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Performance Comparison of Positioning Algorithms for UAV Navigation Purposes based on Estimated Distances

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***Abstract*—A comparison of four algorithms of positioning for unmanned aerial vehicles (UAVs) navigation is presented in this work. In particular, the radical axis, maximum likelihood, Gauss-Newton and Quasi-Newton methods are implemented under the same conditions in a simulation platform. With this platform it is possible to simultaneously evaluate the accuracy of the position estimation and the trajectory tracking of a UAV of each algorithm allowing the comparison between them. In addition, the complexity of the algorithms is addressed and commented. Simulation results show that the studied methods present similar accuracies of the position estimation, being the Gauss-Newton and Quasi-Newton methods with the smallest error range.**

***Index Terms*—UAV, position estimation, radical axis, maximum likelihood, Gauss-Newton, Quasi-Newton, indoor localization, trajectory tracking.**

I. Introduction

One of the most relevant problems for unmanned aerial vehicles (UAVs) navigation in an autonomous way implemented in unknown environments is the acquisition of the UAV position in real time. Although it is common the use of global position satellite systems like the Global Position System (GPS) in outdoor scenarios where it is assumed that the satellite signal is available during all the flight mission, in some cases the satellite link could be blocked and affected by weather conditions [1]. For indoor scenarios the satellite signal is significantly attenuated or even totally lost.

In order to estimate the position of one device or vehicle, some techniques have been developed and have a long history. In essence, the received signal strength (RSS), the angle of arrival (AoA), the time of arrival (ToA), and the time difference of arrival (TDoA) are parameters often used for position estimation purposes [2]. Currently the ultra wide band (UWB) systems offer an excellent solution for positioning in indoor scenarios due to their ultra short time-domain pulses [3]. In general terms, if the time of arrival of a UWB pulse can be known with little uncertainty, then it is possible to estimate the distance between the transmitter and the receiver. Using the estimated distances from a UAV to multiple receivers (commonly called anchors) whose position are known, then, it will be possible to estimate the UAV position employing triangulation techniques [3]. The simplest method to estimate the position of a vehicle is by solving a system of equations formed from the estimated distances from each anchor. The two dimensional (2D) exact solution can be obtained by solving a system of three equations, implying that three distance estimations are obtained from the UAV to three different anchors. For the three dimensional (3D) case, four anchors will be required to estimate the distances between the UAV and the anchors forming so an equation system baseline to determine the UAV position [4]. However, if there are redundant distance estimations associated to a greater number of anchors, then the system is over-determined. In this case several algorithms have been proposed in order to obtain the best position estimation as be possible (see for example [5], [6] and [7]).

This paper focuses on evaluating and comparing some of these algorithms such as radical axis, maximum likelihood, Gauss-Newton and Quasi-Newton. With that in mind, two simulation platform was developed in Matlab where a UAV navigates in a random way in an area with several anchors deployed. The first simulation makes use of the estimated distances from the four nearest anchors to estimate the position of the UAV by each algorithm without consider the UAV dynamics in order to compare them in terms of position accuracy only. On the other hand, in the second simulation a UAV performs a trajectory tracking using the estimated position in order to observe how the accuracy of the each algorithm could affect the UAV navigation. Thus, the paper is organized as follows: in Section II the algorithms for position estimation are presented. The UAV dynamics and the control strategy are presented in Section III. The simulation settings are stated in Section IV. The results are analyzed and compared in Section V, and concluding remarks are given in Section VI.

II. Position estimation algorithms

A. Preliminary concepts

In order to explain the algorithms of position estimation to be evaluated in this paper, let us state the framework over which the analysis of all methods will be carried out. For the sake of the simplicity, let us assume that the position of the anchors deployed are known with coordinates (x_i, y_i) for

$$i = 1, 2, .., N$$

where N is the number of anchors. If the distance from the UAV to each anchor is estimated as $\hat{d}_i$, then, the UAV position (x, y) can be estimated by solving the following system of simultaneous equations formed from the N anchors

$$\begin{aligned} A_1 :&(x - x_1)^2 + (y - y_1)^2 = \hat{d}_1^2 \\ A_2 :&(x - x_2)^2 + (y - y_2)^2 = \hat{d}_2^2 \\ &\vdots \\ A_N :&(x - x_N)^2 + (y - y_N)^2 = \hat{d}_N^2 \end{aligned} \tag{1}$$

Due to the fact that errors can be introduced during the distance estimations (associated to impairments of the air-to-ground channel, for instance), it is not always possible to obtain a unique solution. Nevertheless, based on the information provided by the system of equations in (1), each algorithm studied here is able to estimate the position of the UAV whose estimation error will be related to the errors in the estimated distances. Since the aim of the positioning schemes is to obtain an estimated position minimizing the errors between the actual and estimated positions, firstly it will be necessary to define these range errors as

$$e_i(x, y) = \sqrt{(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2} - \hat{d}_i \tag{2}$$

Then the objective function required for the optimization algorithm of the estimated position p will be the mean squared errors (MSE) between the estimated position and the position of each of the N anchors [3]

$$F(p) = \frac{1}{2N} \sum_{i=1}^{N} e_i^2(x, y) \tag{3}$$

It is worth mentioning that for the analysis of this paper, a 2D scenario will be considered, where we will use $N = 4$ in order to analyze the estimations computed by each algorithm for an over-determined system.

B. Radical Axis

Let us consider that each equation in (1) represents a circle of radius d_i with center at (x_i, y_i), then it is possible to solve the problem though the radical axis concept, where A_i is used to indicate the i-th circle: "*The radical axis of both circumferences formed by the equations A_i and A_j is a line perpendicular to the line drawn between centers; If A_i and A_j touch in two different points, their radical axis coincides with the common chord; if the circumferences A_i and A_j are tangents, the radical axis will be the common tangent, and if A_i and A_j does not have any common point, their radical axis is the locus of one point where the lengths of the tangents drawn from this point to both circles are the same*" [8]. Fig. 1 depicts the three cases of the aforementioned cases of radical axis.

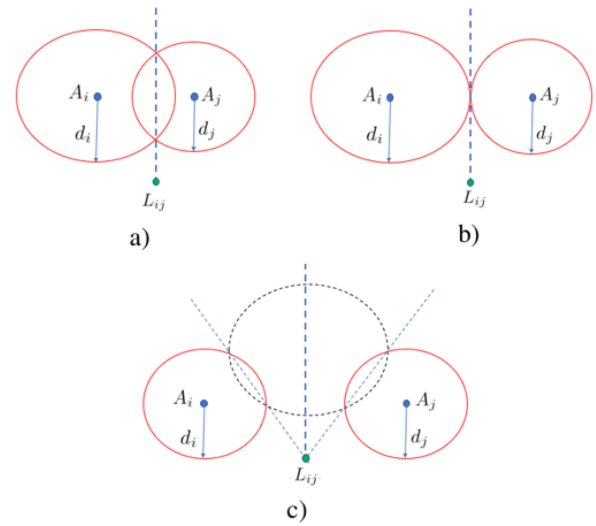

Fig. 1. Radical axis cases of two circles: a) when both circles touch in two points, b) when both circles touch in one point and c) when both circles do not touch each other

The general equation of the radical axis, L_{ij}, of a pair of circles A_i and A_j is derived by a simple subtracting of $A_i - A_j$ resulting in the following equations

$$L_{ij} = A_i - A_j \tag{4}$$

$$\begin{aligned} L_{ij} : (x - x_i)^2 + (y - y_i)^2 - \hat{d}_i^2 \\ -(x - x_j)^2 - (y - y_j)^2 + \hat{d}_j^2 = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} L_{ij} : 2(x_j - x_i)x + 2(y_j - y_i)y \\ +x_i^2 - x_j^2 + y_i^2 - y_j^2 + d_j^2 - d_i^2 = 0 \end{aligned} \tag{6}$$

which could be expressed as

$$L_{ij} : a_{ij}x + b_{ij}y + c_{ij} = 0 \tag{7}$$

where

$$\begin{aligned} a_{ij} =&2(x_j - x_i) \\ b_{ij} =&2(y_j - y_i) \\ c_{ij} =&x_i^2 - x_j^2 + y_i^2 - y_j^2 + d_j^2 - d_i^2 \end{aligned} \tag{8}$$

Thus, the solution for the position estimation problem is given by

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_{RA} = \mathbf{A}^{+}\mathbf{c} \tag{9}$$

where $(\hat{x}, \hat{y})$ is the estimated position and $\mathbf{A}^{+}$ denotes the pseudoinverse of matrix $\mathbf{A}$. For this case with 4 anchors there are six radical axis, hence, $\mathbf{A}$ and $\mathbf{c}$ are given by

$$\mathbf{A} = \begin{bmatrix} a_{12} & b_{12} \\ a_{13} & b_{13} \\ a_{14} & b_{14} \\ a_{23} & b_{23} \\ a_{24} & b_{24} \\ a_{34} & b_{34} \end{bmatrix} \tag{10}$$

$$\mathbf{c} = [c_{12}\ c_{13}\ c_{14}\ c_{23}\ c_{24}\ c_{34}\ ] \tag{11}$$

C. Maximum Likelihood Estimation

The positioning problem has a random nature for diverse factors, and so, a statistical approach can be followed. The maximum likelihood estimation (MLE) is one of the most used methods to estimate the parameters of a statistical model $f(x;\theta)$ and is based on the maximum likelihood principle: "*Given a random sample $X_1, ..., X_n$ and a parametric model $f(x_1, ..., x_n; \theta)$, choose as the estimator $\hat{\theta}$, the value of θ that maximizes the likelihood function* " [9].

The interest of this paper is to use the MLE for position estimation using four anchors. As previously mentioned for the 2D case, it is possible to estimate the UAV position by solving three simultaneous equations as long as the three circumferences generated by the equations touch each other assuring in this way a solution. Under the conditions studied in this work, when there are four ranging results, i.e. by taking $N = 4$ in (1), it is possible to create four sets of 3 distances, which are then used to calculate the possible positions $\rho_i = (\hat{x}_i, \hat{y}_i)$ with $i = 1, 2, 3, 4$. These possible positions ρ_i will be considered as samples data of the probability density function of the actual UAV position $f(\rho \mid (x, y))$. Thus, in order to find the most likely position of the UAV $\hat{P}(x, y)_{ML}$, it will be necessary to maximize the following likelihood function denoted by the operator $\mathcal{L}$ [10]

$$\mathcal{L}(\hat{P}(x, y)_{ML}) = \arg\max \prod_{i=1}^{4} f(P; \rho_i, \sigma) \tag{12}$$

As in most engineering fields, it is possible to assume a Gaussian distribution for independent and identically distributed samples from a random process (see [11], [12] and [13]), hence, the probability density function for each possible position could be expressed by

$$f(P; \rho_i, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\left[\frac{(P-\rho_i)^2}{2\sigma^2}\right]} \tag{13}$$

where σ is the standard deviation. Assuming a homogeneous system, all anchors have the same standard deviation.

D. Gauss-Newton

An optimization problem ocurrs when there is a need of minimize or maximize an objective finding the parameters that give an optimal solution. Today, numerical methods and algorithms have been developed for solve the most common problems [14]. In the position estimation case, it is required to minimize the mean squared errors formulated in the objective function. Some of the algorithms used to find the position where the range errors are minimized include the Gauss-Newton and Quasi-Newton methods.

The Gauss-Newton method consists of linearizing an objective function using a Taylor series expansion around a set of initial parameters k_0 [15]. In this work the objective function (3) will be linearinzing by a second order Taylor approximation, then, we have

$$F(p_k + \mathbf{s}_k) \approx F(p_k) + \mathbf{g}_k^{\mathrm{T}}\mathbf{s}_k + \frac{1}{2}\mathbf{s}_k^{\mathrm{T}}\mathbf{H}(p_k)\mathbf{s}_k \tag{14}$$

where p_k is the current solution in the iteration k which will be updated by the directional vector $\mathbf{s}_k$ until a desired error tolerance ϵ has been reached.

On the other hand, $\mathbf{g}_k$ represents the gradient of the objective function given by

$$\mathbf{g}_k = \mathbf{J}_k^{\mathrm{T}} F(p_k) \tag{15}$$

where $\mathbf{J}_k$ is the Jacobian matrix of the objective function with the first-order partial derivatives and $\mathbf{H}$ denotes the Hessian matrix with the second-order partial derivatives of the objective function [16]. Knowing that $\mathbf{H}$ could be approximated by the product $\mathbf{J}_k^{\mathrm{T}}\mathbf{J}_k$, then, after some algebra manipulations as are shown in [16], the Gauss-Newton recursion equation results as follows

$$F(p_{k+1}) = F(p_k) + \mathbf{s}_k \tag{16}$$

where

$$\mathbf{s}_k = -(\mathbf{J}_k^{\mathrm{T}}\mathbf{J}_k)^{-1}\mathbf{J}_k^{\mathrm{T}} F(p_k) \tag{17}$$

The Gauss-Newton method is an iterative algorithm and its procedure could be described as follows:

Gauss-Newton Algorithm

- Step 0: Set initial values of $(\hat{x}, \hat{y})$, tolerance $\epsilon > 0$ and $k_0 = 0$
- Step 1: Calculate the errors with Equation (2)
- Step 2: Calculate the Jacobian matrix $\mathbf{J}_k$ and gradient g_k from (15). If $\parallel \mathbf{g}_k \parallel \leq \epsilon$, stop
- Step 3: Calculate $\mathbf{s}_k$ from (17)
- Step 4: Update p_k with (16). Set $k = k + 1$ and go to Step 1.

E. Quasi-Newton

The Quasi-Newton algorithm is very similar to the Gauss-Newton method but it is characterized by approximate the Hessian matrix by a symmetric positive definite matrix $\mathbf{B}_k$. In the literature there are different ways to update $\mathbf{B}_k$ in order to reduce the errors with each iteration. One of them is using the Davidon-Fletcher-Powell formula (DFP) given by [3]

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \frac{\mathbf{h}_k \mathbf{h}_k^{\mathrm{T}}}{\mathbf{h}_k^{\mathrm{T}} \mathbf{q}_k} - \frac{\mathbf{B}_k \mathbf{q}_k \mathbf{q}_k^T \mathbf{B}_k}{\mathbf{q}_k^{\mathrm{T}} \mathbf{B}_k \mathbf{q}_k} \tag{18}$$

where

$$\begin{aligned} \mathbf{h}_k =& F(p_{k+1}) - F(p_k) \\ \mathbf{q}_k =& \mathbf{g}_{k+1} - \mathbf{g}_k \end{aligned}$$

The initial matrix $\mathbf{B}_1$ can be any positive definite matrix. Usually the identity matrix is used when there is not a better estimation. It is worth mentioning that for the nature of the DFP formula when a gradient is reached, it is possible to get into trouble of a uncertainty value, for that reason, it is recommendable the use of limits in order to prevent this situation [17].

III. UAV DYNAMICS

In order to test and compare the position estimation of each algorithm for UAV navigation purposes, the UAV trajectory tracking was evaluated by simulation considering tracking trajectory using a well-known dynamic model of a quadrotor and a PD controller as control strategy. The theoretical basic concepts of the dynamic model and the control strategy are exposed in what follows.

A. Dynamic Model [18]

The simplified dynamic model of the rotorcraft obtained by Euler-Lagrange equations is denoted by

$$\begin{aligned} m\ddot{x} =& -u\sin\theta \\ m\ddot{y} =& u\cos\theta\sin\phi \\ m\ddot{z} =& u\cos\theta\cos\phi - mg \\ \ddot{\theta} =& \tau_\theta \\ \ddot{\phi} =& \tau_\phi \\ \ddot{\psi} =& \tau_\psi \end{aligned} \tag{19}$$

where the position of the center of gravity $\xi = [x\ y\ z]^{\mathrm{T}}$ and the attitude of the vehicle represented by the tree Euler angles (pitch, roll and yaw) $\eta = [\theta\ \phi\ \psi]^{\mathrm{T}}$ could be stabilized by the main trust u, the pitch moment τ_θ, the roll moment τ_ϕ and the yaw moment τ_ψ.

B. Control Strategy [19]

From Equation (19), and in order to assure the UAV stabilization and trajectory tracking of a desired position (x_d, y_d, z_d), the main trust and the three angular moments are given by

$$\begin{aligned} u =& -Kp_z(\hat{z} - z_d) - Kd_z(\dot{z}) \\ \tau_\theta =& -Kp_\theta(\theta) - Kd_\theta(\dot{\theta}) - (-Kp_x(\hat{x} - x_d) - Kd_x(\dot{x})) \\ \tau_\phi =& -Kp_\phi(\phi) - Kd_\phi(\dot{\phi}) - (-Kp_x(\hat{y} - y_d) - Kd_y(\dot{y})) \\ \tau_\psi =& -Kp_\psi(\psi) - Kd_\psi(\dot{\psi}) \end{aligned} \tag{20}$$

where Kp_x, Kp_y, Kp_z, Kp_θ, Kp_ϕ, Kp_ψ, Kd_x, Kd_y, Kd_z, Kd_θ, Kd_ϕ and Kd_ψ are constants and $(\hat{x}, \hat{y}, \hat{z})$ is the estimated position, which will be obtained by each of the positioning algorithms explained in this work. Due to the dynamics shown in (19) it is possible to see that the movements on the axis X and Y depends respectively on the angles θ and ϕ, so the control of position need to be applied by τ_θ and τ_ϕ.

IV. SIMULATION SETTINGS

In order to evaluate and compare each positioning algorithm presented in the previous section, a simulation platform was developed. In this platform a mobile station representing the UAV with a mass of 1 kg moves randomly in an area with dimensions of 30×30 m and with a velocity of 3 m/s. At the beginning of the simulation the UAV is positioned at the center of the area and the direction of movement is selected in a random way varying each time that the UAV reaches the border of the area.

We deployed a total of 16 anchors in a square grid array where the anchors are space 20 m apart each other obtaining a maximum distance of 28 m between anchors in the opposite corner. Fig. 2 shows the scenario of simulation described.

The time of simulation is fixed to 15 minutes since that it is the typical operation time for this type of vehicles. The algorithms compute the position estimation each 100 ms, that means, during the time of simulation the position of the UAV is estimated 9,000 times by each algorithm simultaneously.

Regarding the estimated distances, we are considering to use devices based on UWB technology, which, as was commented in Section I, present very accuracy results. Thus, all algorithms use the same estimated distances provided by the four nearest anchors to the UAV. The errors of these estimations are limited by a standard deviation of 0.1 m due that this is the common error specified by UWB devices [20].

For the case of iterative algorithms like the Gauss-Newton and the Quasi-Newton, we firstly run them with 50 iterations. Both algorithms exhibited convergence in less than 20 iterations, thus, this limit was used for the rest of simulations.

In addition, a simulation of the trajectory tracking with the same parameters above presented was developed where the UAV uses each of the algorithm studied to estimate his position and follow a trajectory of reference. Due to the objective of

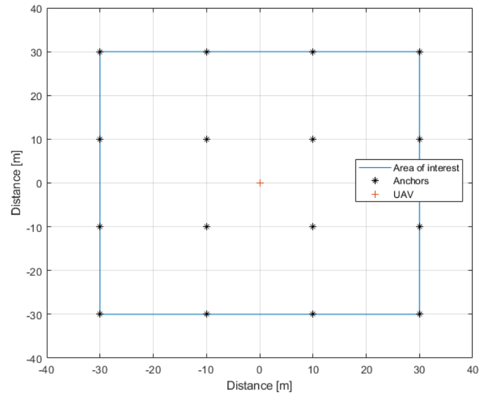

Fig. 2. Simulation scenario where a UAV flies randomly in an area of 30×30 m with a square grid array of anchors.

this work is to compare the precision of the algorithms to estimate the position of the UAV in the plane XY (the altitude can be estimated by the UAV using other sensors such as barometer), then, the UAV altitude and the yaw angle will be stabilized at 1.5 m (same for the anchors) and 0°, respectively. The resulting values for the constants of the control strategy given in Section III are determined by repetitive tests and are shown in Table I.

TABLE I
CONSTANTS OF THE PD CONTROLLER USED IN THE TRAJECTORY TRACKING

	Position			Attitude		
Proportional	Kp_x	Kp_y	Kp_z	Kp_θ	Kp_ϕ	Kp_ψ
	0.13	0.71	2.6	7.74	15.5	1.1
Derivative	Kd_x	Kd_y	Kd_z	Kd_θ	Kd_ϕ	Kd_ψ
	0.54	1.16	3.6	4.7	7.2	2.3

V. RESULTS

In this section, the results obtained by making 1000 simulation runs with the settings previously described are presented. In order to evaluate the accuracy of the estimated position generated by each algorithm, the error of estimation is calculated as the difference between the actual and the estimated position without consider the UAV dynamics. Once time the accuracy of each algorithm is analyzed, then, the UAV performs a trajectory tracking based on the estimated position provided by each algorithm in order to analyze and compare the accuracy with the UAV dynamics.

A. Comparison of the Position Accuracy without UAV Dynamics

In order to compare the estimation errors presented by each algorithm, the MSE determined by (3), mean μ and standard deviation σ of the errors obtained by the histograms generated from 900,000 estimations for each algorithm are shown in Table II. The histograms of the estimation errors resulted by each algorithm are shown in Fig. 3-6.

TABLE II
STATISTICS OF ESTIMATED ERRORS AND MSE FOR THE STUDIED ALGORITHMS

Algorithms	Mean (μ) [cm]	Standard deviation (σ) [cm]	MSE [m^2]
Radical axis	7.26	5.38	1.143
Maximum likelihood	7.66	6.15	1.481
Gauss-Newton	6.91	5.02	0.936
Quasi-Newton	6.91	5.04	0.940

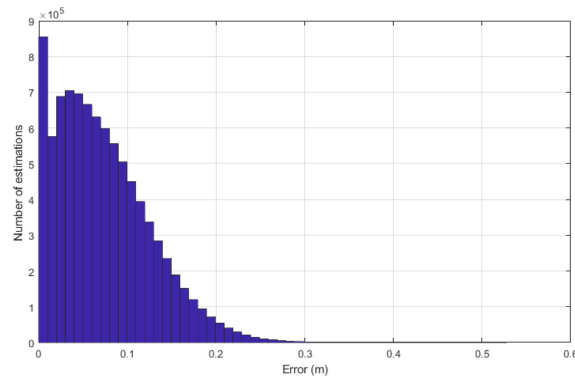

Fig. 3. Histogram of the estimation error for the radical axis algorithm.

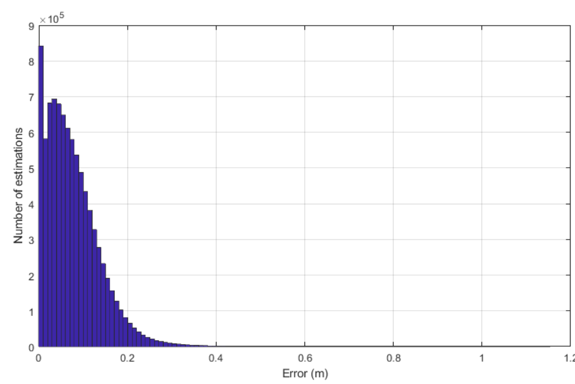

Fig. 4. Histogram of the estimation error for the maximum likelihood algorithm.

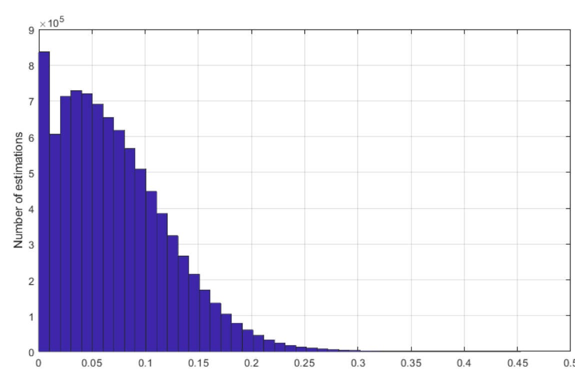

Fig. 5. Histogram of the estimation error for the Gauss-Newton algorithm.

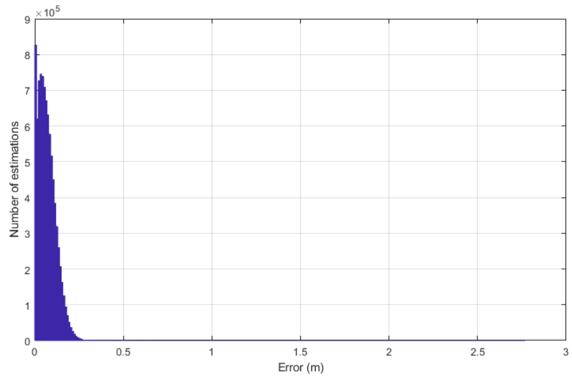

Fig. 6. Histogram of the estimation error for the Quasi-Newton algorithm.

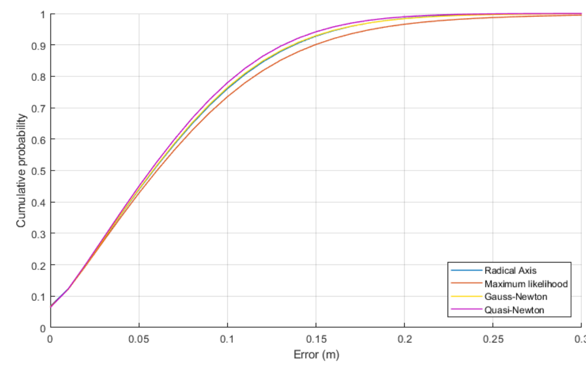

Fig. 7. CDF of the estimation error obtained for each positioning algorithm.

It is interesting to observe that the accuracy of the algorithms is very similar, particularly the Gauss-Newton method and the Quasi-Newton algorithm whose values of mean, standard deviation and MSE are very close (see Table II). We can observe in Fig. 3 that the radical axis algorithm has the lowest range of errors with a maximum value of 0.52 m, however, the dispersion of its values is slightly higher than those of Gauss-Newton method and the Quasi-Newton algorithm resulting in a greater mean and standard deviation. The histogram of the maximum likelihood algorithm shown in Fig. 4 presents a maximum value of 1.15 m, in addition, compared with the other algorithms, the maximum likelihood algorithm has the greatest mean and standard deviation. On the other hand, the histograms of the Gauss-Newton method and the Quasi-Newton method shown respectively in Figs. 5 and 6 have the best values of mean and standard deviation despite of having a maximum value of 0.45 m and 2.76 m. Fortunately, these errors are observed in a very low percentage of the estimations (the worst case presented only 124 estimations with errors greater than 1 m, from a universe of the 9,000,000 estimations).

On the other hand, the simulation results are represented by a cumulative distribution function (CDF) of probability of error in terms of the estimation error as can be seen in Fig. 7. The probability that the error is less than 10 cm is around 68.48% for the maximum likelihood algorithm, 70.90% for the radical axis algorithm, 71.16% for the Gauss-Newton method and 72.66% for the Quasi-Newton algorithm showing a maximum difference around of 4.18%. On the other hand, the probability that the estimation error is less than 0.3 m is of 99.35% for maximum likelihood algorithm, 99.81% for the Quasi-Newton algorithm, 99.88% for the Gauss-Newton method and 99.93% for the radical axis algorithm.

From the obtained results we observe that the accuracies are very similar. Hence, we can state that each algorithm could be used to estimate the UAV position based on the estimated distances from it to anchors whose positions are known. The choice about what algorithm to use, could not only depend of their accuracy but also other aspects as the computation complexity. For example, for the Gauss-Newton and Quasi-Newton algorithms, the continuous iterations represent an increment in the computational cost. On the other hand, for the radical axis method, the determination of the pseudoinverse of a matrix could be difficult into an embedded computer of a UAV. Regarding the maximum likelihood method, its estimation depends on the existence of solution for the system of equations which for large errors in the estimated distance could not always be possible reducing so the available information and consequently the accuracy.

B. Comparison of the Trajectory Tracking considering th UAV Dynamics

Let us now present results of the accuracy of the UAV trajectory tracking. When the UAV uses the estimated position generated by the algorithms studied in this paper, instead of the real position (unknown) to track a reference trajectory, it is possible to analyze and compare the accuracy of each algorithm when the UAV dynamics are considered. Figure 8 shows a reference trajectory generated in random way with the same conditions of simulation explained in Section IV.

In order to observe and compare the trajectory tracking performance obtained with the estimated position by each algorithm, in this paper we focus in analyzing the portion of the trajectory shown in red color in Figure 8 whose result is depicted in Figure 9. It is possible to observe that the accuracy of the algorithms has a great importance in the trajectory tracking. From the curves shown in this figure, it seems that the maximum likelihood algorithm has the poorest performance. In order to be more specific, the statistics (mean and standard deviation) and MSE of the error for the trajectory tracking were obtained for the trayectory portion of this example and whose results are shown in Table III. Thus, the maximum likelihood method presents indeed the largest error of position estimation, and hence largest error is introduced in the trajectory tracking as can be seen in Table III and graphically in Figure 9. On the other hand, provided that to the Gauss-Newton and Quasi-Newton methods have practically the same position accuracy we can observe a similar behavior in terms of the trajectory tracking. In contrast, the radical axis estimation presents a moderate error.

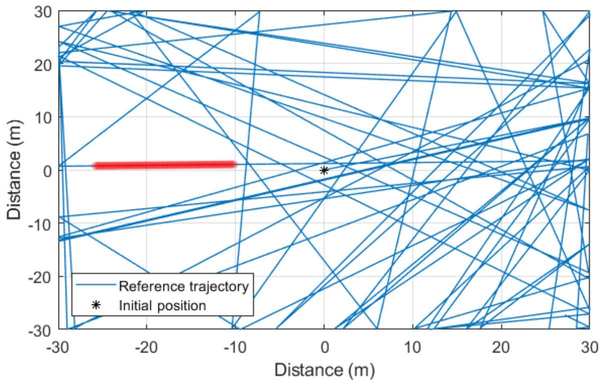

Fig. 8. Reference trajectory generated randomly in one of the simulation runs.

TABLE III
STATISTICS OF ESTIMATED ERRORS OF POSITION AND MSE PRESENTED IN A FLIGHT WITH TRAJECTORY TRACKING.

Algorithms	Mean (μ) [cm]	Standard deviation (σ) [cm]	MSE [m^2]
Radical axis	10.1	9.1	3.35
Maximum likelihood	11.4	10.3	4.44
Gauss-Newton	9.0	8.2	2.71
Quasi-Newton	9.1	8.2	2.72

Finally, by comparing results of Table II with those of Table III, we observe larger error values for the latter because in this case the UAV dynamics and PD controller are taken into account.

VI. CONCLUSION

In this paper, a comparison between four algorithms for position estimation to be used in unmanned aerial vehicles applications was presented. In order to achieve the above, we developed a simulation platform where each algorithm was implemented to estimate the position of a UAV under the same conditions (scenario free of obstructions and an evaluation simultaneous for all algorithms and without consider the UAV dynamics), demonstrating that any of them could be considered for position estimation in real time of these vehicles. However, in terms of accuracy the Gauss-Newton method and Quasi-Newton algorithm have the smallest mean and standard deviation which are comprehensible taking in mind that both find the optimal position minimizing the mean squared errors. For that reason, in terms of accuracy, it is recommendable the use of the Gauss-Newton method or the Quasi-Newton algorithm for estimation position purposes. With that in mind, a trajectory tracking with PD controllers using the estimations of the position by the different algorithms studied in this work was analyzed. From the found results it was demonstrated the influence that the accuracy of the position estimation has in the UAV navigation. In this context, the best results were obtained with the iterative methods (Gauss-Newton and Quasi-Newton) provided that they presented the smallest errors. Finally, it is worth pointing out that all these methods will be analyzed for shadowing and multipath propagation environments. The obtained results will be reported in a further article.

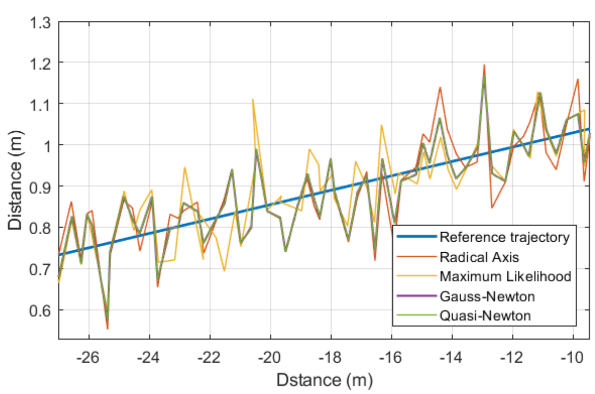

Fig. 9. Trajectory tracking of the UAV using the estimated position by each algorithm during a portion of the simulation run.

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