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H_∞ Control of Switched Affine Systems with Single Delay: a Lyapunov-Krasovskii Approach with Practical Applications

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***Abstract*—This paper is on the robust stabilization of switched affine systems with time-delay. The class of systems under analysis is mainly motivated by the energy management of fuel-cell based power applications. Here, using Lyapunov-Krasovskii functionals, we propose a state-dependent switching controller, which ensures global exponential stability and predefined H_∞ performance guarantees for the system. Effectiveness of the approach is demonstrated via two practical applications. In the first one, the designed switched policy is able to regulate the output voltage of a fuel-cell system despite unknown exogenous signals. In the second one, the switched strategy guarantees avoidance of congestion in a communication network in spite of noise in the measurements.**

***Index Terms*—Time delay systems, switched affine systems, switched control, H_∞ control.**

I. Introduction

A switched system comprises of a family of subsystems and a switching signal. Each subsystem of the family can be interpreted as a mode of operation of the switched system and for every time instant, only one mode can be activated by the switching signal.

Switched systems have attracted tremendous attention in the last decades due to their potential for describing a wide variety of real-world problems in engineering, economics and biology [1]–[4]. Examples can be found in renewable energy systems [5], bacterial growth [6], and communication networks [7]. The literature is rich with a broad range of results on the stability analysis of switched systems, which is commonly assessed by following a Lyapunov framework based on how the system dissipates energy along its trajectories [4], [8]–[13].

Many control systems are influenced by delays, mainly because it takes time to measure, sense, decide, or transmit information among their components [14]. On the one hand, delays are usually associated with deterioration of system's performance and, regardless of the amount of delay, ignoring them without careful consideration of time scales in the system may result in loss of stability [15], [16]. On the other hand, the presence of delay may be beneficial to the dynamics. Indeed, an appropriate amount of delay can increase stability margins, can be deliberately introduced as part of a controller to enhance the performance of a closed loop system, and can help recover stability [17]–[22]. While switched systems offer numerous opportunities for regulating a diverse range of practical applications, their analysis is not trivial and the presence of delays further complicates the challenges by inducing infinite-dimensionality in the model [23], [24].

In this paper, we focus on an important class of switched systems without common equilibria among their modes of operation, frequently referred in the literature to as switched affine systems [25]. In engineered switched affine systems, one of the main interests is to design an appropriate switching rule able to drive the trajectories of the system towards an admissible point of operation. In many applications, e.g. consensus problems in multi-agent systems [26], [27], the goal is not only to reach the operating point but also to do so as fast as possible. Yet, plant uncertainties, measurement errors, and the presence of delays limits one's ability to fulfill this control objective. However, to the best of our knowledge, a robust switching rule for switched affine systems at the interplay between delays, stability, and disturbances has never been investigated in the literature; and some advancements in this direction are reported only recently in the context of bilinear systems and congestion control considering fixed and variable time delays [28]–[30].

In view of the above rationale, in this manuscript, we synthesize a switching rule adopting a Lyapunov-Krasovskii approach, which ensures global exponential stability of the switched system. The design reduces to solving a simple linear matrix inequality (LMI) using classical convex optimization tools. In contrast to [28]–[30], the obtained switching rule is proven to robustly stabilize, with prescribed performance guarantees, a general class of switched affine time-delay systems against exogenous noise signals, disturbances and delays. The obtained switching policy is then implemented to regulate the output voltage of a fuel-cell system and to avoid congestion problems in a communication network.

Notation: The notation is standard. For a real square matrix S, $S > 0$ $(S < 0)$ means that S is symmetric and positive definite (negative definite), and the symbol $\bullet$ denotes the symmetric blocks. The $\mathrm{vec}(S)$ operator creates a column vector from matrix S by stacking its columns. For a given positive integer N, the index set $\mathbb{K}$ is defined as $\mathbb{K} = \{1, ..., N\}$. For $s_i \in \mathbb{R}$, we use $\arg\min_{i\in\mathbb{K}}\{s_i\}$ to denote any index $j \in \mathbb{K}$ such that $s_j = \min_{i\in\mathbb{K}}\{s_i\}$. The space of all square integrable and Lebesgue measurable functions on the interval $[0, \infty)$ is denoted by $\mathcal{L}_2$. The superscript $\top$ indicates transpose.

II. Problem formulation

We consider a retarded type switched affine system with single delay of the form

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma x(t-r) + H_\sigma w(t) + a_\sigma, \tag{1}$$
$$z(t) = Cx(t) + Dx(t-r) + G_\sigma w(t), \tag{2}$$

with initial function $x(\tau) = 0$ for all $\tau \in [-r, 0)$. The switching function $\sigma : \mathbb{R} \to \mathbb{K}$ decides which of the N modes of operation is active, $x(t) \in \mathbb{R}^n$ is the instantaneous state, $z(t) \in \mathbb{R}^m$ is the controlled output with $0 < m \le n$, $w \in \mathcal{L}_2$ with $w(t) \in \mathbb{R}^p$ is an exogenous signal that may include sensor noise or disturbances, $r \ge 0$ is a constant time-delay and $A_i, B_i, H_i, a_i, C, D, G_i$ are constant matrices of appropriate dimensions for each $i \in \mathbb{K}$. We assume that the full state $x(t)$ is available for feedback.

Let $z_d \in \mathbb{R}^m$ be a constant reference for the output $z(t)$. Thus, for any z_d there exists a constant vector $x_e \in \mathbb{R}^n$ such that $z_d = (C+D)x_e$. Hence, we define the output error

$$z_e(t) = z(t) - z_d\,,$$

where $z_d = (C+D)x_e$. Before presenting the problem formulation, we state the following definition.

Definition 1: Given a scalar $\alpha > 0$, we define the performance index

$$J(w) = \int_0^\infty \left(z_e^\top z_e - \alpha^2 w^\top w\right) dt, \tag{3}$$

to measure the attenuation of exogenous signals. Hereafter, α is referred to as the error attenuation, see [14, Section 4.3.1].

Then, the problem we wish to address is:

Problem 1: Find a switching strategy guaranteeing that system (1)-(2) is internally exponentially stable, i.e. exponentially stable for $w = 0$, and $J(w) < 0$ for all non-identically zero $w \in \mathcal{L}_2$.

It is worth mentioning that Problem 1 may be understood as an H_∞ control problem in the sense that we require the minimization of the functional J with respect to the inputs $w \in \mathcal{L}_2$, see [14, Section 4.3.1]. Further, Problem 1 is to be solved without any assumptions on the structure of system matrices.

III. Main result

Aiming at solving Problem 1, let us introduce the error vector $e(t) = x(t) - x_e$, then from (1)-(2) we obtain

$$\dot{e}(t) = A_\sigma e(t) + B_\sigma e(t-r) + H_\sigma w(t) + b_\sigma, \tag{4}$$
$$z_e(t) = Ce(t) + De(t-r) + G_\sigma w(t), \tag{5}$$

where b_σ is the affine term defined by

$$b_\sigma = (A_\sigma + B_\sigma)\, x_e + a_\sigma\,.$$

In this setting, Problem 1 is equivalent to finding σ such that $e \to 0$ exponentially fast with $w = 0$. However, since $b_\sigma \ne 0$ in general, we have that $e = 0$ is not necessarily an equilibrium of (4). On the other hand, assuming that $b_i = 0$ for all $i \in \mathbb{Z}$ imposes strong restrictions on the system's structure and hence, in this investigation we let $b_\sigma \ne 0$. In other words, we assume that x_e is not necessarily an equilibrium for all of the operation modes of (1), but it is an *admissible equilibrium* of (1) in the sense described in [29]. To further clarify the above, we introduce the following definitions.

Definition 2: Let the simplex Λ be defined by

$$\Lambda = \left\{ \lambda \in \mathbb{R}^N \,:\, 0 \le \lambda_i \le 1,\ \sum_{i=1}^N \lambda_i = 1 \right\}.$$

Then, we say that a convex combination S_λ of the $n \times m$ real matrices $S_1, \ldots, S_N$ is given by

$$S_\lambda = \sum_{i=1}^N \lambda_i S_i\,, \quad \lambda \in \Lambda\,.$$

Definition 3: We say that x_e is an *admissible equilibrium* of (1) if $x_e \in X_e$, where

$$X_e = \{x_e \in \mathbb{R}^n \,:\, (A_\lambda + B_\lambda)x_e + a_\lambda = 0,\ \lambda \in \Lambda\}\,.$$

According to [29, Lemma 1], if for a given switching function σ and a given x_e we have that $e(t) \to 0$ as $t \to \infty$, then there exists $\lambda \in \Lambda$ such that $(A_\lambda + B_\lambda)x_e + a_\lambda = 0$. Thus, x_e being an admissible equilibrium is a necessary condition to solve Problem 1.

Before introducing the main Theorem, to simplify the presentation, let us define the following matrices:

$$M_i = \begin{bmatrix} A_i^\top P + PA_i + Q + 2\gamma P & \bullet & \bullet \\ B_i^\top P & -e^{-2\gamma r}Q & \bullet \\ H_i^\top P & 0 & -\alpha^2 I \end{bmatrix} + \begin{bmatrix} C^\top \\ D^\top \\ G_i^\top \end{bmatrix}\begin{bmatrix} C^\top \\ D^\top \\ G_i^\top \end{bmatrix}^\top, \tag{6}$$

$$N_i = \begin{bmatrix} A_i^\top P + PA_i + Q + 2\gamma P & \bullet \\ B_i^\top P & -e^{-2\gamma r}Q \end{bmatrix} + \begin{bmatrix} C^\top \\ D^\top \end{bmatrix}\begin{bmatrix} C^\top \\ D^\top \end{bmatrix}^\top + \begin{bmatrix} (H_i^\top P + G_i^\top C)^\top \\ (G_i^\top D)^\top \end{bmatrix}\left[\alpha^2 I - G_i^\top G_i\right]^{-1}\begin{bmatrix} (H_i^\top P + G_i^\top C)^\top \\ (G_i^\top D)^\top \end{bmatrix}^\top, \tag{7}$$

for all $i \in \mathbb{K}$, where the zero and identity matrices must be taken of adequate dimensions.

Theorem 1: Consider the error system (4)-(5), and let x_e be an admissible equilibrium of (1) with some $\lambda \in \Lambda$. If there exist positive definite matrices P and Q, and scalars $\gamma > 0$ and $\alpha > 0$ satisfying

$$\begin{bmatrix} A_\lambda^\top P + PA_\lambda + Q + 2\gamma P & \bullet & \bullet & \bullet \\ B_\lambda^\top P & -e^{-2\gamma r}Q & \bullet & \bullet \\ H_\lambda^\top P & 0 & -\alpha^2 I & \bullet \\ C & D & G_\lambda & -I \end{bmatrix} < 0\,, \tag{8}$$

then, the switching rule

$$\sigma(t) = \arg\min_{i \in \mathbb{K}} \zeta(t)^\top [N_i \zeta(t) + 2\,\mathrm{vec}(Pb_i, 0)]\,, \tag{9}$$

with $\zeta(t) = [e^\top(t),\ e^\top(t-r)]^\top$, guarantees that:

1) $e = 0$ is globally exponentially stable with a decaying rate γ for $w \equiv 0$;
2) $J < 0$ for any non-identically zero $w \in \mathcal{L}_2$.

Proof: The proof is given in Section IV. ∎

Remark 1: Observe that the switching rule (9) is state-dependent. It then follows that both (1) and (4) are retarded-type differential equations with discontinuous right-hand side and hence, the standard definition of solution is not applicable in this particular problem. Therefore, concepts of generalized solutions for this class of systems are required. In this paper, we adopt the concept of solution provided in [31] (see also [32, p. 28]).

Remark 2: Observe that in general (8) is a bilinear matrix inequality, but by fixing the value of γ it becomes a standard linear matrix inequality.

IV. Proof of Theorem 1

Consider the Lyapunov-Krasovskii functional

$$v(e_t) = e^{\top}(t)Pe(t) + \int_{t-r}^{t} e^{\top}(s)e^{2\gamma(s-t)}Qe(s)ds\,, \qquad (10)$$

where the function e_t is given by $e(\tau)$ for all $\tau \in [t-r, t]$. Observe from Remark 1 that e is an absolutely continuous function of time. Moreover, the smoothness properties of (10) guarantee that $v(e_t)$ is also an absolutely continuous function of time, hence, its time derivative exists almost anywhere. Therefore, standard Lyapunov-Krasovskii stability theorems are still valid for our case [33]. In order to achieve exponential stability, it is sufficient to show that $\dot{v}(e_t) + 2\gamma v(e_t) < 0$. Hence, we start by taking the derivative of the functional (10) along the trajectories of the error system (4):

$$\dot{v}(e_t) = 2e^{\top}(t)P\dot{e}(t) + e^{\top}(t)Qe(t) - e^{\top}(t-r)e^{-2\gamma r}Qe(t-r) - 2\gamma\int_{-r}^{0} e^{\top}(t+\theta)e^{2\gamma\theta}Qe(t+\theta)d\theta\,, \qquad (11)$$

where we have used the chance of variable $\theta = s - t$ into the integral. Let $\xi(t) = \left[e^{\top}(t), e^{\top}(t-r), w^{\top}(t)\right]^{\top}$, then (11) is rewritten as

$$\dot{v}(e_t) + 2\gamma v(e_t) = \xi^{\top}M_{\sigma}\xi + 2e^{\top}Pb_{\sigma} - z_e^{\top}z_e + \alpha^2 w^{\top}w, \quad (12)$$

by adding and subtracting the term $2\gamma e^{\top}Pe + z_e^{\top}z_e + \alpha^2 w^{\top}w$ where M_{σ} is of the form (6). To continue with the proof we require the following lemma:

Lemma 1: The following holds

$$\sup_{w\in\mathcal{L}_2} \xi^{\top}M_{\sigma}\xi \leq \zeta^{\top}N_{\sigma}\zeta\,,$$

where $\zeta(t) = [e^{\top}(t),\ e^{\top}(t-r)]^{\top}$.

Proof: The proof follows by noticing that regarding $\xi^{\top}M_{\sigma}\xi$ as a function of w, it exhibits a maximum at

$$w = \Omega_1^{-1}\Omega_2^{\top}\ e(t) + \Omega_1^{-1}\Omega_3^{\top}\ e(t-r), \qquad (13)$$

where $\Omega_1 = \alpha I - G_{\sigma}^{\top}G_{\sigma}$, $\Omega_2 = PH_{\sigma} + C^{\top}G_{\sigma}$ and $\Omega_3 = D^{\top}G_{\sigma}$. ■

Then, from Lemma 1 and (12) we have that

$$\dot{v}(e_t) + 2\gamma v(e_t) \leq u_{\sigma}(\zeta) - z_e^{\top}z_e + \alpha^2 w^{\top}w\,, \qquad (14)$$

where

$$u_{\sigma}(\zeta) = \zeta^{\top}N_{\sigma}\zeta + 2\zeta^{\top}\mathrm{vec}(Pb_{\sigma}, 0)\,. \qquad (15)$$

Let us now analyse the convex combination $u_{\lambda}(\zeta) = \zeta^{\top}N_{\lambda}\zeta + 2\zeta^{\top}\mathrm{vec}(Pb_{\lambda}, 0)$. First note that performing the Schur complement in (8) with respect to the fourth block-row and block-column, and repeating the process into the result with respect to the third block-row and block-column, we obtain the fact that $N_{\lambda} < 0$. Second, since x_e is an admissible equilibrium we have that $b_{\lambda} = 0$, hence $u_{\lambda}(\zeta) < 0$ for all $\zeta \neq 0$. Third, since

$$u_{\lambda}(\zeta) = \sum_{i\in\mathbb{K}} \lambda_i u_i(\zeta) < 0\,,$$

and $\lambda_i \geq 0$ for all $i \in \mathbb{K}$, it follows that there exists at least one $i \in \mathbb{K}$ for which $u_i(\zeta) < 0$. Therefore, adopting the switching rule

$$\sigma(t) = \arg\min_{i\in\mathbb{K}} u_i(\zeta), \qquad (16)$$

(note that (9) follows from (15) and (16)) we have from (14) that

$$\dot{v}(e_t) + 2\gamma v(e_t) \leq \min_{i\in\mathbb{K}} u_i(\zeta) - z_e^{\top}z_e + \alpha^2 w^{\top}w < -z_e^{\top}z_e + \alpha^2 w^{\top}w. \qquad (17)$$

Hence, $e = 0$ is exponentially stable whenever $w \equiv 0$. Now, from (17) we have that $\dot{v}(e_t) < -z_e^{\top}z_e + \alpha^2 w^{\top}w$. Thus, by integrating both sides of this inequality from 0 to ∞, and given that $v(e_0) > 0$ and $\lim_{t\to\infty} v(e_t) = 0$, we obtain

$$J(w) < 0 \qquad (18)$$

for any non-identically zero $w \in \mathcal{L}_2$, which completes the proof.

Remark 3: Notice in the above proof that the exponential stability of the system is ensured in the absence of exogenous perturbations; e.g. with $w = 0$. On the other hand, for any arbitrary finite energy signal $w \in \mathcal{L}_2$ we guarantee that the performance index $J(w)$ is strictly negative. That is, the design of $\sigma(t)$ in (9) effectively solves Problem 1.

V. Case studies

In the sequel we test the above developments via numerical simulations. To this end, we present the output voltage regulation of a fuel cell system and the congestion control of a communication network. The LMI in Theorem 1 is solved in Matlab using SeDuMi [34] and Yalmip [35].

A. Output voltage regulation of a fuel-cell system

Fuel-cell systems comprise, in general, of three main components: 1) a hydrogen fuel-cell stack, 2) a switch-mode dc-dc power converter, and 3) electrical loads, see Fig. 1. Since fuel-cells are low-voltage generators, the objective of the power stage is to increase the output voltage to an appropriate level for the load. Hence, a boost cd-cd converter is employed. From Fig. 1, one can see that the switching signal $\sigma \in \{1, 2\} = \mathbb{K}$ is used to determine whether the fuel-cell stack is connected to the load or not. Further, we consider that the input voltage v_{in} is given by

$$v_{in}(t) = E_{in} + w(t), \qquad (19)$$

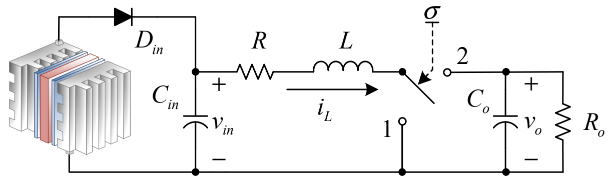

Fig. 1. Fuel-cell system composed by a hydrogen fuel-cell stack, a boost cd-cd converter and a resistive load. Rated output power $P_o = 853.33$ W.

TABLE I
NOMINAL VALUES OF THE CONVERTER'S PARAMETERS

Nominal load, R_o	2.7	Ω
Output capacitor, C_o	136	μF
Input capacitor, C_{in}	5600	μF
Inductor, L	85	μH
ESR of inductor, R	10	mΩ

where $E_{in} = 24$ V is the nominal output voltage of the fuel-cell stack, and $w(t)$ is an unknown voltage variation associated with changes in fuel-cell's temperature and humidity variations of the environment [36].

Adopting the inductor current i_L and the output capacitor voltage v_o as state variables, namely $x = [i_L, v_o]$, and under the above considerations, the fuel-cell system may be written in the form of (1) where B_i, D and G_i are zero for all $i \in \mathbb{K}$, $C = [0, 1]$, and

$$A_1 = \begin{bmatrix} -R/L & 0 \\ 0 & -1/(R_oC_o) \end{bmatrix}, \; A_2 = \begin{bmatrix} -R/L & -1/L \\ 1/C_o & -1/(R_oC_o) \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}, \; H_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \text{ and } a_1 = a_2 = \begin{bmatrix} E_{in}/L \\ 0 \end{bmatrix}.$$

The nominal values of the converter's parameters are listed in Table I. Following [9], we have scaled the matrices $\{A_i, H_i, a_i\}$ by a factor of 0.01. Then, choosing an exponential decay $\gamma = 0.01$ and a desired equilibrium point

$$x_e = [35.928514, 48]^\top \in X_e, \tag{20}$$

which is admissible with $\lambda = [0.507520, 0.492480]^\top$, we have that (8) holds with

$$P = \begin{bmatrix} 0.024639 & -0.008935 \\ -0.008935 & 0.039216 \end{bmatrix}, \; Q = \begin{bmatrix} 0.002631 & 0.002606 \\ 0.002606 & 0.002642 \end{bmatrix},$$

and $\alpha = 1.7603$. Equipped with P and Q, we next implement the switching strategy σ in (9) to regulate the output voltage of the fuel-cell system.

The result[1] shown in Fig. 2 top panel, demonstrates that the trajectories of the system remain regulated despite external signals suddenly appearing at $t \in [2, 3]$. Indeed, for $w(t) = \sin(16\pi t)$ the computed cost $J(w) = -1.456089$ is strictly negative. Finally, note from the bottom panel of Fig. 2 that, the average value of σ corresponds to a duty cycle of 0.5 indicating that the input voltage is scaled by a factor of two, namely, $v_o = 48$ V.

[1]The system trajectories are obtained using the Euler integration method coded in Matlab with a fixed step of 100 μs. It is worthy of mention that a more sophisticated fourth order Runge-Kutta method yields the same result.

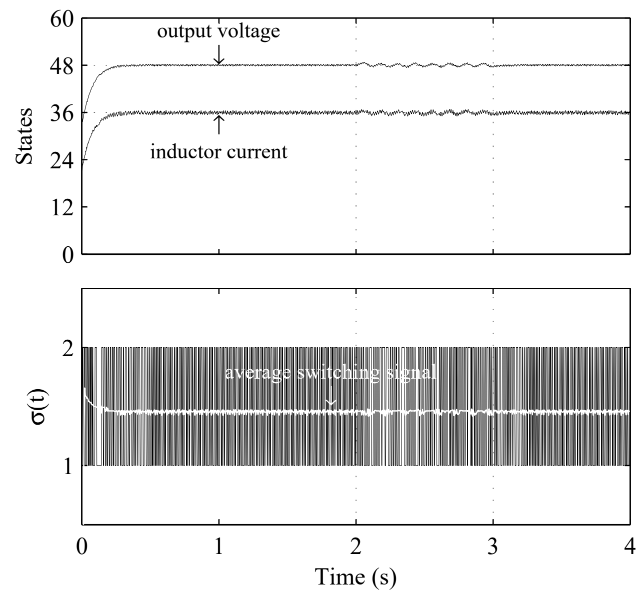

Fig. 2. Fuel-cell system response using switching rule $\sigma(t)$ in (9) at nominal output power $Po = 853.33$ W. The desired equilibrium point is computed as $x_e = [35.92, 48.00]^\top$ with $E_{in} = 24$ V and nominal load $Ro = 2.7$ Ω. A disturbance $w(t) = \sin(16\pi t)$ appears at time $t \in [2, 3]$. Data is down-sampled for clarity of display.

Remark 4: Notice that when B_i and D are zero, system (1)-(2) recovers the delay-free switched affine system studied in [37]. It is worthy of mention that the control approach proposed here is also able to guarantee both global exponential stability and H_∞ performance for the aforementioned delay-free systems.

B. Congestion control of a communication network

This example is inspired from [28], where the authors studied the problem of congestion control of a single bottleneck network model from the point of view of piecewise affine systems. Here, we revisit the problem from an H_∞ control perspective considering that the dynamics of both the amount of data stored at the bottleneck of the router $q(t)$ and the sending rate of data source $p(t)$ are given by

$$\dot{q}(t) = \begin{cases} p(t-r) - \mu, & q(t) > 0, \\ \max(0, p(t-r) - \mu), & q(t) = 0, \end{cases} \tag{21}$$
$$\dot{p}(t) = u(t), \tag{22}$$

where μ is the capacity of the bottleneck router and $r = 1$ is the average round trip delay. Adopting $x_1 = q - q_d$ and $x_2 = p - \mu$ as state variables, where q_d is the desired amount of data in the router, and considering the PD-like controller $u = -k_1x_1 - k_2x_2$, system (21)-(22) can be written[2] as in

[2]The reader is referred to [28] for details on how to embed system (21)-(22) into a switched piecewise affine system.

TABLE II
NOMINAL VALUES FOR THE CONGESTION CONTROL PROBLEM

Capacity of the router, μ	0.3
Average roundtrip delay, r	1 s
Desired amount of data, q_d	0.2
Proportional gain, k_1	1
Derivative gain, k_2	1

(1) with $\mathbb{K} = \{1, 2, 3\}$, $D = [0, 0]$, $G_1 = 0$, $G_2 = G_3 = 1$, $C = [1, 0]$, $a_1 = a_2 = [-q_d, 0]^\top$, $a_3 = [0, 0]^\top$, and

$$A_1 = A_2 = \begin{bmatrix} -1 & 0 \\ -k_1 & -k_2 \end{bmatrix}, \ A_3 = \begin{bmatrix} 0 & 0 \\ -k_1 & -k_2 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ B_2 = B_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, H_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

For the simulations, the nominal system parameters are listed in Table II. Choosing an exponential decay $\gamma = 0.01$ and a desired equilibrium

$$x_e = [-0.107692, 0.107692]^\top \in X_e, \tag{23}$$

which is admissible with $\lambda = [0.4, 0.3, 0.3]^\top$, condition (8) holds with

$$P = \begin{bmatrix} 17.651302 & 3.252260 \\ 3.252260 & 15.411501 \end{bmatrix}, \ Q = \begin{bmatrix} 3.928106 & 4.867708 \\ 4.867708 & 17.303440 \end{bmatrix},$$

and $\alpha = 4.0876$. Using P and Q, we next implement σ in (9) to solve the congestion problem.

The result in Fig. 3 shows that both the amount of data in the buffer $q = x_1 + q_d > 0$ and the sending rate of the data source $p = x_2 + \mu > 0$ reach the desired equilibrium point $x_e + [q_d, \mu]^\top = [0.092307, 0.407692]^\top$. The above may be clearly observed in Fig. 4 where the error trajectories decay exponentially fast towards the origin. Also, note that whenever p and q contain measurement noise w, see Fig. 3 at $t \in [8, 10]$, the index $J(w) = -4.323448$ confirms that the exogenous signal w is being effectively attenuated.

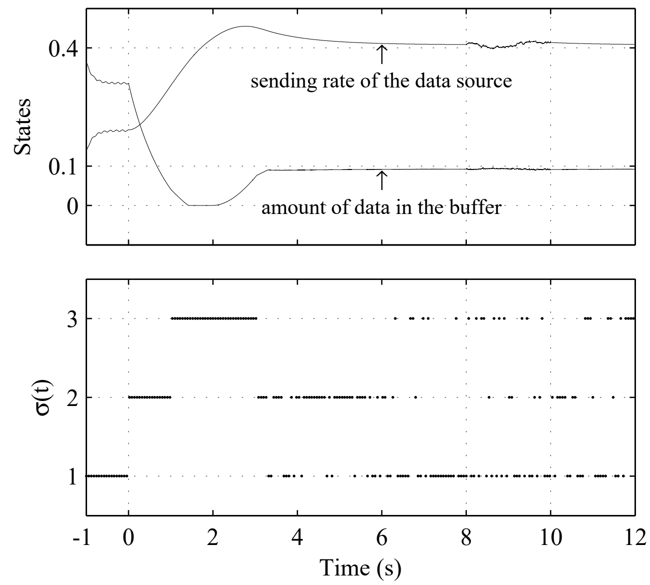

Fig. 3. Congestion control of a communication network using switching rule $\sigma(t)$ in (9). The desired amount of data in the buffer is set to $y_d = 0.2$. High frequency measurement noise $w(t)$ appears at time $t \in [8, 10]$. The top panel shows the states of the system. The bottom panel depicts the switching signal σ. Data is down-sampled for clarity of display.

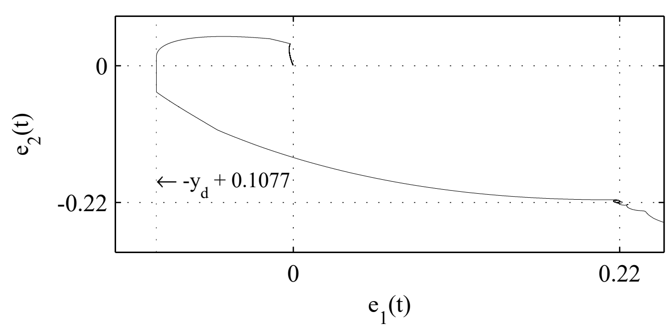

Fig. 4. Error trajectories of system (21)-(22) decaying exponentially fast towards the origin as ensured by Theorem 1.

VI. CONCLUSION

For a class of switched affine time-delay systems, a state-dependent switching policy is designed to minimize the effects of exogenous signals. A Lyapunov-Krasovskii framework is constructed to guarantee the exponential stability of the system at hand, ultimately landing in a global switching strategy that solves the H_∞ control problem. This strategy relies on both the feasibility of a simple LMI condition and the availability of the complete vector state. The study is complemented with simulation results validating the proposed approach by means of two practical applications demonstrating that, using the designed switching rule, the output voltage delivered to the power load in a fuel-cell system remains properly regulated despite unknown input voltage variations; and congestion problems can be avoided in a communication network in the presence of measurement noise.

Future research directions include the experimental verification of the approach, the consideration of time-delay in the switching signal, and extending the result to cover parametric variations and actuator time-delays.

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