achieves up to 90% of probability to have an error less than 10 cm in a crowded area of 30 x 30 m. Finally, the results show the advantage of this algorithm respect to the performance of each method working separately.

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