Velocity trajectory tracking control: an Adaptive Ohnishi’s Disturbance Observer approach

Luis Luna∗, Erick Asiain†, Rubén Garrido‡ and Mario Lopez§
Automatic Control Department
CINVESTAV-IPN
Mexico City, Mexico
Email: *jluna@cinvestav.mx, †easiain@ctrl.cinvestav.mx, ‡garrido@ctrl.cinvestav.mx, §mlopezc@ctrl.cinvestav.mx

Abstract—The online estimation of the input gain in servo systems through an adaptive algorithm is carried out to design a Proportional controller equipped with a Disturbance Observer proposed by K. Ohnishi, to counteract constant perturbations affecting a servo system while executing velocity trajectory tracking tasks. This work relaxes the condition of knowing exactly the value of the input gain through a Gradient update law plus projection, the latter precluding singularities in the control law. A stability analysis allows concluding that all the closed loop signals remain bounded and that the velocity tracking error converges to zero. The effectiveness of the proposed controller is assessed by performing real-time experiments on a laboratory prototype.

Index Terms—Disturbance observer, PD control, controller tuning, servo system, adaptive control.

I. INTRODUCTION

Motion control systems are of primary importance when used for velocity or position control in some applications such as assembly robots [1], mobile robots [2] and solar trackers [3], among other technologies.

The study of effective control methods that exploit the high-speed and high accuracy positioning/tracking performance of servo systems has been the subject of sustained interest for many years [4]. The main problems, especially in terms of velocity control, are the occurrence of changes in system parameters and the presence of disturbances [5]. Besides, there are several internal and external disturbances such as friction on the rotor and load changes, respectively, that potentially affect the output performance and stability of servo systems [6]. To face these problems, it is necessary to apply some control techniques to achieve high performance without requiring exact knowledge of the servo model and at the same time dealing with uncertainties and external disturbances.

The strategies applied to the servo systems for velocity control reported in the literature are the Proportional Integral Derivative (PID) Controllers [7], artificial neural networks [8], Sliding Mode control [9] and Adaptive controllers [10]. In the field of Active Disturbance Rejection Control (ADRC), the work done by [11] proposed a Generalized Proportional Integral (GPI) Observer is intended to control the DC motor velocity and current sharing of the parallel DC/DC buck converters. On the other hand, some works combine the previous techniques like [12], which studies an Adaptive-Fuzzy-PID control scheme equipped with a Disturbance Observer (DOB). The disturbance Observer (DOB) proposed by Ohishi, Ohnishi and Miyachi is another ADRC philosophy that has been applied mainly in motion control. The DOB has been one of the most widely used robust control tools since K. Ohnishi proposed it in 1983 [4]. In this scheme, input and output measurements and a nominal model of an unperturbed plant are used to reconstruct the disturbances. The disturbance estimate is then injected into the plant input to counteract the effects of the real disturbance [13]. In practically all the DOB schemes, the input gain of the plant under control is assumed exactly known. On the other hand, in some references, an adaptive mechanism is added to the DOB framework, but the servo parameters are not explicitly estimated [14], or when it is done, no arguments are given to guarantee closed-loop stability [15], [16].

The main contribution of this paper is the online estimation of the servo system input gain with an update law and its use in the design of the controller and the DOB. Exact knowledge of the input gain is not necessary, and it is assumed that only an upper and a lower bound of this term are known. Furthermore, it is shown that the proposed adaptive scheme guarantees that all the closed-loop system signals remain bounded, and the velocity error converges to zero in the case of constant disturbances.

This work is organized as follows. In Section II, the mathematical model of a servo system for velocity control is presented. In Section III, an adaptation law is obtained for estimating the servo system input gain, which is used in the design of the proposed controller. Section IV presents the stability proof of the closed-loop system. Finally, experiments in a test bed allow assessing the performance of the controller. The conclusions are given at the end of the work.

II. SERVO SYSTEM MODEL

The following equation gives the model of the servo system

\[ J \ddot{x} = ku + \rho \]

where \( J \) is the servo system inertia, \( k \) is a parameter associated with the motor torque constant and the gain of the amplifier driving the motor, \( \rho \) is a constant disturbance, the acceleration and velocity of the servo system are \( \dot{x} \) and \( x \) respectively,
and the control input voltage corresponds to \( u \). Model (1) is rewritten as

\[
\dot{x} = bu + d \\
b = k/J \\
d = \rho/J
\]

The parameter \( b > 0 \) is unknown and the disturbance \( d \) is constant.

### III. CONTROL LAW

Assume that \( b \) is known and let the control law \( u \) be

\[
u = \frac{1}{b} (K_p e + \dot{r} - \dot{d})
\]

where \( e \) is the velocity error, \( r \) is the reference, \( K_p \) is a positive constant, and \( \dot{d} \) is an estimate of \( d \), which is estimated by means of a DOB [14] (see Fig. 1) where \( \dot{d} \) is the disturbance estimate

\[
\dot{\hat{d}} = -\beta \dot{\hat{d}} + \beta (\dot{x} - bu)
\]

The estimator (5) is a first order low-pass filter with transfer function

\[
G_{DOB} = \frac{\beta}{s + \beta}
\]

with input \( \dot{x} - bu \) and output \( \dot{\hat{d}} \). Parameter \( \beta > 0 \) settles the DOB cutoff frequency. Replacing (3) into (5) produces

\[
\dot{\hat{d}} = -\beta \dot{\hat{d}} + \beta (-\dot{e} - K_p e + \dot{d})
\]

Consequently

\[
\dot{\hat{d}} + \beta \dot{\hat{d}} = -\beta K_p e
\]

Now, define the auxiliary variable \( z \) as

\[
z = \hat{d} + \beta e
\]

whose time derivative is

\[
\dot{z} = \dot{\hat{d}} + \beta \dot{e}
\]

Equating (8) and (10) yields

\[
\dot{z} = -\beta K_p e
\]

Therefore, an alternative for computing \( \hat{d} \) is

\[
\hat{d} = z - \beta e
\]

\[
\dot{\hat{z}} = -\beta K_p e
\]

Substituting (12) into (3) permits obtaining

\[
u = \frac{1}{b} (K_p e + \dot{r} - z + \beta \dot{e})
\]

Now, assume that only an estimate \( \hat{b} \) of \( b \) is available. Hence, control law (14) is computed as:

\[
u = \frac{1}{\hat{b}} (K_p e + \dot{r} - z + \beta \dot{e})
\]

Adding and subtracting \( \hat{b} u \) in (2) leads to

\[
\dot{x} = \hat{b} u + d - \hat{b} u
\]

Replacing (15) into (16) leads to

\[
\dot{x} = K_p e + \dot{r} - z + \beta \dot{e} + d - \hat{b} u
\]

which is equivalent to

\[
\dot{e} + \lambda e = v + \hat{b} u
\]

\[
\lambda = \beta + K_p > 0
\]

\[
v = z - d
\]

### IV. STABILITY PROOF

Let the next Lyapunov function candidate be

\[
V = \frac{1}{2} e^2 + \frac{1}{2\gamma} \hat{b}^2 + \frac{1}{2\beta K_p} v^2
\]

Its time derivative corresponds to

\[
\dot{V} = e \dot{e} + \frac{1}{\gamma} \hat{b} \dot{\hat{b}} + \frac{1}{\beta K_p} \dot{v} \dot{v}
\]

The time derivative of \( \dot{v} \) is

\[
\dot{\dot{v}} = -\beta K_p e
\]

which is obtained from (13). Substituting (19) and (24) into (23) boils down to

\[
\dot{V} = -\lambda e^2 + \hat{b} \left[ \frac{1}{\gamma} \dot{\hat{b}} + \beta e \right]
\]

If the gradient law for estimating \( \hat{b} \) is defined as

\[
\dot{\hat{b}} = \hat{b} - \gamma \dot{\hat{e}}
\]

then (25) finally becomes

\[
\dot{V} = -\lambda e^2
\]

It is clear from (27) that \( V_0 = V(0) \geq V \) and \( e, v \) and \( \dot{\hat{b}} \) are bounded so do \( x \) and \( z \). Consequently, as long as \( \hat{b} \neq 0 \) is fulfilled, \( \dot{e} \) and \( u \) remain bounded. Using Barbalat’s lemma [17] permits proving convergence of \( e \) to zero. To this end, the integration of (27) with respect to time leads to

\[
V - V_0 = -\lambda \int_0^t e^2(\tau)d\tau
\]
Since $V_0 \geq V$, it follows that
\[ \lambda \int_0^t e^2(\tau)d\tau = V_0 - V \leq 2V_0 \] (29)
which boils down to
\[ \int_0^t e^2(\tau)d\tau \leq \frac{2V_0}{\lambda} < \infty \] (30)
From (30) and the boundedness of $e$ and $\dot{e}$, it is concluded that $e$ converges to zero.

It is worth noting that there exists a singularity in the control law (15) when $\dot{b} = 0$, i.e. a division by zero. A way to face this problem is by implementing a parameter projection in the adaptation law (26).

Define another auxiliary variable as
\[ \zeta = -ue \] (31)
which enables (26) to be rewritten as
\[ \dot{\hat{b}} = \gamma \zeta \] (32)
Besides, let $\hat{\Psi} \subset \mathbb{R}$ be a convex set such that $b \in \Psi \subset \hat{\Psi}$ where $\Psi$ is defined as
\[ \Psi = \{ b \mid 0 < b_{\min} \leq b \leq b_{\max} < \infty \} \] (33)
Here $b_{\min}$ and $b_{\max}$ are respectively the lower and upper bounds of the parameter $b$. Define also
\[ \Psi_e = \{ b \mid b_{\min} - \epsilon \leq b \leq b_{\max} + \epsilon \} \] (34)
Constant $\epsilon > 0$ is selected to fulfill $\text{Pr}(\gamma \zeta)$ as [18]
\[
\text{Pr}(\gamma \zeta) = \begin{cases} 
\gamma \zeta, & \text{if } b_{\min} \leq \dot{b} \leq b_{\max} \\
\gamma \zeta, & \text{if } \dot{b} > b_{\max} \text{ and } \zeta \leq 0 \\
\gamma \zeta, & \text{if } \dot{b} < b_{\min} \text{ and } \zeta \geq 0 \\
\gamma \zeta, & \text{if } \dot{b} > b_{\max} \text{ and } \zeta > 0 \\
\gamma \zeta, & \text{if } \dot{b} < b_{\min} \text{ and } \zeta < 0
\end{cases} \] (35)
The terms $\tilde{\zeta}$ and $\hat{\zeta}$ are defined as
\[
\tilde{\zeta} = \left[ 1 + \frac{b_{\max} - \dot{b}}{\epsilon} \right] \zeta, \quad \hat{\zeta} = \left[ 1 + \frac{\dot{b} - b_{\min}}{\epsilon} \right] \zeta
\]
Choosing $b_{\min} - \epsilon > 0$ ensures $\hat{b} \neq 0 \forall b \in \Psi_e$. Wherefore the adaption law (32) can be written as
\[
\dot{\hat{b}} = \text{Pr}(\gamma \zeta), \quad \zeta = -ue
\] (36)
Using the adaption law (36) does not modify the previous stability result. To verify this fact the term $\gamma \zeta$ is decomposed as
\[ \gamma \zeta = \text{Pr}(\gamma \zeta) + (\gamma \zeta)_\perp \] (37)
where
\[
(\gamma \zeta)_\perp = \begin{cases} 
0, & \text{if } b_{\min} \leq \dot{b} \leq b_{\max} \text{ or } \dot{b} > b_{\max} \text{ and } \zeta \leq 0 \text{ or } \dot{b} < b_{\min} \text{ and } \zeta \geq 0 \\
\left[ \frac{\dot{b} - b_{\max}}{\epsilon} \right] \gamma \zeta, & \text{if } \dot{b} > b_{\max} \text{ and } \zeta > 0 \\
\left[ \frac{b_{\min} - \dot{b}}{\epsilon} \right] \gamma \zeta, & \text{if } \dot{b} < b_{\min} \text{ and } \zeta < 0
\end{cases}
\] (38)
From (38), one can prove that
\[
(\dot{b} - b)(\gamma \zeta)_\perp = b(\gamma \zeta)_\perp \geq 0
\] (39)
Substituting (36) into (25) and bearing in mind that $\dot{\hat{b}} = \dot{\hat{b}}$ leads to
\[ \dot{V} = -\alpha e^2 + \frac{1}{\gamma} b[-\gamma \zeta + \text{Pr}(\gamma \zeta)] \] (40)
Replacing (37) into the above equality yields
\[ \dot{V} = -\alpha e^2 - \frac{1}{\gamma} b(\gamma \zeta)_\perp \] (41)
Thus, from (39) it follows that
\[ \dot{V} = -\alpha e^2 - \frac{1}{\gamma} b(\gamma \zeta)_\perp \leq -\alpha e^2 \] (42)
Finally, the control law obtained from the present analysis is
\[ u = \frac{1}{b} \left( K_p e + \dot{r} - \ddot{d} \right) \] (43)
\[ \dot{d} = -\beta \dot{d} + \beta \left( \dot{x} - b u \right) \] (44)
\[ \dot{\hat{b}} = \text{Pr}(\gamma \zeta), \quad \zeta = -ue \] (45)
The next proposition resumes the stability result obtained previously, and the Fig. 2 depicts a block diagram of the proposed controller.

**Proposition 1:** Consider the servo system model (2) in closed-loop with the control law (43). If (44) computes the estimate $\dot{d}$ of the real disturbance $s$ and (45) estimates the parameter $b$, whose lower and upper bounds are known, then all the closed-loop signals stay bounded, and $e$ converges asymptotically to zero.

Note that the values $b_{\min}$ and $b_{\max}$ used in the parameter projection (35) may be obtained from off-line parameter identification or through the technical data of the servo system. One may also wonder if a fixed value $\hat{b}$ obtained from a trial and error may be an alternative to a parameter estimation. In this regard, a good guessing of the value would come from a good knowledge of the servo system parameters. Therefore, if the fixed value $\hat{b}$ is too large then the control signal would be too weak. On the other hand, small values of $\hat{b}$ would produce large control signals and it is possible that the closed-loop system have an oscillatory behavior or become unstable.
It is also possible to incorporate in the model (1) a term modeling viscous friction. However, it adds another differential equation for estimating the viscous friction coefficient. This will issue will be studied in future work.

V. EXPERIMENTAL SETUP

The servo system consists of a DC motor, a power amplifier operating in current mode, and a velocity sensor. The experimental setup is composed of the following:

- A Clifton Precision brushed DC motor model JDTH-2250-DQ-1C driving a brass disk.
- An Servotek tachogenerator model SA-7388F-1 used to measure the servo angular velocity.
- A Copley Controls power amplifier model 413 working as a current source.
- A Servo To Go data card.
- A box for electrically isolating the power amplifier from the data acquisition card.
- Mathworks Matlab/Simulink software and Quanser Consulting WINCON real-time software.
- The Simulink Euler-ode1 method with an integration time of 1 ms.

The next transfer function is applied to the tachogenerator output to mitigate high-frequency measurement noise where variable $x_f$ is the filtered velocity measurement

$$\mathcal{L}\{x_f\} = \frac{8100}{s^2 + 180s + 8100}$$

Note that the cut-off frequency of this filter is 90 rad/s, which is well-above the largest desired pole of the closed-loop system that is equal to $K_p = 4.1667$. Therefore, it is expected that the filter dynamics does not affect closed-loop stability.

Closed-loop performance is assessed through the Integral of the Squared Errors ($ISE$), the Integral of the Absolute value of the Error ($IAE$), the Integral of the Absolute value of the Control ($IAC$), and the Integral of the Absolute value of the Control Variation ($IACV$). The latter is employed to check the variation and oscillation of the control signal. These performance indexes are defined as

$$ISE = \int_{0}^{3} (e)^2 dt$$
$$IAE = \int_{17}^{20} |e| dt$$
$$IAC = \int_{2}^{\tau_1} |u| dt$$
$$IACV = \int_{\tau_1}^{\tau_2} \frac{du}{dt} dt$$

The time window for evaluating the above indexes is defined in the time instances $\tau_1 = 0s$ and $\tau_2 = 20s$.

In the experiments the reference $r$ is the sine wave signal $r = 30 \sin (0.2\pi t) + 960 \, \text{r/min}$. The time derivative $\dot{r}$ is obtained through the next filter

$$\mathcal{L}\{\dot{r}\} = \frac{300s}{s + 300}$$

VI. TRACKING TRAJECTORY TASK IN REAL TIME EXPERIMENTS

In this section, the controller studied in Sections III and IV is experimentally assessed. For comparison purposes, the parameter $b$ is estimated off-line using the Least Squares (LS) method, which produces $\hat{b}_{LS} = 43.73$. The value of this estimate remains almost the same for different values of the servomotor angular velocity.
The input gain \( b \) of the linear model (2) fulfills \( b_{\min} = 5 \) and \( b_{\max} = 120 \). The value of \( \epsilon \) is set to 0.01. Several initial conditions for \( \hat{b}(0) \) are tested to show the performance of the control scheme.

Table I resumes the outcomes of the experiments. The closed-loop responses, the control signals, and velocity errors graph are depicted in Fig. 4, Fig. 5 and Fig. 6 respectively. The estimated gains \( b \) are shown in Fig. 7 and Fig. 8, as well as the estimated disturbances in Fig. 9.

For this set of experiments, Table I indicates that the ISE reaching its minimum value when \( \hat{b}(0) = 50 \). Note that when the initial conditions are close to the value of \( b_{LS} = 43.73 \), i.e when \( \hat{b}(0) = 40, \hat{b}(0) = 50 \) and \( \hat{b}(0) = 60 \), the overshoots in the transitory response are smaller compared with those obtained with other initial conditions, see Fig. 4 and Fig. 6.

It is worth remarking that if the initial condition is small, for example, when \( \hat{b}(0) = 20 \), the obtained overshoot is significant, as shown in Fig. 4. For evaluating the stationary error, the IAE index is employed and it shows that all the experiments have a similar performance.

On the other hand, the values of the IACV index in Table I show that the control signal variations are smaller using large values in the initial condition \( \hat{b}(0) \), which happens when \( b(0) \) takes values between 40 and 80.

Besides, the values of the IACV index in Table I show that the control signal variations are higher using small values in the initial condition \( \hat{b}(0) \), that is when using \( \hat{b}(0) = 20 \) and \( \hat{b}(0) = 30 \), see Fig. 5. Observe that when \( \hat{b}(0) = 20 \), the higher variation in the control signal is obtained. The closed-loop responses and the velocity errors in the transitory state, as shown respectively in Figs. 4 and 6, reflect the effects of a small initial condition such as large overshoots. In the case of the IAC index, the results in each experiment are akin, see Fig. 5. Nevertheless, for the initial condition \( \hat{b}(0) = 20 \), the value of this index is higher. Finally, note that nearly all the estimates of \( \hat{b} \) converges to a neighborhood of \( b_{LS} \) except for \( b(0) = 20 \), see Figs. 7 and 8. Nonetheless, the disturbance estimate through the adaptive DOB (44) compensates for this difference, as shown in Fig. 9.

**TABLE I**

<table>
<thead>
<tr>
<th>( \hat{b}(0) )</th>
<th>ISE</th>
<th>IAE</th>
<th>IACV</th>
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</table>

**VII. Conclusion**

This paper presents theoretical and experimental results on the on-line estimation of the servo system input gain with an adaptive algorithm, which is used in the design of a controller relying on a Disturbance Observer. The proposed algorithm is aimed to velocity tracking in servo systems. Only upper and lower bounds on the servo system input gain are known beforehand. Assuming constant disturbances, it is shown that the velocity error converges to zero, and all the closed-loop signals are bounded. The experimental outcomes allow verifying satisfactory performance of the proposed scheme. Future work includes evaluating the control law under large changes in the desired velocity and to include the estimation and compensation of the viscous friction torques.
Fig. 7. Evolution of the estimated gains $\hat{b}$ for different initial conditions $\hat{b}(0)$.

Fig. 8. Evolution of the estimated gains $\hat{b}$ for different initial conditions $\hat{b}(0)$.

Fig. 9. Estimated disturbance $\hat{d}$ for different initial conditions $\hat{b}(0)$.

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