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Control Scheme for Rotary Base Inverted Pendulum by Means of Nested Saturation Functions

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***Abstract*—This paper develop the design of control scheme for the Rotary Base Inverted Pendulum (RBIP) system which consist in two closed-loop controls: First, a proportional-derivative (PD) control law for tracking trajectory at the angle of the base is apply; then, a closed-loop control consider angle of the base as artificial control and use nested saturation functions to stabilize pendulum's angle and horizontal displacement. The control scheme use the desired trajectory as artificial control input, through tracking trajectory control, and shows a methodology to express the system as chain of integrator to propose a controller by means of nested saturation functions. Numerical simulations are presented to show the effectiveness of control scheme proposed and is contrasted with a linear feedback controller.**

***Index Terms*—Non-linear control, nested saturation, under-actuated system, inverted pendulum, tracking trajectory.**

I. Introduction

Automatic control theory is a research field related to others such as mechatronics, robotics, electronics, economics, chemistry, among others [1]–[4] and allow processes or systems to perform a specific function as expected; that is why it's a fundamental element in many applications.

The analysis and control of under-actuated mechanical system is an attractive topic research in control theory as they appear in many practical applications such as robotic systems, aerospace systems and marine systems [23]. A classical under-actuated mechanical system is the inverted pendulum type that has been used as benchmark in automatic control proposals. There are different configurations for inverted pendulum systems as: wheeled pendulum, spherical pendulum, Furuta pendulum, inertial wheel inverted pendulum, acrobot, pendubot; and in recent years, flying inverted pendulum. The classic control problem is the stabilization of inverted pendulum system in unstable equilibrium point.

The importance to study this kind of systems is because some others show an inverted pendulum behavior [6]. The control of humanoid robots is a challenging task, because of its complexity, but many works simplify this problem using the analogy between bipedal gait and inverted pendulum [7], [8]. In recent years, some electrical individual mobile transportation devices have been developed which are a mobile wheeled inverted pendulum [9]. A robotic arm is a double inverted pendulum fully actuated [10]. Some works have shown unmanned aerial vehicles exhibit mathematically similar behavior to an inverted pendulum system [11].

Several works have been developed to stabilize inverted pendulum systems by divers control approaches. In Olivares et al. [12] uses a linear approximation around the unstable equilibrium point in flywheel inverted pendulum system to design a linear controller in two steps: the output is controlled with a simple PID controller then, a second linear controller in an outer loop provides the global stability. An improvement of this control propose is shown in [13] where an observer-based state feedback control is implement in flywheel inverted pendulum system. Hehn et al. [14] introduced the flying inverted pendulum system, conformed by an inverted pendulum placed on the top of the unmanned aerial vehicle; the paper explores the static and dynamic equilibrium of the system and proposed Linear-Quadratic Regulator (LQR) control for stabilization. Spong presented the partial feedback linearization technique in [15] to stabilize the Acrobot and Pendubot systems and simulations results are presented. Aguilar-Ibañez et al. [16] solves the stabilization problem of strongly damping inertia wheel pendulum, around its unstable equilibrium, by using nested saturation functions; the proposed control strategy makes the closed-loop system globally asymptotically and locally exponentially stable even in physical damping presence. Aguilar-Ibanez et al. [17] used direct Lyapunov based control for stabilization of Furuta Pendulum system usign a suitable partial feedback linearization and proposing a candidate Lyapunov function; stabilizing controller is derived from candidate Lyapunov function.

Related to Inverted Pendulum on a Cart system (IPC) many interesting works can be found in literature. Pathak et al. [18] obtain a partial feedback linearization of the IPC system and design two controllers in two levels to control velocity and position, stability analysis is proved by Lyapunov second method. Aguilar et al. [19] present a nonlinear control force for IPC system with partial feedback linearization, to linearize the actuated coordinate of the inverted pendulum, and then, a suitable Lyapunov function is formed to obtain a stabilizing feedback controller; the obtained closed loop system is locally asymptotically stable around its unstable equilibrium point. Huang et al. [20] proposes a high-order disturbance observer for IPC system in combination with sliding mode control for

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balance and speed control; the boundness of the estimation error and stability of the closed-loop control system is achieved through the appropriate selection of sliding surface coefficients. As well, an interesting work is developed by Aguilar-Ibañez et al. [21] were a nonlinear controller is presented for IPC system where is expressed as a chain of integrators with a nonlinear perturbation allowing to use a nested saturation control technique to stabilize the pendulum to the top position with zero displacement of the cart; the stability analysis is archive by second Lyapunov method.

This work presents the Rotary Base Inverted Pendulum system (RBIP), which is a variety of wheeled inverted pendulum which consist in a planar inverted free swing pendulum with a mobile base in the horizontal direction, but is actuated using a pair of forces in its ends and can rotate around its center of gravity. This change in the system adds one degree of freedom in the system, the angle of the base. Considering dynamic model of the system, this paper presents a control scheme in two stages: first, close-loop control is proposed for tracking trajectory in the angle of the base, so then, using the angle of the base as artificial control, the dynamic model is expressed as chain of integrator and a control law by means of nested saturation functions is used for stabilization of horizontal displacement and pendulum's angle.

This paper is organized as follows: Section II describes the dynamic model of the RBIP. Section III shows the control law to use the base's angle as artificial control input and describes several coordinate transformations to describe the system as chain of integrator. Using this representation, a control law by means of a nested saturations functions is proposed for stabilization. Section IV presents numerical simulations to verify the performance of the closed-loop controller. Finally, section V is devoted to conclusions.

II. Dynamic Model RBIP

The RBIP, shown in figure 1, is a under-actuated mechanical system that consists in a base with two forces, f_1 and f_2, located at its ends and an inverted pendulum is attached to base's center of gravity. The base can be moved along the horizontal axis through the forces which are generated by pair of motors with a propeller. The dynamic equations were modeled by Euler-Lagrange equations and expressed as

$$M_{+}\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = U \tag{1}$$

with

$$M_{+} = \begin{bmatrix} (m+M) & l_p m cos(\theta) & 0 \\ l_p m cos(\theta) & (i_p + l_p^2 m) & 0 \\ 0 & 0 & iv \end{bmatrix}$$

$$C = \begin{bmatrix} -l_p m sin(\theta)\dot{\theta} \\ 0 \\ 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ -g l_p m sin(\theta) \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} u_1 sin(\theta) & 0 & u_2 \end{bmatrix}^{\mathsf{T}} \quad q = \begin{bmatrix} x & \theta & \alpha \end{bmatrix}^{\mathsf{T}}$$

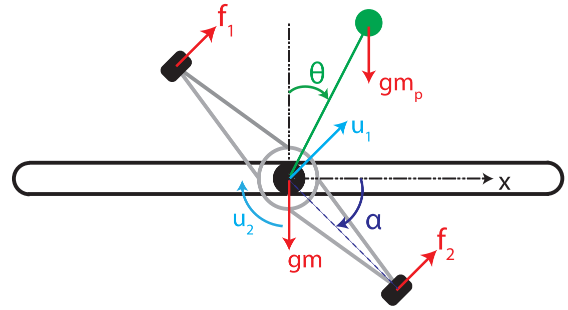

Fig. 1. RBIP system.

where M_{+} is a non-singular symmetric matrix, C is the coriolis matrix, G is the gravity matrix, U is the control input matrix, q are the generalized coordinates, m is the pendulum's mass, M is the base's mass, I is the base's inertia, i_p is the pendulum's inertia, g is the gravity acceleration, l_p is the pendulum's length, u_1 is the sum of forces $f_1 + f_2$ and u_2 is the difference of forces $L(f_2 - f_2)$, L is the base's length. u_1 and u_2 are the control inputs for RBIP system. Without loss of generality, lets consider the following: $L = 1$, $I = 1$, $i_p = 0$, so, the generalized coordinates can be expressed as

$$\begin{aligned} \ddot{x} &= \frac{a}{m + 2M - mcos(2\theta)} \\ \ddot{\theta} &= \frac{b}{-l_p(m+M) + l_p m cos(\theta)^2} \\ \ddot{\alpha} &= u_2 \end{aligned} \tag{2}$$

where $a = 2u_1 sin(\alpha) - gmsin(2\theta) + 2l_p m sin(\theta)\dot{\theta}^2$ and $b = u_1 cos(\theta) sin(\alpha) + sin\theta(-g(m+M) + l_p m cos(\theta)\dot{\theta}^2)$.

Control problem: Considering the RBIP system (2) the control scheme is designed to stabilize the system at origin, $x = \theta = \alpha = 0$, through control inputs u_1 and u_2 by means of nested saturation functions when pendulum initialize in the upper half plane.

III. Control Scheme

This work presents a control scheme to stabilize the RBIP system. The control scheme is divided into two control laws: an inner linear proportional-derivative (PD) control for tracking trajectory of α angle, then using desired trajectory α_d as artificial control, the system is expressed as chain of integrator plus non-linear perturbation and control by means of nested saturation function is proposed for stabilization. Figure 2 shows a diagram of the control scheme proposed.

A. Control for angle α

Considering the third equation of RBIP system (2), control input u_2 acts directly to $\ddot{\alpha}$, so, let's define the tracking trajectory error as follows.

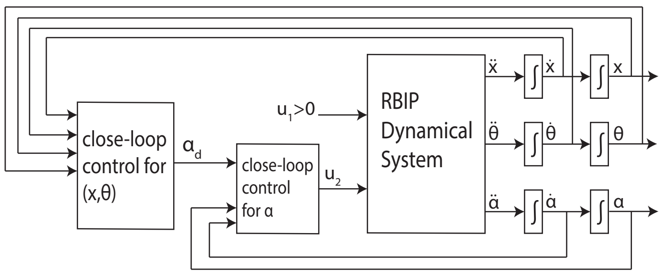

Fig. 2. Close-loop system.

$$e_\alpha = \alpha - \alpha_d$$

where α_d is the desired trajectory twice differentiable, so the error dynamics is

$$\ddot{e}_\alpha = u_2 - \ddot{\alpha}_d \tag{3}$$

Defining control input u_2 as

$$u_2 = \ddot{\alpha}_d - k_1 e_\alpha - k_2 \dot{e}_\alpha \tag{4}$$

Combining equations (4) and (3), the error dynamics can be expressed as

$$\ddot{e}_\alpha = -k_1 e_\alpha - k_2 \dot{e}_\alpha \tag{5}$$

The control parameters k_1, k_2 are proposed such that characteristic polynomial of error dynamics, $s^2 + k_2 s + k_1$, is Hurwitz, so $\lim_{t\to\infty} \mid e_\alpha \mid = 0$.

B. Control for (x,θ)

The control input u_2 (4) allows the α coordinate to follow a desired α_d trajectory, so it's possible replace α for α_d and use it as artificial control input [23], [24] for (x,θ) system, in combination with u_1. Defining α_d as

$$\alpha_d = \arcsin(v_0) \tag{6}$$

$$v_0 = \frac{c}{u_1} \tag{7}$$

where $c = l_p m v_1 cos(\theta) + (m + M)sec(\theta)(-l_p v_1 + gsin(\theta)) - l_p msin(\theta)\dot{\theta}^2$. Is clear that $u_1 > 0$. Defining the following scalar transformations and expressing the system with state variables x_1, x_2,θ_1, θ_2 obtained.

$$\hat{x} = x/g \quad \hat{\theta} = \frac{\theta l_p}{g} \quad v_1 = \frac{vg}{l_p}$$
$$x_1 = \hat{x} \quad x_2 = \dot{x}_1 \quad \theta_1 = \hat{\theta} \quad \theta_2 = \dot{\theta}_1 \tag{8}$$

The sub-system (x,θ) can be expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -vsec(\theta) + tan(\theta) \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= v \end{aligned} \tag{9}$$

The system (9) is the same as partial linearized model of the Inverted Pendulum on a Cart (IPC) [21] and the above representation is validated for all $\theta_1 \in (\frac{\pi}{2}, -\frac{\pi}{2})$, the upper half plane. The main idea is to express the system (9) as a chain integrator with a non-linear perturbation and to propose a control law by means of nested saturation functions.
According to Decoupling Theorem [25], the following coordinate transformation is introduced.

$$\begin{aligned} z_1 &= Log\left(\frac{1 + tan(\theta_1/2)}{1 - tan(\theta_1/2)}\right) + x_1 \\ z_2 &= \frac{1}{cos(\theta)} + x_2 \\ \omega_1 &= tan(\theta_1) \\ \omega_2 &= sec^2(\theta_1)\theta_2 \\ v_f &= sec^2(\theta_1)v + 2\theta_2^2 tan(\theta_1)sec^2(\theta_1) \end{aligned} \tag{10}$$

which lead to

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \omega_2 + \omega_2^2 \frac{\omega_1}{(1+\omega_1^2)^{3/2}} \\ \dot{\omega}_1 &= \omega_2 \\ \dot{\omega}_2 &= v_f \end{aligned} \tag{11}$$

The above system has the chain integrator form plus non-linear perturbation. The nested saturation technique was introduced by Teel [26] and has been used for the stabilization of a linear integrators chain and controlling mini-flying machines [27]. In order to use the nested saturation technique, following linear transformation is introduced

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \omega_1 \\ \omega_2 \end{bmatrix} \tag{12}$$

Then, system (11) is transformed into

$$\begin{aligned} \dot{q_1}_1 &= v_f + q_2 + q_3 + q_4 + 3\delta_a(q) \\ \dot{q_2}_2 &= v_f + q_3 + q_4 + \delta_a(q) \\ \dot{q_3}_1 &= v_f + q_4 \\ \dot{q_4}_2 &= v_f \end{aligned} \tag{13}$$

where the perturbation $\delta_a(q) = q_4^2 G(q_3 - q_4)$ and $G(\omega) = \frac{\omega}{(1+\omega^2)^{3/2}}$[1].
Based on Stabilizing controller may be readily proposed as

$$v_f = -q_4 - k\sigma_\beta\left(\frac{q_3 + \sigma_\gamma(q_2 + \sigma_\delta(q_1))}{k}\right) \tag{14}$$

where k is a positive constant and $\sigma_m(s)$ is a linear saturation function.

[1]Note that $\mid G(q_3 - q_4) \mid \leqslant k_0 = \frac{2}{3^{3/2}}$

$$\sigma_m(s) = \begin{cases} s & if \quad | s | \leqslant m \\ msign(s) & if \quad | s | > m \end{cases}$$

According with the work of Aguilar et al. [21], the states $q_i, i = (1, 2, 3, 4)$ in system (13) in close-loop with controller v_f (14) are bounded in finite time with following constrains on parameters α, β and γ.

$$\alpha = r \quad \beta = r/2 \quad \gamma = 3r/14 \tag{15}$$

for all $0 < r < 1$.
And after finite time the controller v_f is no longer saturated, that is

$$v_f = -q_1 - q_2 - q_3 - q_4$$

and the closed-loop system turns out to be, as

$$\begin{aligned} \dot{q}_{11} &= -q_1 + 3\delta_a(q) \\ \dot{q}_{22} &= -q_1 - q_2 + \delta_a(q) \\ \dot{q}_{31} &= -q_1 - q_2 - q_3 \\ \dot{q}_{42} &= -q_1 - q_2 - q_3 - q_4 \end{aligned} \tag{16}$$

Therefore, as shown in [21], the system (13) is globally asymptotically stable and locally exponentially stable in closed-loop with controller v_f (14) in the sense of Lyapunov when the control parameters satisfy the restriction (15) and the following inequalities are fulfill

$$q_1^2 + q_2^2 - k_0(3q_1 + q_2)^2 > 0; \quad -1 + k_0 q_4^2 \leqslant -1 + 14k_0k^2 < 0$$

Proposition 1. Considering the RBIP system (2), with pendulum initialized in the upper half plane $\theta \in (-\pi/2, \pi/2)$, in closed-loop with the controllers:

$$u_2 = \ddot{\alpha}_d - k_1 e_\alpha - k_2 \dot{e}_\alpha$$
$$u_1 > 0$$

with definitions provided in (6), (7), (8), (10), (12) and (14), such that control parameters k_1 and k_2 are proposed so that characteristic polynomial of error dynamics, $s^2 + k_2 s + k_1$, is Hurwitz and control parameters α, β and γ satisfies the restrictions (15).
Then, the close-loop system is semiglobally stable and locally exponentially stable in the sense of Lyapunov.

IV. Numerical simulation

The efficiency of the proposed control scheme was tested by numerical simulations implemented in Matlab software with following system parameters: M=1 kg, m=0.3 kg, l_p=0.3 m, g=9.81 m/s^2. The controller parameters were set as u_1=10 N, k_1=-1, k_2=-1.8, β=0.99, γ=0.49, δ=0.214 and k=0.1589. The initial conditions were set to $x(0) = 0$ m, $\dot{x} = 0$ m/s, $\theta(0) = 0.81$ rad, $\dot{\theta}(0) = 0$ rad/s, $\alpha(0) = 0$ rad, $\dot{\alpha}(0) = 0$ rad/s.

The control scheme proposed in this work is compared with classical feedback control law with the form

$$u_2 = -Kx$$

where $K = [-10, \quad -15, \quad -409, \quad -63, \quad 251, \quad 63,]$ is a gain matrix designed by LQR technique as shown in [28] and $x = (x, \dot{x}, \theta, \dot{\theta}, \alpha, \dot{\alpha})$ is the states vector.

Figure 3 show the closed-loop system with control scheme proposed, in continuous line (−), and LQR controller, in discontinuous line (−−). When control scheme proposed is used, stabilization time is lower than when LQR controller is apply. As well, closed-loop system with LQR controller shows undesired oscillation phenomena that control scheme proposed in this work doesn't exhibit.

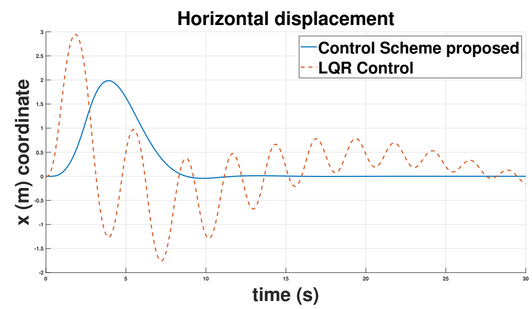

Fig. 3. Closed-loop response of the x coordinate.

Figure 4 show the closed-loop response for θ angle with control scheme proposed, in continuous line (−), and LQR controller, in discontinuous line (−−). As can be seen, the oscillation phenomena is present when LQR control is used. The control scheme proposed shows less stabilization time but amplitude is higher than LQR is implemented.

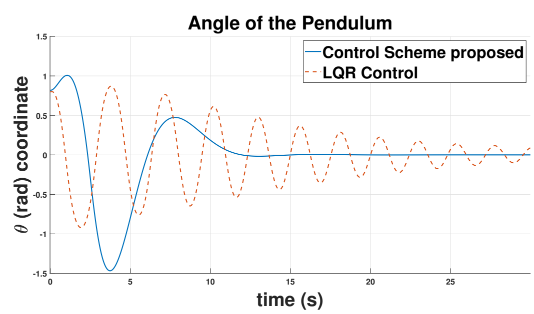

Fig. 4. Closed-loop response of the θ coordinate.

Figure 5 shows the response of the controller u_2 with control scheme proposed, in continuous line (−), and LQR controller, in discontinuous line (−−). Both cases shows the peaking phenomena, due to the α_d derivatives in the case of control scheme proposed but the amplitude of the peaking phenomena is smaller when control scheme proposed is applied than when LQR control is used.

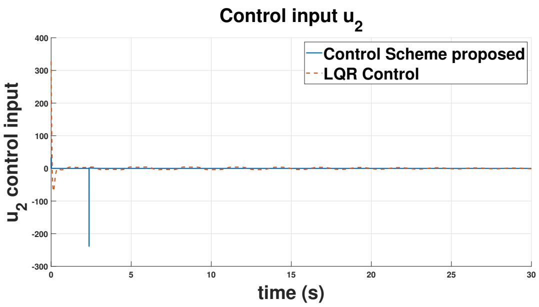

Fig. 5. Controller response u_2.

In general, the control scheme proposed in this paper move the base's angle to generate a movement in horizontal and keep the pendulum's angle close to zero. When the pendulum's angle is close to zero, the displacement in the base tries to stop and return slowly to origin keeping the pendulum's angle close to zero.

V. Conclusions

In this work, a control scheme for RBIP system is proposed that consists in a combination of two controllers: a linear PD control for tracking trajectory in the α angle and control by nested saturation functions that generates the desired trajectory α_d. The methodology presented shows that the RBIP system can be expressed as an inverted pendulum on a cart system, so, the control proposal is based on a previous work. Numerical simulations show the stability to zero for the whole system in close-loop with the control scheme proposed that is contrasted with a linear feedback controller. The close-loop system with control scheme proposed exhibit a less stabilization time without oscillation phenomena, but controller u_2 presents undesired peaking phenomena.

As future work, tracking trajectory control for α must be improved to avoid the peaking phenomena and to propose a robust control for internal and external disturbances.

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