Model Predictive Torque Control of an Induction Motor with Discrete Space Vector Modulation

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Abstract—In order to improve the performance of model predictive torque control (MPTC) for induction motor (IM), this paper proposed the use of discrete space vector modulation (DSVM). In traditional MPTC, the application of one voltage vector is performed during the whole control cycle, which requires a small sampling time for proper operation. The application of the voltage vector can be performed in a discrete manner by using the DSVM to increase the sampling time required for the control. Nevertheless, DSVM increase considerable the computational burden of the controller, for this reason, this paper proposed a methodology to reduce the number of voltage vectors of DSVM evaluated in MPTC to keep computational burden feasible. Simulation results on a commercial IM are presented to prove the effectiveness of the proposed method.

Index Terms—predictive control, induction motor, torque control, discrete space vector modulation

I. INTRODUCTION

Induction Motors (IMs) are by far the most popular motors in the industry, this is mainly because of characteristics such as robustness, simple design, durability, and low cost [1]. In general, IMs are used in applications of constant speed where high dynamic torque responses are not required, such as fans and pumps [2], [3]. In order to obtain an accurate performance of the IM, a close loop control is necessary for the speed, torque, or current control. This has motivated the use of high-performance control schemes such as field-oriented control (FOC) [4], direct torque control (DTC) [5], and model predictive control (MPC) [6], [7].

Compared to FOC and DTC, MPC is a more intuitive control approach for power electronics; there is no need of rotational transformations; and non-linearities can be considered in the control design [8]. These are the reasons of recent popularity of MPC for the control of the IM. Moreover, since a voltage source inverter (VSI) is commonly used to feed the voltage of the IM, a limited number of control actions is obtained, and MPC becomes a constrained optimization control commonly known as finite set MPC (FS-MPC).

In FS-MPC the effect of each voltage vector in the future behavior of the controlled variables during the whole control cycle is evaluated, and the voltage vector which minimize the cost function is selected as the optimal control action. However, this implies the use a short sampling time to reduce the torque and flux ripples, which has limited the application of FS-MPC in industrial drive systems. An optional approach to reduce torque and flux ripple without increasing the sampling time is to apply more than one voltage vector during each control cycle. In this way, several control strategies to modulate FS-MPC have been presented, such as mean torque control [9], dead beat torque control [10], and variable switching control [11]. When using these strategies, the simplicity of FS-MPC is lost and complex calculations are introduced.

The simplicity of FS-MPC can be preserved by using the discrete space vector modulation (DSVM) introduced in [12]. Rather to calculate the application time of the voltage vector during each sampling time as in conventional SVM, in DSVM fixed virtual vectors are used. These virtual vectors are fixed in direction and magnitude and are combined with FS-MPC to preserve simplicity and to improve performance of the control. In this way, the application of multiple voltage vector during each sampling time reduce torque and flux ripple.

The main drawback of FS-MPC with DSVM is that computational burden is exponentially increased with the number of virtual vectors selected. Therefore, a simplification of
the evaluated voltage vectors during each sampling time is mandatory, in [12] hysteresis comparators are used, in [13] torque and flux variations, in [9] dead beat control, and in [14] look up tables. These methods must take into account flux and torque deviation for the selection of the optimal voltage vector, for this reason, this paper proposes a new methodology for the simplification of the FS-MPC with DSVM. In the proposed method, only the torque deviation is used to select the number of voltage vectors evaluated in each control cycle, which allows achieving the control objectives accurately with a reasonable computational burden. The performance of the proposed methodology is assessed on simulation environment for a commercial IM.

II. THEORETICAL BACKGROUND

A. Inverter topology

A two-level voltage source inverter (VSI) is commonly used to feed the voltage in the machine. The topology of the VSI is shown in Fig 1. Assuming that the switching devices can accept only one of the two possible states "on (1)" or "off (0)", the VSI has only eight possible switching states, generating eight voltage space vectors (VSV). Each VSV produce a voltage $u_v$ given by:

$$u_v = \begin{cases} \frac{2}{3}U_{DC} \cdot e^{j(v-1)\frac{\pi}{3}} & \text{when } v = 1, 2, \ldots, 6 \\ 0 & \text{when } v = 0, 7 \end{cases}$$

(1)

where $U_{DC}$ is the voltage in the DC-link and $v$ is the VSV evaluated.

B. IM equations

The state space model of the IM in the $\alpha-\beta$ reference frame can be described in its compact form by using space vector representation by the following equations [15]:

$$\dot{\psi}_s = R_s i_s + \frac{d}{dt} \psi_s$$

(2)

$$0 = R_r i_r + \frac{d}{dt} \psi_r - j\omega_e \psi_r$$

(3)

$$\dot{\psi}_s = L_s i_s + L_m i_r$$

(4)

$$\dot{\psi}_r = L_m i_s + L_r i_r$$

(5)

$$T_e = \frac{3}{2}p\Re\{\psi_s^* \cdot i_s\}$$

(6)

$$J \frac{d}{dt} \omega_m = T_e - T_L$$

(7)

Where $\psi_s$, $i_s$, and $\psi_s$ are the stator voltage, current and flux vectors respectively; $i_r$, $\psi_r$ are the rotor current and flux vectors respectively; $R_s$ and $R_r$ are the stator and rotor resistances respectively; $L_s$, $L_r$ and $L_m$ are the stator, rotor, and magnetizing inductances respectively; $T_e$ and $T_L$ are the electromagnetic and load torques respectively; $\omega_e$ and $\omega_m$ are the electrical and mechanical speeds respectively; $J$ is the inertia of the motor; $^*$ is the complex conjugate, and $p$ is the number of pole pairs. Each space vector “is formulated as the combination of the real $\alpha$ and the imaginary $\beta$ component, for instance, $\psi_s = \psi_s^* + j\psi_s^*.$

The relation between the mechanical speed and the electrical speed is given by:

$$\omega_e = p \cdot \omega_m$$

(8)

C. Conventional FS-MPC

A simplified scheme of the conventional FS-MPC for an IM drive is shown in Fig 2. Conventional FS-MPC is commonly performed into three steps: estimation of the machine variables, prediction of the machine variables for each VSV generated by the VSI, and minimization of the cost function for the selection of the next control action.

In the first step, the measured stator currents $i_{s_u,s_v}$ are transformed into the $\alpha-\beta$ reference frame through the Clark transformation. Then, the currents $i_{s_u,s_v}$, and the electrical speed $\omega_e$ are used to estimate the values of the rotor and stator flux of the IM. Hence, by using (2)-(5), the estimated rotor flux $\hat{\psi}_r$, and the estimated stator flux $\hat{\psi}_s$ can be described respectively as:

$$\frac{d}{dt} \hat{\psi}_r = R_r \frac{L_m}{L_r} \hat{\psi}_s - \left( \frac{R_r}{L_r} - j\omega_e \right) \hat{\psi}_r$$

(9)

$$\frac{d}{dt} \hat{\psi}_s = \frac{L_m}{L_r} \hat{\psi}_s + \sigma \hat{i}_s L_s$$

(10)

Where $\sigma = 1 - \frac{L_m}{L_r}.$

FS-MPC is inherently a discrete-time controller, therefore, the mathematical model of the IM needs to be discretized. To this end, Forward Euler, Backward Euler or exact discretization is most commonly employed [16]. The Euler approach is very popular as computational load remain cheap while a sufficient accuracy is obtained, thus, the already pronounced computational cost of MPC is not increased. If reader is interested in a better approximation in the discretization of the
continuous model they can refer to [17]. Since the proposed control scheme will increase the computational burden, the Forward Euler method is selected to discretize the model, however, the stability is preserved due to the small sampling time used. In this way, by using (2)-(6), the predictive values of stator flux \( \psi_s^p \) and torque \( T_p^e \) at the sampling instant \( k + 1 \) respectively, can be obtained as:

\[
\psi_s^p(k + 1) = \psi_s^e(k) + T_s \left( \psi_s^e(k) - R_s i_s^e(k) \right) \quad (11)
\]

\[
T_p^e = \frac{3}{2} p \omega_s \left( \psi_s^e(k + 1)^* i_s^r(k + 1) \right) \quad (12)
\]

where \( T_s \) is the sampling time, and the values of \( \psi_s^e(k) \) and \( i_s^r(k) \) can be obtained from (9)-(10), which results in:

\[
\psi_s^e(k) = \psi_s^e(k - 1) + T_s \left( R_s \frac{L_m}{L_r} - \left( R_r - j\omega_s \right) \psi_s^r(k - 1) \right) \quad (13)
\]

\[
i_s^r(k) = \frac{L_m}{L_r} \psi_s^r(k) + \sigma L_s i_s^e(k) \quad (14)
\]

As noted in (12), torque prediction depends of the current prediction \( i_s^r \) in sampling instant \( k + 1 \). Hence, the stator current can be predicted using the equivalent equation of the stator and rotor dynamics of a cage type IM given by [18]:

\[
i_s^r(k + 1) = \left( 1 - \frac{T_s}{\tau_r} \right) i_s^r(k) + \frac{T_s}{\tau_r \sigma} \left\{ \frac{k_r}{\tau_r} - jk_r \omega_s(k) \right\} \psi_s^e(k) + u_s^e(k) \quad (15)
\]

where \( k_r = \frac{L_m}{R_r} \); \( R_r = R_s + k_r R_r^2 \); \( \tau_r = \frac{L_r}{R_r} \); \( L_r = \sigma L_s \) and \( \tau_r = \frac{L_r}{R_r} \).

In conventional FS-MPC, the prediction of the machine variables need to be performed for each VSV to evaluate the cost function, then, the selection of the VSV which minimize the tracking error between the reference and the actual value of the controlled variables is performed. And the resulted VSV is applied in the next control cycle. A common choice of the cost function is to use the \( L_2 \)-norm, which ensures practical stability [19]. The cost function is given by:

\[
g = |T_{ecf} - T_r(k + 1)|^2 + \lambda \left( |\psi_{srcf}| - |\psi_s(k + 1)| \right)^2 \quad (16)
\]

where \( g \) is the value of the cost function; \( T_{ecf} \) is the reference torque; \( \psi_{srcf} \) is the reference flux; and \( \lambda \) is a factor to balance the tradeoff between torque and flux.

III. PROPOSED FINITE SET MODEL PREDICTIVE CONTROL

Conventional FS-MPC has the problem of large flux and torque ripple when a low sampling frequency is used, therefore, high sampling times are required for proper operation of FS-MPC. To overcome this drawback, the combination of FS-MPC with DSVM (MPC-DSVM) can be carried out. MPC-DSVM has the advantage of lower sampling frequency, with a similar performance to conventional FS-MPC. MPC-DSVM uses the same procedure to perform the control of the IM than conventional FS-MPC: estimation, prediction and optimization of a cost function. However, the main difference is that MPC-DSVM applies a virtual vector (VV) in each control cycle.

In MPC-DSVM the sampling time is subdivided into equivalent time intervals, and by the combination of real VSV, the application of a VV in each control cycle is performed. By using VV, the sampling time can be increased, hence, there is more time to perform different algorithms such as on-line parameter estimation, sensorless operation, or control implementation in low cost platforms. However, the computational burden is exponentially increased with the number of time intervals selected in the DSVM, i.e., the higher the time intervals the higher the computational burden. To reduce the computational burden, a simplification in the number of evaluated VV can be carried out by evaluating the torque deviation as will be shown in the next subsections.

A. DSVM

In a two-level VSI, the eight VSV can be represented in the \( \alpha \beta \) plane as shown in Fig. 3a (red vectors: \( V_1, V_2, ..., V_6 \) and two \( V_{17} \)). These VSV are fixed in magnitude and direction, however, a linear combination of the real VSV can be used to synthetize a VV. The new VV is a combination of the real VSV in prefixed time intervals, as described by [13]:

\[
V^v = \sum_{j=1}^{n} t_j V^r_j \quad (17)
\]

\[
t_1 + t_2 + ... + t_n = T_s \quad (18)
\]

\[
V^r_j \in \{V_1, V_2, ..., V_6\} \quad (19)
\]
In principle, flux variation is not considered for the selection of the admissible VVs, however, torque prediction is performed based on flux and current predictions, therefore, flux performance in inherently considered in the proposed methodology. According to the sector and the torque deviation, a look up table can be predefined for the admissible VV. For instance, according to Fig. 3a and Fig. 3b if the stator flux relies in sector 1, and the torque need to be increased ($\delta T_e > 0$), the admissible VVs are the ones located in sector 2 and sector 3. Conversely, if the torque need to be decreased ($\delta T_e < 0$), the admissible VVs are the ones located in sector 5, and sector 6 as shown in Table 1. This reduce the number of evaluations to only 14 VV. The remain sectors of the LUT can be determined in a similar way.

The proposed control algorithm can be summarized as follow:

- Measure $i_s(k)$, $V_{dc}(k)$, and $\omega_m(k)$
- Apply the optimal VV previously calculated: $\tilde{V}_{vir}$
- Estimate $\hat{\psi}_s$ and $\hat{\psi}_r$
- Select the admissible VV from table I.
- Predict $\psi_s(k+1)$, $\psi_r(k+1)$, and $T_p(k+1)$ for each VV, and evaluate the cost function given by (16).
- Select the VV which minimize the cost function.

### C. Speed control design

A simple design of the speed controller can be obtained following a Lyapunov approach. For the speed control lets define the error $e_m$ between the reference speed $\omega_{mref}$ and the real speed $\omega_m$ as:

$$e_m = \omega_{mref} - \omega_m + k_w \int (\omega_{mref} - \omega_m) \, dt$$  \hspace{1cm} (22)

where $k_w$ is positive definite gain. The error dynamic can be obtained from (22) as:

$$e_m = \omega_{mref} - \omega_m + k_w (\omega_{mref} - \omega_m) = \omega_{mref} - \frac{\omega_{mref}}{J} + \frac{T_p}{J} + k_w (\omega_{mref} - \omega_m)$$  \hspace{1cm} (23)

Let’s proposed the following Lyapunov function $V$:

$$V = \frac{1}{2} e_m^2$$  \hspace{1cm} (24)

To guarantee the stability of the controller, the condition $\dot{V} \leq 0$ need to be accomplished, and we have:

$$\dot{V} = e_m \dot{e}_m$$

$$= e_m (\omega_m (\omega_{mref} - \frac{T_p}{J} + \frac{T_p}{J} + k_w (\omega_{mref} - \omega_m))$$  \hspace{1cm} (25)

By selecting $T_L$ as the control action $u$, $u$ can be proposed as:

$$u = J \left( k_1 e_m + \omega_{mref} + \omega_m \right)$$  \hspace{1cm} (26)

where $k_1$ is positive definite gain, and $T_L$ is estimated from (7). By replacing $u$, (25) becomes

$$\dot{V} = e_m \left( \omega_m - \frac{T_p}{J} + \frac{T_p}{J} + k_w (\omega_{mref} - \omega_m) \right)$$

$$= -k_1 e_m^2$$  \hspace{1cm} (27)

From (27), If the gain $k_1 > 0$ then $\dot{V} \leq 0$, the system is asymptotically stable.
IV. Simulation Results

In order to verify the effectiveness of the proposed control, the system shown in Fig. 4 is implemented in Matlab/Simulink programming environment. The performance of the proposed control is compared with the conventional FS-MPC. The machine under test is a commercially available IM whose parameters are listed in table II. For both control schemes, the speed control is carried out based on (26); and the factor of the cost function is set to $\lambda = 2$. For all control schemes the IM is magnetized to create the stator flux and reduce initial currents, this is done by applying the VSV V2 several times until a 70% of the nominal stator flux is obtained, then, the close loop control of the IM is performed. In practice, the measured stator currents are commonly filtered by a low pass filter to avoid noise problems during prediction of the machine variables.

The performance of the IM in steady state under the conventional FS-MPC and the proposed MPC-DSVM is shown in Figs. 5-6. A sampling time of 50 $\mu$s for the FS-MPC and a sampling time of 120 $\mu$s for the proposed control are used. This sampling time lead to an approximate equal switching frequency. In this test the reference speed is set to 150 rad/s. The load torque is equal to 0 Nm during the startup, and at a time of 0.75 s a load torque of 7 Nm is applied. The results show a fast dynamic response of the torque and flux control during the startup, which lead to an accurate tracking of the reference speed. It can be noted that under perturbations the speed remains stable and torque is increased to compensate the load. It can be seen from Figs. 5-6 that performance of FS-MPC and MPC-DSVM is similar, however, the proposed MPC-DSVM lead to a higher sampling time, which can be used to perform more complex control algorithms.

The second test is performed to evaluate the transient response of the system under MPC-DSVM when a sampling time of 50 $\mu$s is used. The results of the evaluation of MPC-DSVM under speed control is shown in Fig. 7. In this test the speed is initially set to 150 rad/s and at a time of 1s is changed to -150 rad/s. For the load torque, a torque of 0 Nm is used during the startup, then load torque is varied from 0 to 5 Nm and from 0 to 7 Nm in different instants of time. It can be observed from simulation, that the performance of the controller is effective for the speed tracking against torque load perturbations, and that torque and flux ripples are smaller compared to FS-MPC.

### Table II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>0.053</td>
<td>$\psi_{nom}$</td>
<td>1.0 Wb</td>
</tr>
<tr>
<td>$I_{nom}$</td>
<td>0.085</td>
<td>$T_{nom}$</td>
<td>7.4 Nm</td>
</tr>
<tr>
<td>$L_s$</td>
<td>0.5192</td>
<td>$J$</td>
<td>0.011787 kgm²</td>
</tr>
<tr>
<td>$L_{m}$</td>
<td>0.4893</td>
<td>$\omega_{nom}$</td>
<td>1415 r/min</td>
</tr>
</tbody>
</table>
the implementation of the proposed control scheme. This issue will be investigated in the future.

REFERENCES


Finally, a comparison with the method proposed in [13] is carried out. The same parameters used for the transient state evaluation in previous test are used. It can be seen from Figs. 7-8, that similar results are obtained from the proposed method and the one presented in [13], therefore, a quantitative evaluation of the torque and flux ripple is performed based on the flux and torque deviation from the reference values, hence, the standard deviation is used and expressed as percentage. Under $T_s = 50\mu s$, this test results in a torque ripple of 3.74% for the proposed method, and a torque ripple of 3.70% of the method presented in [13]. On the other hand, for the proposed method the flux ripple is equal to 0.93%, and for the method presented in [13] is equal to 1.05%.

The results obtained demonstrate the effectiveness of the proposed control scheme. The proposed MPC-D SVM presents a similar performance of FS-MPC but with a significant higher sampling time. In comparison with [13], the proposed method results in a similar torque performance, but a slightly better flux performance. Moreover, while [13] uses flux and torque deviation for the selection of the admissible VV, the proposed method only uses torque deviation, and from 37 VV only 14 VV are required for evaluation.

V. CONCLUSIONS

In this paper a simplified approach for the MPC-D SVM of an IM drive is proposed. By using the torque deviation and the sector where stator flux vector relies, a simplification in the number of VVs used to predict the behavior of the machine variables is performed, therefore, a reduction of the computational burden is aimed to be obtained. Furthermore, a simple speed control design is proposed to improve robustness of the system. From the simulation results, good performance under steady and transient state is obtained, and the response is not significantly affected by the proposed simplification of the admissible VV selection. It has been shown that the proposed speed control lead to fast torque dynamic. The results motivate