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A Disturbance Observer Based Control scheme using an Active Disturbance Rejection Controller: An underactuated moving crane example.

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***Abstract*—This article proposes a Disturbance Observer Based Controller (DOBC) improved by the inclusion of an Active Disturbance Rejection Controller (ADRC), in classical transfer function form, as a feedback controller. Typically, the DOB controller uses a disturbance observer (DO) obtained by subtracting the low pass filtered plant inverse from the similarly filtered input signal. After direct disturbance cancellation, the feedback controller part of the DOB controller scheme is usually prescribed in an independently free manner. The use, in classical transfer function form, of a Reduced Order Extended State Observer based Active Disturbance Rejection Controller (ROESO-ADRC) as a feedback controller, in place of an unrelated controller, is shown to vastly improve the total disturbance attenuation in the closed loop system, thus enhancing the output reference trajectory tracking performance. The method is tested in the trajectory tracking control of an under-actuated, nonlinear, moving crane.**

***Index Terms*—ADRC, disturbance observer, under-actuated system**

I. INTRODUCTION

The presence of disturbances constitutes a problem that directly affects the performance and even the stability of controlled systems. Usually, it is not possible, or far too expensive, to measure the multiple disturbances affecting the plant, so as to include them in a control law and proceed to attenuate their adverse effects.

The approximate determination of the total disturbances, and their cancellation via feedback, or, alternatively, by direct means, has resulted in a variety of robust control schemes of varying efficiency and popularity. Two of the most recently cited robust control schemes are: the disturbance observer-based control (DOBC) and the active disturbance rejection control (ADRC). Their close relations have been somewhat controversial and competitive. Su and his collaborators, in [11], pointed out that a DO is equivalent to an extended state observer. In Sira-Ramírez *et al.* [6], it is shown that the ROESO-ADRC scheme, in classical transfer function form, is completely equivalent to the DOBC, using, as a feedback controller, an estimated state feedback controller. The redundancy involved in a disturbance observer based controller in combination with a ROESO-ADRC, results in a significantly improved closed loop attenuation of the input disturbance estimation error effects, as compared to those obtained via the DOBC, or the ADRC, schemes by themselves.

Using a frequency domain analysis, this paper establishes the robustness features of the ROESO-ADRC scheme in combination with the DO already encripted in the ROESO-ADRC. The method is addressed, from now on, as: DOB+ADRC scheme (instead of, ROESO-ADRC-DOB). The performance of the proposed scheme is tested and analyzed in the context of the output reference trajectory tracking problem, defined on an underactuated, non-feedback linearizable (i.e., non-flat), system.

Section II presents the proposed DOB+ADRC control design for a generic fourth-other pure integration perturbed plant. The robustness features of the DOB+ADRC scheme are analyzed from the frequency domain viewpoint. Section III presents the model of an under-actuated smooth moving crane and the design of the proposed controller to perform a rest-to-rest output reference trajectory tracking maneuver. The performances of the ADRC and the proposed DOB+ADRC are simulated and compared. Finally, section IV presents conclusions of this paper and possible extensions of this work.

II. DISTURBANCE ESTIMATION BASED CONTROL SCHEMES: ROESO-ADRC AND DOBC

The ROESO-ADRC is a robust control scheme that allows, through a reduced order extended state observer, the simultaneous on-line estimation the of system's unmeasured states and of the input total disturbance. The total disturbance includes endogenous as well as exogenous disturbances. The feedback control scheme consists of an estimated state feedback controller with a disturbance cancellation effort incorporated into the control law [3] [4].

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On the other hand, the DOBC is a control strategy that, similarly to ADRC, provides the estimation of input-matched disturbances by means of a disturbance observer. Such an observer is obtained by subtracting the low pass filtered plant inverse signal from the similarly filtered input signal. After direct disturbance cancellation, the feedback controller part of the DOBC scheme is separately prescribed.

It turns out, as a result of an equivalence previously found between the two schemes under description (ROESO-ADRC and DOBC), that the ROESO-ADRC scheme already contains a DO, exactly of the same low pass filter form, as that usually prescribed in DOBC schemes (See [6]). Furthermore, the ROESO-ADRC scheme, automatically prescribes the controller part as an estimated state feedback controller (See [5]).

A. Reduced order extended state observer based ADRC

Consider a fourth order disturbed system in simplified form:

$$y^{(4)} = \beta u + \xi(t), \tag{1}$$

where y is a noise-free low-frequency output, β is a known scalar gain, u is the control input and ξ is an unknown but bounded, sufficiently smooth signal, which represents a total disturbance term containing all endogenous disturbances such as: unmodeled dynamics, ignored nonlinearities, plant parameter variations, etc., also containing all exogenous disturbances such as: sensing noise, environmental or other system influences, etc. For design and implementation purposes, only the variables u and y are supposed to be available. It is desired to perform a trajectory tracking maneuver, represented by a smooth trajectory y^*. A nominal control input signal u^* may be used based on the full system model, on the unperturbed version of the simplified system (1). The input and output tracking errors are, respectively, defined as follows: $e_u = u - u^*$, $e_y = y - y^*$. The error dynamics, in state space representation, is obtained as:

$$e_y^{(4)} = \beta e_u + \xi(t) \tag{2}$$

A reduced order observer is usually proposed on the basis of the knowledge of the unmeasured variable, $\dot{e}_y = e_{y_2}$ using the state space representation of (2). The scheme is useful for analysis purposes only, since, equivalently, the proposed reduced order observer is later cast in terms of the actual measured output tracking error $e_y = e_{y_1}$. The reduced order observer is given by:

$$\begin{aligned}
\dot{\hat{e}}_{y_2} &= \hat{e}_{y_3} + \lambda_3(e_{y_2} - \hat{e}_{y_2}) \\
\dot{\hat{e}}_{y_3} &= \hat{e}_{y_4} + \lambda_2(e_{y_2} - \hat{e}_{y_2}) \\
\dot{\hat{e}}_{y_4} &= \beta e_u + z + \lambda_1(e_{y_2} - \hat{e}_{y_2}) \\
\dot{\hat{z}} &= \lambda_0(e_{y_2} - \hat{e}_{y_2})
\end{aligned} \tag{3}$$

From (2) and (3), it is possible to write the estimation error dynamics $\tilde{e}_{y_i} = e_{y_i} - \hat{e}_{y_i}$ as follows,

$$\begin{aligned}
\dot{\tilde{e}}_{y_2} &= \tilde{e}_{y_3} - \lambda_3 \tilde{e}_{y_2} \\
\dot{\tilde{e}}_{y_3} &= \tilde{e}_{y_4} - \lambda_2 \tilde{e}_{y_2} \\
\dot{\tilde{e}}_{y_4} &= e_\xi - \lambda_1 \tilde{e}_{y_2} \\
\dot{\hat{e}}_\xi &= \dot{\xi} - \lambda_0 \tilde{e}_{y_2}
\end{aligned} \tag{4}$$

Where the input-output relationship between the estimation error of $\tilde{e}_{y_2}$ and ξ can be obtained like,

$$\tilde{e}_{y_2}^{(4)} + \lambda_3 \tilde{e}_{y_2}^{(3)} + \lambda_2 \tilde{e}_{y_2}^{(2)} + \lambda_1 \tilde{e}_{y_2}^{(1)} + \lambda_0 \tilde{e}_{y_2} = \dot{\xi} \tag{5}$$

It is possible to appropriately select the set of constant values λ_i such that the characteristic polynomial of the estimation error dynamics becomes Hurwitz, and, therefore, the behavior of the unperturbed system is guaranteed to be asymptotically exponentially stable. Using the Laplace transform of (5), one obtains the transfer functions between the state estimation errors and the disturbance signal as follows:

$$\begin{aligned}
\tilde{e}_{y_2}(s) &= \left[\frac{s}{s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0}\right]\xi(s) \\
\tilde{e}_{y_3}(s) &= \left[\frac{s(s + \lambda_3)}{s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0}\right]\xi(s) \\
\tilde{e}_{y_4}(s) &= \left[\frac{s(s^2 + \lambda_3 s + \lambda_2)}{s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0}\right]\xi(s) \\
\tilde{e}_\xi(s) &= \left[\frac{s(s^3 + \lambda_3 s^2 + \lambda_2 s + \lambda_1)}{s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0}\right]\xi(s)
\end{aligned} \tag{6}$$

The last relation constitutes the disturbance estimation error sensitivity transfer function while the first three are the state estimation errors sensitivity transfer functions.

A control law may be now proposed using the estimated, unmeasured, states as well as the disturbance estimate z,

$$e_u = \frac{1}{\beta}\left(-\gamma_3 \hat{e}_{y4} - \gamma_2 \hat{e}_{y3} - \gamma_1 \hat{e}_{y2} - \gamma_0 \hat{e}_y - z\right). \tag{7}$$

The closed-loop system obtained from (2) and (7), results in the following sensitivity transfer function,

$$e_y(s) = \left[\frac{s^4 + \kappa_7 s^3 + \kappa_6 s^2 + \kappa_5 s}{s^8 + \kappa_7 s^7 + \cdots + \kappa_2 s^2 + \kappa_1 s + \kappa_0}\right]\xi(s), \tag{8}$$

where,

$$\begin{aligned}
&s^8 + \kappa_{72} s^7 + \kappa_{62} s^6 + \cdots + \kappa_{22} s^2 + \kappa_{12} s + \kappa_{02} = \\
&(s^4 + \lambda_{32} s^3 + \cdots + \lambda_{02})(s^4 + \gamma_{32} s^3 + \cdots + \gamma_{02})
\end{aligned} \tag{9}$$

and $\lambda' s$ and $\gamma' s$ must be selected, respecting the separation principle, such that the closed loop system is characterized by a Hurwitz polynomial. The corresponding transfer function for the overall controller is given by,

$$C(s) = \frac{1}{\beta}\left[\frac{\kappa_4\, s^4 + \kappa_3\, s^3 + \kappa_2\, s^2 + \kappa_1\, s + \kappa_0}{s\ (s^3 + \kappa_7\, s^2 + \kappa_6\, s + \kappa_5)}\right] \tag{10}$$

The transfer function in (10) turns out to be the same controller transfer function found for the DOBC scheme using an estimated states feedback control law [6].

B. A DO with ROESO-ADRC design

An improvement to the classic form of the ROESO-ADRC scheme can be implemented, as proposed by Aguilar-Orduña in [7], adding a DO and using, as the controller, a ROESO-ADRC scheme, in transfer function representation. The scheme is shown in Figure 1.

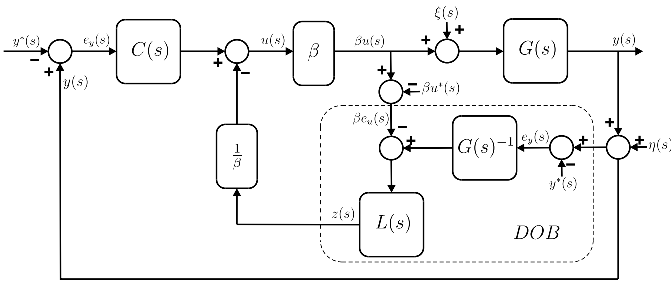

Fig. 1: ADRC-DOB Scheme.

For a fourth order system, in classical DO schemes, the estimation of the disturbances is proposed as:

$$z(s) = L(s)(G(s)^{-1}e_y - \beta e_u) \tag{11}$$

Where $G(s)^{-1} = s^4$ is the inverse of the nominal plant transfer function $G(s) = \frac{1}{s^4}$, and $L(s)$ is the disturbance estimation sensitivity function associated with the ROESO-ADRC scheme, which, for the fourth order case, is obtained as:

$$\begin{aligned} z(s) &= \xi(s) - e_\xi(s) \\ &= \left[\frac{\lambda_0}{s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0}\right]\xi(s) = L(s)\xi(s) \end{aligned} \tag{12}$$

The DO provides, via a suitable low pass filter (which, as already shown, may also be directly obtained from a ROESO-ADRC scheme) an accurate but imperfect estimation of the total disturbance term ξ affecting the plant. The injected plant, obtained by subtracting from the simplified perturbed plant the total disturbance estimate, conforms the basis to propose a feedback control law. An ADRC, designed for the injected plant, estimates and further attenuates the low frequency components of the remaining disturbance error as part of a combined control strategy.

To analyze the robustness features of the DOB+ADRC scheme, consider the fourth-order injected plant

$$e_y^{(4)} = \beta e_u + \xi(t) - z = \beta e_u + e_{\xi(t)} \tag{13}$$

The injected plant exhibits *as a new total disturbance*, the estimation error achieved by the DO scheme. If an ADRC scheme, with the same observer parameters used in the DO scheme, is now proposed on the injected plant, the same formulae, previously derived for the ROESO-ADRC scheme, directly apply.

The new estimate of, $e_\xi(:= \zeta)$, is denoted by $\hat{\zeta}$; the disturbance estimation error sensitivity function is obtained as, $e_\zeta(s)$. The DOB+ADRC disturbance estimate sensitivity function is given by,

$$\begin{aligned} \hat{\zeta}(s) = \hat{e}_\xi(s) &= \left[\frac{\lambda_0}{s^4 + \lambda_3 s^3 + \cdots + \lambda_0}\right]\zeta(s) \\ &= L(s)(1 - L(s))\xi(s) \end{aligned} \tag{14}$$

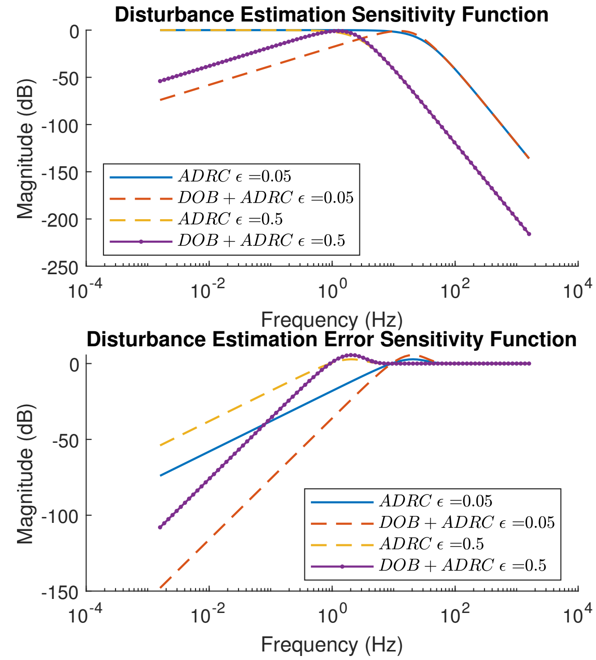

Fig. 2: Bode responses for representative transfer functions

while the disturbance estimation error sensitivity function is just,

$$e_\zeta = (1 - L(s))\zeta = (1 - L(s))^2\xi(s) \tag{15}$$

An enhanced attenuation, improved by a factor of two, of the total disturbance is obtained, as the following magnitude Bode graphs clearly demonstrate.

According to the foregoing, to compute the bode graphs, given a DO, equivalent to a ROESO [6], [7], a ROESO based ADRC is designed, then expressions (14) and (15) are valid. The designed parameters of the DO and the ROESO, in the ADRC scheme, are selected according to the following Hurwitz polynomial

$$p(s) = \left(s^2 + 2\zeta\frac{\omega_n}{\epsilon}s + \frac{\omega_n}{\epsilon}^2\right)^2; \quad 0 < \epsilon < 1 \tag{16}$$

while for the controller part the design parameters obey

$$p(s) = \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)^2 \tag{17}$$

From the graphs in Fig. 2, it follows that the use of a ROESO-ADRC, in classical transfer function form, as the controller in the DOBC scheme, generally improves the closed loop response. This is depicted in the bode graphs of the disturbance estimation sensitivity function and the disturbance estimation error sensitivity function.

It is clear that, given a specific value for ϵ, the magnitude bode plot for the disturbance estimation sensitivity function in equation (14) shows an accurate estimation of the remaining estimation error, obtained from the DO scheme, for all frequencies. The result is a more precise estimation of the total disturbance term in the overall scheme.

Moreover, from the disturbance estimation error sensitivity function, the attenuation of the influence of the total disturbance term is twice the one obtained from a single ROESO-ADRC scheme or from an equivalent DO scheme. In summary, these graphs show a relevant improvement in the estimate of the total disturbance term, and consequently, the classical ROESO-ADRC, controller exhibits an improvement in its performance. These results are verified in the following section by simulations on a case study.

III. An underactuated moving crane with flexibility example

Consider the under-actuated flexible joint mobile manipulator system shown in Figure 3. The system consists of a moving crane with a two-link arm joined by a torsional spring of constant k. The mechanism is controlled by two input signals τ and F, where each one of these inputs is associated to the power delivered by a DC motor. The generalized position variables of the system are x, the car horizontal position; θ_1, the angle of the first link measured from the vertical; and θ_2 the angular position of the second link, also measured from the vertical.

We propose as control objective the positioning of the mechanism end effector from an initial position $(\bar{X}_{init}, \bar{Y}_{init})$ to a final desired equilibrium $(\bar{X}_{final}, \bar{Y}_{final})$. The system dynamics is governed by the smooth lagrangian given by [8]:

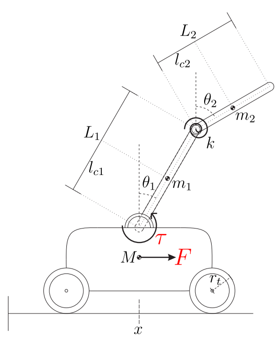

Fig. 3: The moving crane with flexibility.

$$\mathcal{L} = \frac{1}{2}\dot{q}^T \mathcal{M}(q)\dot{q} - (\mathcal{V}(q) - q^T Bu) \tag{18}$$

Where,

$$\mathcal{M}(q) = \begin{bmatrix} M + m_1 + m_2 & (L_1\, m_2 + l_{c1}\, m_1)\cos(\theta_1) & l_{c2}\, m_2 \cos(\theta_2) \\ (L_1\, m_2 + l_{c1}\, m_1)\cos(\theta_1) & m_2\, {L_1}^2 + m_1\, {l_{c1}}^2 + I_1 & L_1\, l_{c2}\, m_2 \cos(\theta_1 - \theta_2) \\ l_{c2}\, m_2 \cos(\theta_2) & L_1\, l_{c2}\, m_2 \cos(\theta_1 - \theta_2) & m_2\, {l_{c2}}^2 + I_2 \end{bmatrix} \tag{19}$$

Meanwhile,

$$\mathcal{V}(q) = m_1 g L_{c1}\cos\theta_1 + m_2 g\,(L_1\cos\theta_1 + L_{c2}\cos\theta_2) + \frac{1}{2}k(\theta_2 - \theta_1)^2 \tag{20}$$

However, this system is not feedback linearizable in exact form, thus is not flat [9] [10]. An alternative to control the system is to use a tangent second order linearization. The set of differential equations for the incremental linearized system $x_\delta = x - \bar{x}$, $\theta_{1\delta} = \theta_1 - \bar{\theta_1}$, $\theta_{2\delta} = \theta_2 - \bar{\theta_2}$, $F_\delta = F - \bar{F}$, $\tau_\delta = \tau - \bar{\tau}$ near the equilibrium point $(\bar{x}, \bar{\theta_1}, \bar{\theta_2}, \bar{F}, \bar{\tau}) = (0, 0, 0, 0, 0)$ is given by [8]:

$$\begin{bmatrix} \ddot{x}_\delta \\ \ddot{\theta}_{1_\delta} \\ \ddot{\theta}_{2_\delta} \end{bmatrix} = -\mathcal{M}^{-1}(\bar{q})\mathcal{K}(\bar{q}) \begin{bmatrix} x_\delta \\ \theta_{1_\delta} \\ \theta_{2_\delta} \end{bmatrix} + \mathcal{M}^{-1}(\bar{q}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_\delta \\ \tau_\delta \end{bmatrix} \tag{21}$$

Where,

$$\mathcal{M}(\bar{q}) = \begin{bmatrix} M + m_1 + m_2 & L_1\, m_2 + l_{c1}\, m_1 & l_{c2}\, m_2 \\ L_1\, m_2 + l_{c1}\, m_1 & m_2 + m_1\, {l_{c1}}^2 + I_1 & L_1\, l_{c2}\, m_2 \\ l_{c2}\, m_2 & L_1\, l_{c2}\, m_2 & m_2\, {l_{c2}}^2 + I_2 \end{bmatrix} \tag{22}$$

$$\mathcal{K}(\bar{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & m_1 g L_{c1} + m2 g L_1 + k & -k \\ 0 & -k & m_2 g L_{c2} + k \end{bmatrix} \tag{23}$$

Now, given the flat outputs for the linearized system:

$$\begin{aligned} y_1 &= (M + m_1 + m_2)x_\delta + (m_1 L_{c1} + m_2 L_1)\theta_{1\delta} + m_2 L_{c2}\theta_{2\delta} \\ y_2 &= m_2 L_{c2} x_\delta + m_2 L_1 L_{c2}\theta_{1\delta} + (m_2 L_{c2}^2 + I_2)\theta_{2\delta} \end{aligned} \tag{24}$$

it is possible, using the methodology described in [3], to write the system in Brunovsky's canonical form given by,

$$\begin{aligned} y_1^{(2)} &= F_\delta; \\ y_2^{(4)} &= \beta_{22}\tau_\delta + \xi_2 \end{aligned} \tag{25}$$

Where the variable term ξ_2 contains the ignored linearized system's dynamic, all not-modeled dynamics and exogenous disturbances. Additionally, includes all F_δ terms in order to use the methodology presented for SISO systems in sections II-A and II-B, finally obtaining a decoupled plant.

Now, considering the dynamic equations for a *i-th* DC motor,

$$\begin{aligned} L_{mi}\frac{d}{dt}i_a &= -R_i i_a - k_{mi}\dot{\theta}_m + E_i \\ J_i\ddot{\theta}_m &= -B_i\dot{\theta}_m + k_{mi} i_a - \tau_{mi} \end{aligned} \tag{26}$$

If the inductance value in (26) is assumed $L \approx 0$ due to the difference between mechanic and electrical time constants in the model, the motor dynamics is reduced to a single differential equation,

$$J_i\ddot{\theta}_m = -B_i\dot{\theta}_m - \frac{k_{mi}}{R_i}\left(k_{mi}\dot{\theta}_m - E_i\right) - \tau_{mi} \tag{27}$$

Substituting (27) into the dynamics given by (25), and considering $\tau_{m1} = \tau_\delta$, $\tau_{m2} = F_\delta r_t$, a full system simplified reduced dynamics is obtained described by,

$$\begin{aligned} y_1^{(2)} &= \frac{k_{m2}}{R_2 r_t}E_2 + \xi_1(t) = \beta_{11}^* E_2 + \xi_1(t) \\ y_2^{(4)} &= \beta_{22}\frac{k_{m1}}{R_1}E_1 + \xi_2(t) = \beta_{22}^* E_1 + \xi_2(t) \end{aligned} \tag{28}$$

TABLE I: Simulation parameters.

Parameter	Value	Parameter	Value
M	400$[Kg]$	L_{mi}	0.658 $[mH]$
m_1	100$[Kg]$	k_{mi}	3.61 $[N \cdot m/A]$
m_2	50$[Kg]$	B_{mi}	0.1$[N \cdot m \cdot s]$
l_{c1}	0.75$[m]$	J_{mi}	0.01$[Kg \cdot m^2]$
L_1	2$[m]$	R_{mi}	1.76$[\Omega]$
l_{c2}	0.75$[m]$	L_2	1.5$[m]$
k	800$[N \cdot m/rad]$	g	9.85$[m/s^2]$
I_1	0.5$[Kg \cdot m^2]$	I_2	0.3$[Kg \cdot m^2]$
r_t	0.25$[m]$	—	—

Where E_1 is the voltage of the motor that actuates the first link, and E_2 is the motor's voltage that actuates the wheel of the crane.

For this pair of SISO subsystems is possible to design a ROESO-ADRC controller such that their transfer functions are found to be,

$$\begin{bmatrix} \beta_{11}^* & 0 \\ 0 & \beta_{22}^* \end{bmatrix} \begin{bmatrix} E_2 \\ E_1 \end{bmatrix} = \begin{bmatrix} y_{1\delta}^{(2)*} - \dfrac{\kappa_{21}\, s^2 + \kappa_{11}\, s + \kappa_{01}}{s\ (\kappa_{31} + s)}(y_{1\delta} - y_{1\delta}^*) \\ y_{2\delta}^{(4)*} - \dfrac{\kappa_{42}\, s^4 + \kappa_{32}\, s^3 + \kappa_{22}\, s^2 + \kappa_{12}\, s + \kappa_{02}}{s\ (s^3 + \kappa_{72}\, s^2 + \kappa_{62}\, s + \kappa_{52})}(y_{2\delta} - y_{2\delta}^*) \end{bmatrix} \tag{29}$$

where the corresponding characteristic polynomials are given by,

$$\begin{aligned} &s^4 + \kappa_3 s^3 + \kappa_2 s^2 + \kappa_1 s + \kappa_0 = \\ &(s^2 + \lambda_{11} s + \lambda_{01})(s^2 + \gamma_{11} s + \gamma_{01}) \end{aligned} \tag{30}$$

$$\begin{aligned} &s^8 + \kappa_{72} s^7 + \kappa_{62} s^6 + \cdots + \kappa_{22} s^2 + \kappa_{12} s + \kappa_{02} = \\ &(s^4 + \lambda_{32} s^3 + \cdots + \lambda_{02})(s^4 + \gamma_{32} s^3 + \cdots + \gamma_{02}) \end{aligned} \tag{31}$$

And each Hurwitz polynomial design is given by the appropriately selection of ω_{ni}, ζ_i, ϵ_i values such that,

$$\begin{aligned} (s^2 + \lambda_{11} s + \lambda_{01}) &= (s^2 + 2\zeta_1 \omega_{n1} + \omega_{n1}^2) \\ (s^2 + \gamma_{11} s + \gamma_{01}) &= \left(s^2 + 2\zeta_1 \frac{\omega_{n1}}{\epsilon_1} + \frac{\omega_{n1}^2}{\epsilon_1^2}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} (s^4 + \lambda_{32} s^3 + \cdots + \lambda_{02}) &= (s^2 + 2\zeta_2 \omega_{n2} + \omega_{n2}^2)^2 \\ (s^4 + \gamma_{32} s^3 + \cdots + \gamma_{02}) &= \left(s^2 + 2\zeta_2 \frac{\omega_{n2}}{\epsilon_2} + \frac{\omega_{n2}^2}{\epsilon_2^2}\right)^2 \end{aligned} \tag{33}$$

Finally, a comparison between ROESO-ADRC and DOB+ADRC schemes is desired. After implementation of both schemes on the moving crane system, an Integral Square Index (ISE) is going to be used, being this a criteria inversely proportional to the precision and accuracy of the control strategies. This performance index is defined as:

$$ISE_{y_i} = \int_0^t (y_{i\delta}(\tau) - y_{i\delta}^*(\tau))^2 d\tau \tag{34}$$

A. Simulation Results

We consider a moving crane with the parameters given in Table I. A control task is adopted entitling the position control of the end effector. We start from an equilibrium point: $(\bar{X}_{init}, \bar{Y}_{init}) = (0, L_1 + L_2 = 3.5)$, with corresponding generalized positions: $(\bar{x}, \bar{\theta}_1, \bar{\theta}_2) = (0, 0, 0)$, to a desired final desired end effector position: $(\bar{X}_{final}, \bar{Y}_{final}) = (2.5, 2.5)$, corresponding to the positions $(x, \theta_1, \theta_2) = (0.139, 0.593, 0.974)$. This control task can be achieved performing trajectory an output reference tracking for the flat outputs $y_{1\delta}^*$ and $y_{2\delta}^*$, using suitable Bèzier polynomials.

Figure 4 depict a smooth trajectory tracking task in the presence of one exogenous disturbance on each control channel, given by the functions $\xi_{ex1} = 10e^{-\sin(0.2t)^2}\sin(4t)\cos(t) + 5_{t \geq 5s}$ and $\xi_{ex2} = e^{-\sin(0.2t)^2}\sin(4t)\cos(t) + 0.5_{t \geq 5s}$.

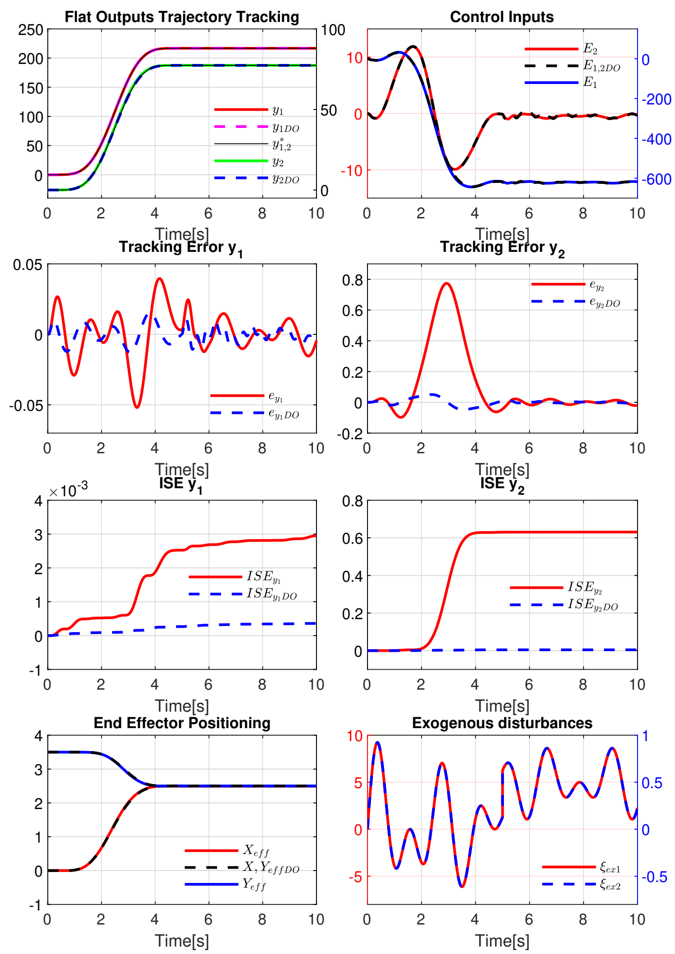

Fig. 4: Comparison between ROESO-ADRC and DOB+ADRC.

It is clear, according to Figure 4, that both control schemes achieve the desired task of output reference trajectory tracking, in the presence of exogenous disturbance signals. This is achieved in spite of the schemes being constituted by linear controllers, which are based on a simplified tangent linearization of the non-flat system. However, making a comparison between both trajectory tracking error responses, for each flat output, it is evident that a considerable improvement is achieved by using a DOB+ADRC scheme. This fact is clearer from both ISE criteria; showing that a disturbance observer acts as a complement on the second observer, i.e., the estimation obtained via ROESO-ADRC is the result of the difference between the actual disturbance and the DO estimation, and as a consequence, the resulting system exhibits a behavior which is closer to that of a pure integration system with respect to a ROESO-ADRC scheme.

B. Robustness on the parameters uncertainty

The performance of the proposed scheme is still dependent on the input gains. These, typically, depend on the internal plant parameters. This section is aimed to demonstrate, through an ISE criteria analysis, that the design used on the moving crane is robust with respect to unforseen changes in the gain parameters, β_i, $i = 1, 2$, appearing in equation (29).

We propose the following "sweeping" for the values of β_{11} and β_{22}, such that both gains experience the same changes at the same time, and for each new value of β_i, the ISE criteria for both outputs will be assessed and plotted. This is shown in Figure 5, in a way that the gain variations will be clear.

$$0.5\beta_i \leq \beta_i \leq 1.5\beta_i \tag{35}$$

According to Figure 5, ISE criteria for outputs $y_{1\delta}$ and $y_{2\delta}$

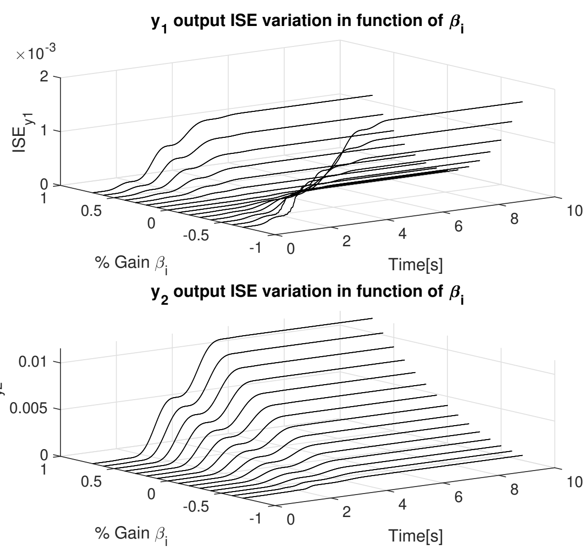

Fig. 5: Variation of ISE parameters in function of variations of β_i.

remain within the same orders of magnitude, exhibiting stable behaviors and a finite constant value for all tested values of the β_i's. Considering the results obtained for $y_{1\delta}$, the best performance occurs for the nominal value of the β_i, as shown in 5. The ISE criteria will increase for larger, or smaller, values of β_i, but the limit value of the index remains finite. On the other hand, for the flat output, $y_{2\delta}$, a better performance is obtained as β_i is smaller. This can be explained due to the magnitude values for the control input E_2 that, in addition to a high gain, might generate larger deviations at the moment in which the trajectory variations are considerable. For a lower gain, the control input behaves less aggressively on the system, allowing better tracking for higher rates of change on the trajectory.

Finally, it is clear that the DOB+ADRC control scheme exhibits robustness against considerably higher, simultaneous, variations on the control input gains and, as a consequence, it allows to propose controller designs that achieve the control tasks, without knowing with precision all expressions of the system in input-output form.

IV. Conclusions

Using a frequency domain analysis, based on Bode plots, we have shown that DOB+ADRC schemes exhibit a remarkable degree of robustness to input disturbances. This resulted in an improvement over the robustness obtained with a traditional DOB controller, or a ROESO-ADRC scheme, by themselves. Via computer simulations, we verified the performance of the proposed scheme, using a non-linear, MIMO, non-flat mechanical system. The design of the control scheme was based on the linearized, decoupled, and simplified, input-output, perturbed model of the system but, ultimately, the controller was applied to the nonlinear plant. An objective assessment of the effectiveness of the proposed controller, to plant parameter variations, was carried through the use of the ISE performance index, with excellent results. In future works, we plan to extend the proposed approach to nonlinear discrete time systems and to nonlinear, MIMO, exactly linearizable (i.e., flat) systems, via dynamic feedback.

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