

# Open-Ended Coaxial Probe Technique for the measurement of the ionic strength due to magnesium sulfate heptahydrate in water

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**Abstract**— The total dissolved solids (TDS) is a parameter considered for different agencies and organizations. The TDS is typically determined by the ionic strength (I). The magnesium sulfate heptahydrate is an electrolyte that produces  $\text{SO}_4^{2-}$  and  $\text{CaCO}_3$  in water and changes its ionic strength. The elimination of therapeutic agents, beverages and other domestic products not used at home, which contain magnesium sulfate heptahydrate, and the agricultural use of fertilizers are sources of magnesium sulfate heptahydrate in the environment. In this work, the complex relative permittivity of liquid samples with different ionic strength levels due to magnesium sulfate heptahydrate dissolved in water is presented. The study covers an ionic strength range from 0 M to 10.553 M. The measurements are performed with an open coaxial probe in a range from 500 MHz to 20 GHz and the Havriliak-Negami model parameters are presented for the different samples. The dielectric constant does not exhibit as high change as loss factor in relation to ionic strength. The loss factor at 500 MHz is used to determine the ionic strength due to magnesium sulfate heptahydrate. The fitting curve shows a second-grade polynomial whose ionic strength goes from 0.0008 M to 8.1179 M and the loss factor at 500 MHz goes from 2.4 to 213.82. Therefore, the open coaxial probe can be used for measuring a wide ionic strength range.

**Keywords**— *permittivity, ionic strength, sensor, uncertainty, microwave, sensitivity*

## I. INTRODUCTION

Water quality is considered as a measure of the physical, chemical, biological and microbiological characteristics of water. The importance of its analysis is exhibited in different cases as in the verification of a water sample quality according to standards, assessment of a system which is used to water treatment and so on [1].

The total dissolved solids (TDS) is a parameter considered for different agencies and organizations such as Environmental Protection Agency (EPA) from USA and World Health Organization (WHO) [2, 3]. The TDS is typically determined by the ionic strength [4, 5, 6]. In turn, the ionic strength (I) is determined in different ways. The

electrical conductivity meter is employed for measuring in real time up to 0.32 M of ionic strength in a solution [7]. A disadvantage of this meter is that the electrode polarization phenomenon occurs in the frequencies in which a common electrical conductivity meter works out [8]. This phenomenon causes the electrodes of the meter undergo high corrosion. Therefore, the manufacturer of the meter must use special covering on the electrodes [9]. The gravimetric method is used to determine higher levels of ionic strength than an electrical conductivity meter [3]. In this method, the liquid sample must be conveyed to the laboratory, for this reason, there is no real time measurement [10].

On the other hand, magnesium sulfate heptahydrate ( $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$ ) is a natural mineral compound of magnesium and sulfate that is available under several brands, such as Epsom salt [11]. It is an important component of therapeutic agents [12], beer [13], drinking water [14] and fertilizers [15]. The elimination of therapeutic agents, beer, drinking water and other domestic products not used at home, which contain magnesium sulfate heptahydrate, and the agricultural use of fertilizers are sources of magnesium sulfate heptahydrate in the environment. The magnesium sulfate heptahydrate produces the ions  $\text{SO}_4^{2-}$  and  $\text{Mg}^{2+}$  in the water. This last one is considered in terms of  $\text{CaCO}_3$ .  $\text{SO}_4^{2-}$  and  $\text{CaCO}_3$  are limited by the Mexican Norm NOM-127-SSA1-1994, EPA and WHO. High levels of  $\text{SO}_4^{2-}$  and  $\text{CaCO}_3$  are regulated because can cause damages to the skin, lungs, eyes and digestive system [16, 17, 18].

Regarding permittivity, the study of the permittivity of different types of substances in water has been carried out previously, such as the case of E. coli cells [19], clay [20], chlorine [21], calcium carbonate [22], among others. The use of permittivity for ionic strength determination due to magnesium sulfate heptahydrate has already been done [5], but the range of detection wasn't established. This characteristic is important since it allows to know what type of water (underground, drinking or sea water) can be analyzed by the Open-Ended Coaxial Probe Technique.

In this paper, the study of the complex relative permittivity of magnesium sulfate heptahydrate samples dissolved in water by means of an open coaxial probe is presented. The study covers concentrations of magnesium sulfate heptahydrate ranging from 0 mg/L to 650 000 mg/L, which corresponds an ionic strength range from 0 M to 10.5532 M. These concentrations were measured in the frequency range from 500 MHz to 20 GHz. For each sample, the Havriliak-Negami model parameters are presented in the measurement range. Changes in the dielectric constant and loss factor in relation to ionic strength and frequency range were studied. Also, the measurement uncertainty analysis is presented, which was used to determine the sensitivity of the coaxial probe. The relationship between ionic strength and loss factor was established. The objective of this work was to dielectrically characterize several liquid samples with different ionic strengths in order to determine that the open coaxial probe can be used for measuring a wide ionic strength range and provide information for designing other permittivity detectors in the microwave range that measure ionic strength higher than 0.32 M and in real time.

## II. DIELECTRIC PARAMETERS

The complex relative permittivity ( $\epsilon^*$ ) to that of free space can be expressed in terms of the dielectric constant ( $\epsilon'$ ) and loss factor ( $\epsilon''$ ) as given in (1).

$$\epsilon^* = \epsilon' - \epsilon'' \quad (1)$$

The dielectric constant is defined as the measure of the ability of a material to store electromagnetic energy [23]. The dielectric loss factor is characterized by the amount of electromagnetic energy converted into heat in a material [24]. The loss factor includes contributions due to dipole rotation and ionic conduction in the microwave range [25].

The most common model used to describe the electrical behavior of aqueous solutions or tissues is the Havriliak-Negami model, which is an empirical modification of the Debye relaxation model and represents the asymmetry and amplitude of the dielectric dispersion curve:

$$\epsilon^* = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{[1 + (j\omega\tau)^{1-\alpha}]^\beta} - \frac{j\sigma_{dc}}{\omega\epsilon_0} \quad (2)$$

where  $\epsilon_\infty$  and  $\epsilon_s$  are the dielectric constants under DC and at infinity frequency, respectively.  $\sigma_{dc}$  is the DC conductivity.  $\tau$  is the characteristic relaxation time of the medium, which is the time required for dipoles to become oriented in the presence of an electric field or the time needed to disorient the dipoles after the electric field is removed [26, 27].  $\alpha$  and  $\beta$  are empirical variables that account for the distribution of the relaxation time and the asymmetry of the relaxation time distribution, respectively. The relaxation frequency indicates the frequency in which a group of dipoles of material no longer follows to the electrical field and in which the loss factor reaches a maximum. This frequency occurs at  $\omega = 1/\tau$  [28, 29].

## III. METHODOLOGY

### A. Experimental matrix

Table 1 shows the composition of the samples of magnesium sulfate heptahydrate (2500-01, J. T. Baker, USA) in distilled-deionized water taken into account in this work with their respective denominations. It is important to mention that considering the endless amount of possible water pollutant

combinations, the currently work is an application-specific, where a solution with known composition is tested.

TABLE I CONCENTRATION OF  $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$  IN DEIONIZED-DISTILLED WATER

| Concentration (mg/L)                      |                |                    | I (M)  | Liquid sample ID |
|---|----------------|--------------------|--------|------------------|
| $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$ | $\text{CaO}_3$ | $\text{SO}_4^{2-}$ |        |                  |
| 0   | 0              | 0                  | 0      | WD1              |
| 37  | 7.3362         | 14.4271            | 0.0006 | WD2              |
| 50  | 9.9138         | 19.4961            | 0.0008 | WD3              |
| 100                                       | 19.8277        | 38.9922            | 0.0016 | WD4              |
| 150                                       | 29.7415        | 58.4882            | 0.0024 | WD5              |
| 200                                       | 39.6554        | 77.9843            | 0.0032 | WD6              |
| 250                                       | 49.5692        | 97.4804            | 0.0041 | WD7              |
| 513                                       | 101.7160       | 200.0298           | 0.0083 | WD8              |
| 750                                       | 148.7077       | 292.4412           | 0.0122 | WD9              |
| 1026                                      | 203.4896       | 400.1727           | 0.0167 | WD10             |
| 1250                                      | 247.8461       | 487.4021           | 0.0203 | WD11             |
| 1560                                      | 309.1989       | 608.0555           | 0.0253 | WD12             |
| 1750                                      | 346.9846       | 682.3629           | 0.0284 | WD13             |
| 2051                                      | 406.9772       | 800.3415           | 0.0333 | WD14             |
| 3079                                      | 610.4668       | 1200.5142          | 0.0500 | WD15             |
| 5000                                      | 991.3845       | 1949.6083          | 0.0812 | WD16             |
| 6158                                      | 1220.9316      | 2401.0245          | 0.1000 | WD17             |
| 7000                                      | 1387.9383      | 2729.4516          | 0.1137 | WD18             |
| 8000                                      | 1586.2152      | 3119.3733          | 0.1299 | WD19             |
| 9237                                      | 1831.4559      | 3601.6518          | 0.1500 | WD20             |
| 10000                                     | 1982.7689      | 3899.2166          | 0.1624 | WD21             |
| 12316                                     | 2441.9207      | 4802.1621          | 0.2000 | WD22             |
| 50000                                     | 9913.8447      | 19496.083          | 0.8118 | WD23             |
| 100000                                    | 19827.689      | 38992.166          | 1.6236 | WD24             |
| 150000                                    | 29741.534      | 58488.249          | 2.4354 | WD25             |
| 200000                                    | 39655.378      | 77984.332          | 3.2471 | WD26             |
| 250000                                    | 49569.223      | 97480.415          | 4.0589 | WD27             |
| 300000                                    | 59483.068      | 116976.49          | 4.8707 | WD28             |
| 350000                                    | 69396.913      | 136472.58          | 5.6825 | WD29             |
| 400000                                    | 79310.757      | 155968.66          | 6.4943 | WD30             |
| 450000                                    | 89224.602      | 175464.74          | 7.3061 | WD31             |
| 500000                                    | 99138.447      | 194960.83          | 8.1179 | WD32             |
| 550000                                    | 109052.29      | 214456.91          | 8.9297 | WD33             |
| 600000                                    | 118966.13      | 233952.99          | 9.7414 | WD34             |
| 650000                                    | 128879.98      | 253449.08          | 10.553 | WD35             |

### B. Measurement of dielectric properties

In this work, a wide-frequency range open-coaxial probe was used to determine the permittivity of liquid samples [30]. The setup was composed by a coaxial probe (Keysight Technologies, Slim Form Probe 030, USA) attached to a vector network analyzer (Keysight Technologies, E8361A PNA, USA) by means of a 50 $\Omega$  flexible cable. The calibration of the system was performed using the 8507x series software (Keysight Technologies, 8507x, USA) and the coaxial probe kit (Keysight Technologies, 85070E Dielectric Probe Kit, USA). The probe kit is composed of an open circuit, a short circuit and 150 mL of distilled-deionized water at room temperature (25 °C) poured in a 200 mL glass vessel. After calibration, the samples were poured in the 200 mL glass vessel and the coaxial probe was submerged 2 cm into them. It is important to mention that, for calibration and measurements, the probe was submerged 2 cm in order to

produce a distance of about  $\lambda/8$  from the probe tip to the base of the sample at the lowest frequency [30]. The permittivity measurements were performed at 25 °C.

### C. Uncertainty of permittivity measurements

The uncertainty of measurement is a range of values within which the true value is found [31]. It is divided into two types: type A and B. Standard uncertainty type A is due to repeatability [32]. Type B uncertainty is due to systematic errors [33]. The sources of type B uncertainty may vary, but, for the measurement of permittivity by means of the open coaxial probe, uncertainties by calibration and deviation are considered [27]. The combined uncertainty is calculated from the square root of the sum of the squares of the standard uncertainty by repeatability, calibration and deviation. It is important to mention that the permittivity of the samples at the frequency range from 500 MHz to 6 GHz was measured by quintuplicate in order to perform uncertainty calculations.

## IV. RESULTS AND DISCUSSION

### A. Permittivity models of liquid samples

The measurements of the complex permittivity of the 35 liquid samples with different ionic strength due to magnesium sulfate heptahydrate considered in Table 1 were carried out. The frequency range covered from 500 MHz to 20 GHz with 401 points. For each of the samples, the parameters of the Havriliak-Negami model were extracted and are shown in Table 2. The dielectric constant shows an adjustment of  $R^2 \geq 0.87$  for samples from WD1 to WD32; and  $R^2 \leq 0.70$  for samples from WD33 to WD35, which shows that the model is not effective for high concentrations. On the other hand, the loss factor has an adjustment factor of  $R^2 \geq 0.91$  for all samples. The adjustment factor  $R^2$  for the liquid samples can be seen in Table 3.

TABLE II HAVRILIAK-NEGAMI MODEL PARAMETERS OF THE LIQUID SAMPLES WITH DIFFERENT IONIC STRENGTH DUE TO MAGNESIUM SULFATE HEPTAHYDRATE

| Liquid sample ID | Havriliak-Negami model parameters |              |                     |   |         |        |
|------------------|-----------------------------------|--------------|---------------------|---|---------|--------|
|                  | $\epsilon_\infty$                 | $\epsilon_s$ | $\sigma_{dc}$ (S/m) | A | $\beta$ | T (ps) |
| WD1              | 5                                 | 79.00        | 0.00                | 0 | 1       | 8.15   |
| WD2              | 5                                 | 78.89        | 0.01                | 0 | 1       | 8.41   |
| WD3              | 5                                 | 78.48        | 0.02                | 0 | 1       | 8.14   |
| WD4              | 5                                 | 78.19        | 0.02                | 0 | 1       | 8.03   |
| WD5              | 5                                 | 78.18        | 0.03                | 0 | 1       | 8.14   |
| WD6              | 5                                 | 78.08        | 0.03                | 0 | 1       | 8.15   |
| WD7              | 5                                 | 78.49        | 0.04                | 0 | 1       | 8.15   |
| WD8              | 5                                 | 78.60        | 0.11                | 0 | 1       | 8.38   |
| WD9              | 5                                 | 78.89        | 0.13                | 0 | 1       | 8.15   |
| WD10             | 5                                 | 78.86        | 0.15                | 0 | 1       | 8.30   |
| WD11             | 5                                 | 78.14        | 0.11                | 0 | 1       | 8.15   |
| WD12             | 5                                 | 79.02        | 0.10                | 0 | 0.99    | 8.49   |
| WD13             | 5                                 | 78.33        | 0.13                | 0 | 1       | 8.15   |
| WD14             | 5                                 | 79.37        | 0.18                | 0 | 1       | 9.05   |
| WD15             | 5                                 | 78.22        | 0.21                | 0 | 1       | 8.30   |
| WD16             | 5                                 | 77.54        | 0.30                | 0 | 1       | 8.14   |
| WD17             | 5                                 | 77.45        | 0.39                | 0 | 0.96    | 8.18   |
| WD18             | 5                                 | 77.26        | 0.41                | 0 | 0.97    | 8.14   |
| WD19             | 5                                 | 76.88        | 0.45                | 0 | 0.97    | 8.14   |

|      |      |       |      |      |      |       |
|------|------|-------|------|------|------|-------|
| WD20 | 5    | 77.27 | 0.60 | 0    | 1    | 8.14  |
| WD21 | 4.50 | 77.75 | 0.62 | 0    | 0.96 | 8.14  |
| WD22 | 5    | 76.47 | 0.74 | 0    | 1    | 8.10  |
| WD23 | 4.50 | 77.36 | 1.70 | 0.05 | 1    | 8.57  |
| WD24 | 4.50 | 74.02 | 2.70 | 0.06 | 1    | 8.57  |
| WD25 | 4.50 | 71.87 | 3.50 | 0.07 | 1    | 8.78  |
| WD26 | 4.50 | 76.13 | 4.20 | 0.14 | 0.93 | 11.13 |
| WD27 | 4.50 | 78.10 | 4.80 | 0.19 | 0.97 | 11.13 |
| WD28 | 4.50 | 76.98 | 5.20 | 0.19 | 1    | 12.01 |
| WD29 | 4.50 | 75.66 | 5.60 | 0.21 | 1    | 12.01 |
| WD30 | 4.50 | 75.67 | 5.70 | 0.21 | 1    | 12.01 |
| WD31 | 4.50 | 70.86 | 5.80 | 0.20 | 0.94 | 13.07 |
| WD32 | 4.50 | 70.00 | 6.03 | 0.23 | 1    | 13.00 |
| WD33 | 4.50 | 70.00 | 6.15 | 0.21 | 1    | 11.63 |
| WD34 | 4.50 | 70.00 | 6.38 | 0.20 | 1    | 11.63 |
| WD35 | 4.50 | 70.00 | 6.43 | 0.21 | 1    | 11.63 |

TABLE III ADJUSTMENT OF HAVRILIAK-NEGAMI MODEL WITH RESPECT TO MEASUREMENT DATA

| Liquid sample ID | $R^2$ for $\epsilon'$ | $R^2$ for $\epsilon''$ |
|------------------|-----------------------|------------------------|
| WD1              | 0.99                  | 0.99                   |
| WD2              | 0.99                  | 0.99                   |
| WD3              | 0.99                  | 0.99                   |
| WD4              | 0.99                  | 0.99                   |
| WD5              | 0.99                  | 0.99                   |
| WD6              | 0.99                  | 0.99                   |
| WD7              | 0.99                  | 0.99                   |
| WD8              | 0.99                  | 0.99                   |
| WD9              | 0.99                  | 0.99                   |
| WD10             | 0.99                  | 0.99                   |
| WD11             | 0.99                  | 0.99                   |
| WD12             | 0.99                  | 0.99                   |
| WD13             | 0.99                  | 0.99                   |
| WD14             | 0.99                  | 0.99                   |
| WD15             | 0.99                  | 0.99                   |
| WD16             | 0.99                  | 0.99                   |
| WD17             | 0.99                  | 0.99                   |
| WD18             | 0.99                  | 0.99                   |
| WD19             | 0.99                  | 0.99                   |
| WD20             | 0.99                  | 0.99                   |
| WD21             | 0.99                  | 0.99                   |
| WD22             | 0.99                  | 0.99                   |
| WD23             | 0.99                  | 0.98                   |
| WD24             | 0.98                  | 0.98                   |
| WD25             | 0.98                  | 0.99                   |
| WD26             | 0.97                  | 0.98                   |
| WD27             | 0.92                  | 0.96                   |
| WD28             | 0.92                  | 0.97                   |
| WD29             | 0.88                  | 0.97                   |
| WD30             | 0.88                  | 0.97                   |
| WD31             | 0.95                  | 0.99                   |
| WD32             | 0.87                  | 0.98                   |
| WD33             | 0.70                  | 0.96                   |
| WD34             | 0.30                  | 0.93                   |
| WD35             | 0.06                  | 0.91                   |

## B. Analysis of permittivity

### 1) Dielectric constant

Fig. 1a shows the measurements of the dielectric constant from 500 MHz to 20 GHz of the liquid samples WD1, WD24, WD27 and WD32 whose ionic strengths are 0 M, 1.6236 M (low ionic strength), 4.0589 M (middle ionic strength) and 8.1179 M (high ionic strength), respectively. In the frequency range from 9 GHz to 12 GHz, there is the highest change of dielectric constant. In order to analyze the change of the dielectric constant in relation to the ionic strength, the analysis is carried out based on equation (3) that represents the percentage of change of the dielectric constant ( $\Delta\epsilon'$ ) of a SUT (sample under test) considering the dielectric constant of this SUT ( $\epsilon'_{SUT}$ ) in relation to the dielectric constant of the sample WD1 ( $\epsilon'_{WD1}$ ), which corresponds to distilled-deionized water.

$$\Delta\epsilon' = \left( \frac{\epsilon'_{SUT} - \epsilon'_{WD1}}{\epsilon'_{WD1}} \right) 100\% \quad (3)$$

The percentage of change of the dielectric constant of the SUT's is shown in Fig. 1b. It's shown that the higher the ionic strength, the less the dielectric constant. This behavior has been seen in solutions of NaCl in pure water [34]. This is because dissolved ions bind water molecules. The degree of binding is directly related to the nuclear charge effect that depends on the size and charge of dissolved ions. In this case, the degree of binding causes a reduction in polarization of water and a decrease in the overall dielectric constant [35].

### 2) Loss factor

Fig. 2a shows the measurements of the loss factor from 500 MHz to 20 GHz of the liquid samples WD1, WD24, WD27 and WD32. At 500 MHz, there is the highest change of loss factor. To analyze the change of the loss factor in relation to the ionic strength, the analysis is carried out based on equation (4) that represents the percentage of change of the loss factor ( $\Delta\epsilon''$ ) of a SUT considering the loss factor of this SUT ( $\epsilon''_{SUT}$ ) in relation to the loss factor of the sample WD1 ( $\epsilon''_{WD1}$ ), which corresponds to distilled-deionized water.

$$\Delta\epsilon'' = \left( \frac{\epsilon''_{SUT} - \epsilon''_{WD1}}{\epsilon''_{WD1}} \right) 100\% \quad (4)$$

The percentage of change of the loss factor of the SUT's is shown in Fig. 2b. It's shown that the higher the ionic strength, the higher the loss factor. This is because the predominant role of ionic conduction to the loss factor at low microwave frequencies [36].

## C. Analysis of measurement uncertainty

The combined uncertainty of the measurement of the dielectric constant for all samples was calculated for a frequency range from 9 GHz to 12 GHz. The lowest and highest combined uncertainty are 1.02% and 1.70%, respectively. The combined uncertainty of the loss factor measurement for all samples was calculated for a frequency range from 500 MHz to 3 GHz. The lowest and highest combined uncertainties are 0.11% and 1.41%, respectively. The frequency ranges from 9 GHz to 12 GHz and 500 MHz to 3 GHz were selected because there are the highest changes for dielectric constant and loss factor of the liquid samples, respectively. The combined uncertainty of the measurement of dielectric constant and loss factor of the liquid samples was similar to that reported for a solution of NaCl in pure water [37].

## D. Determination of concentration measurement range by permittivity

A very important characteristic of a measuring device is its sensitivity since it indicates the lowest measurement it can detect [38, 39]. This characteristic was determined using measurement uncertainty. Sensitivity can be found considering the dielectric constant or the loss factor.

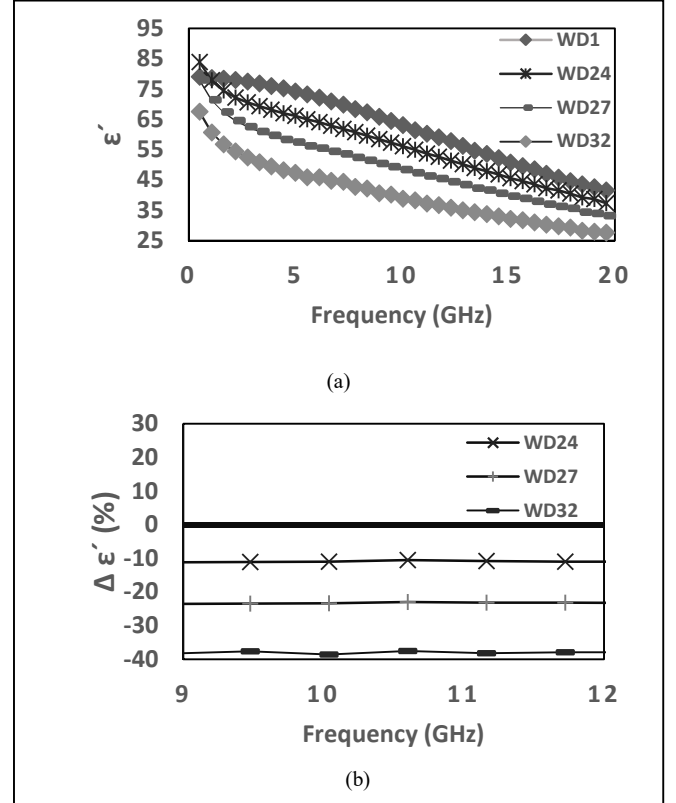


Figure 1. Behavior of the dielectric constant of the representative SUT's (WD1, WD24, WD27 y WD32): (a) dielectric constant of the representative SUT's from 500 MHz to 20 GHz and (b) percentage change of dielectric constant of representative SUT's in a frequency subrange from 9 GHz to 12 GHz respect to dielectric constant of WD1.

The loss factor presents higher change than the dielectric constant when varying to the ionic strength, especially at 500 MHz. This can be observed by comparing fig. 1 and fig. 2. Therefore, the loss factor is used at an analysis frequency of 500 MHz to establish the sensitivity. In fig. 3, loss factors at 500 MHz for low ionic strength levels and their combined uncertainty ranges are presented. The combined uncertainty interval of the sample WD1 ( $\epsilon'' = 2.18$ ), which is distilled-deionized water, overlaps with that of the sample WD2 ( $\epsilon'' = 2.15$ ), but not with that of the sample WD3 ( $\epsilon'' = 2.40$ ) nor with that of the sample WD4 ( $\epsilon'' = 2.91$ ). Therefore, from the sample WD3 (0.0008 M of ionic strength), the SUT's show a significant change in the loss factor with respect to the sample WD1. Since the sample WD3 is a SUT with a value of ionic strength of 0.0008 M, this is the sensitivity of the open coaxial probe. In fig. 4, the highest concentrations are considered to determine the maximum concentration to be measured. It can be noted that, for samples WD30 ( $\epsilon'' = 197.21$ ), WD31 ( $\epsilon'' = 207.25$ ) and WD32 ( $\epsilon'' = 213.82$ ), the loss factor keeps increasing. For samples WD33 ( $\epsilon'' = 211.28$ ), WD34 ( $\epsilon'' = 212.17$ ) and WD35 ( $\epsilon'' = 210.69$ ), the loss factor stops increasing.

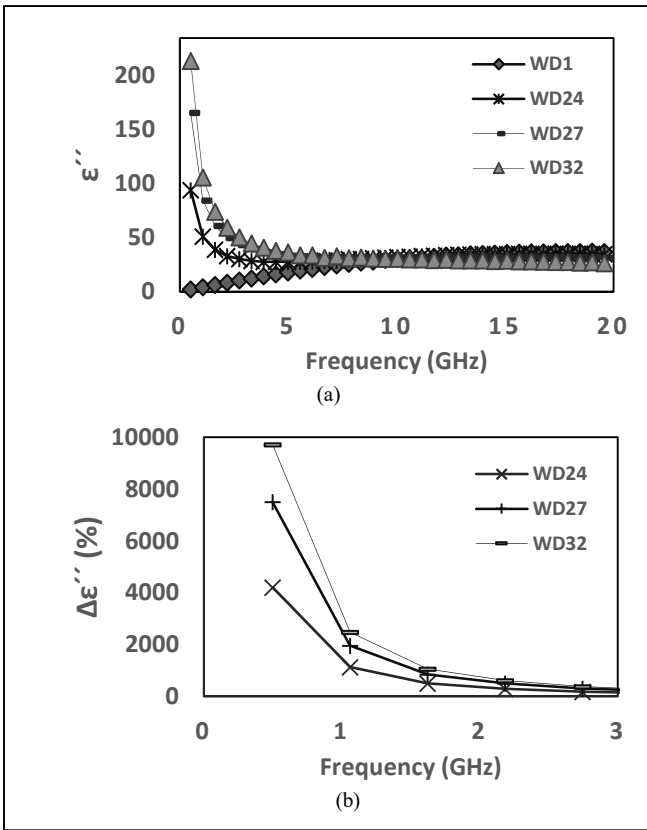


Figure 2. Behavior of the loss factor of the representative SUT's (WD1, WD24, WD27 y WD32): (a) loss factor of the representative SUT's from 500 MHz to 20 GHz and (b) percentage change of loss factor of representative SUT's in a frequency subrange from 500 MHz to 3 GHz respect to loss factor of WD1.

Consequently, it can be established that the maximum concentration to be detected by means of the loss factor corresponds to WD32, which has a value of ionic strength of 8.1179 M. Fig. 5 shows the loss factor of SUT's with ionic strength levels (I) from 0.0008 M to 8.1179 M at 500 MHz. It is important to mention that there is a good correlation between discrete data and fitting curve; which can be conformed since  $R^2 = 0.99$ .

$$I \text{ (M)} = 0.0002 (\epsilon'')^2 - 0.0039 \epsilon'' + 0.0765 \quad (5)$$

Therefore, the open coaxial probe can be used for measuring a wide ionic strength range (0.0008 M to 8.1179 M). On the other hand, according to Guide to the Expression of Measurement Uncertainty, accuracy is the degree of agreement between the result of a measurement and a true value of the measurand. The accuracy of the open coaxial probe was computed comparing the permittivity measured of the water against the permittivity reported in literature of the water. The accuracy of loss factor at 500 MHz was 16 %. The information presented in this work is useful for designing of other permittivity microwave sensors that determine ionic strength. As previously mentioned, the electrical conductivity meter is employed for measuring from 0 M to 0.32 M. In order to determine ionic strength levels higher than 0.32 M, the gravimetric method is used. A permittivity microwave sensor would be able to measure a wide range of ionic strength. Moreover, it wouldn't be affected by the electrode polarization, which occurs in an electrical conductivity meter, and can adjust to a portable device for measuring in real time, which is not possible in the gravimetric method. Real-time

measurement systems are essential to implement dynamic work strategies because they provide timely data to the decision-making process. It is worth to mention that the vector network analyzer employed with the open coaxial probe is expensive, but, in further researches, it could be replaced by low cost vector network analyzers [40]. In the future, the next challenge would be to determine ionic strength due to multiple water electrolytic pollutants possibly present in a sample.

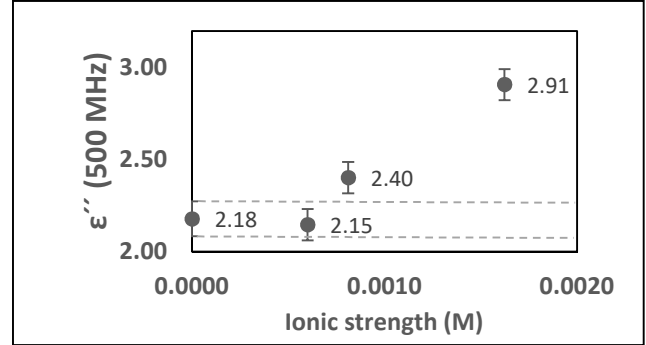


Figure 3. Loss factor at 500 MHz for low ionic strength levels.

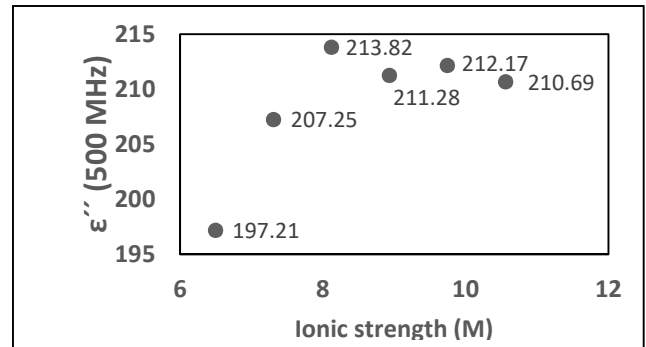


Figure 4. Loss factor at 500 MHz for high magnesium sulfate heptahydrate concentrations.

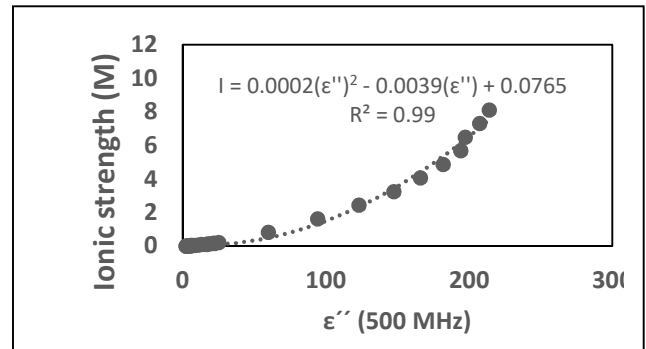


Figure 5. Trend between loss factor ( $\epsilon''$ ) at 500 MHz and Ionic strength (I).

## V. CONCLUSION

In this work, the complex dielectric permittivity, measured with an open coaxial probe, is presented from 500 MHz to 20 GHz for several samples with different levels of ionic strength due to magnesium sulfate heptahydrate in distilled-deionized water. The ionic strength can be determined by using the loss factor at 500 MHz. There is a high correlation between discrete data and the fitting curve obtained from the loss factor and ionic strength. It was determined that the open coaxial probe can be used for measuring a wide ionic strength range. Moreover, the results of this work could be used for

the design of other permittivity sensors in the range of microwaves focused on measuring a wide ionic strength range and in real time.

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