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Sliding Mode Control of a Quadrotor with Suspended Payload: a Differential Flatness Approach

A.H. Martinez-Vasquez
Departament of Electrical Engineering,
Mechatronics section,
CINVESTAV-IPN
Mexico City, Mexico
Email: adrianmatinez@cinvestav.mx

Rafael Castro-Linares
Departament of Electrical Engineering,
Mechatronics section,
CINVESTAV-IPN
Mexico City, Mexico
Email: rcastro@cinvestav.mx

A.E Rodriguez-Mata.
CONACYT-Tecnológico
Nacional de México
Instituto Tecnológico de Culiacán
Culiacán, Sinaloa México
Email: aerm86@hotmail.com

***Abstract*—This article addresses the problem of transporting a slung load with a cable with a quadrotor unmanned aerial vehicle UAV in the frame $x-z$. The proposed solution introduces a nonlinear model system and a robust sliding mode control approach for a trajectory tracking control of the quadrotor with minimum slung load swing. A differential flatness approach is introduced for parametrization of the horizontal dynamics of the system and a sliding mode approach is proposed to control the system by means of thrust force and torque due to the motors of the quadrotor. Numerical results are presented to evaluate the performance of the control strategy.**

I. INTRODUCTION

Nowadays, specialists using unmanned aerial vehicles (UAVs) are carrying out interesting research in this field. Currently, a novel line of study includes the use of UAVs to load, handle and transport objects. There are numerous applications in which transportation cargo with UAV is extremely useful, such as package delivery in urban areas. Based on the distinct types of transport routes, the aerial delivery method can be classified into three categories, namely the gripper handling robot arm, and the cable suspended track [9].

Many approaches have been widely used to control quadrotor UAVs with the slung load. In [3], a new control law for a quadrotor carrying a suspended mass is proposed based on the idea of nested saturation approach. A nonlinear high-level modeling and control strategy based on feedback linearization for a quadrotor with a suspended load is presented in [4]. In [5] and [6] studies of a quadrotor with a cable-suspended load are presented. In both articles, the quadrotor with a cable suspended system is shown to be differentially flat and the notion of a differentially flat hybrid system is introduced. Such a notion enables trajectory generation for the flat hybrid system associated to the quadrotor that allows to handle the case where the tension in the cable goes to zero. In [7] dynamic programming is applied to solve swing-free trajectory tracking for a quadrotor with a suspended load. In [8] adaptive control techniques are used for transporting uncertain payloads with a quadrotor. An extended state observer [11] is used to estimate the disturbance in the quadrotor plant, which is caused by the dynamics of the payload. An online anti-swing trajectory approach is proposed in [10] where the authors focus on the trajectory planning aiming to have both swing elimination and trolley positioning. Also sliding mode control, super-twisting integral sliding mode control, and the backstepping method are used to stabilize a quadrotor with payload in presence of wind gust [12]. A second-order sliding mode control is proposed in [13] where the robustness of the system against both the unknown disturbance effects and the interconnected payload is obtained.

In this paper, a differentially flatness approach is proposed for the trajectory tracking control of a quadrotor UAV with slung load in the frame $x - z$. The approach is based on a nonlinear model and its linear approximation. First, a sliding mode controller is designed to control the quadrotor altitude using the nonlinear model. Then, a flat output is obtained from the linear approximation of the quadrotor model in order to parametrize its horizontal dynamics position in terms of the system states, including the slung load and another sliding mode controller is designed to control this parametrized dynamics position. The design is different from other techniques based on the differential flatness concept and leads to a simpler controller which achieves trajectory tracking in the presence of disturbances with a minimum swing od the payload.

This article is organized as follows. Section II presents the model of a quadrotor with a hanged payload and its linear approximation together with the parametrized dynamics based on differential flatness. Section III describes the control strategy using sliding mode techniques. In Section IV some simulation results are given. Finally, conclusions are drawn in Section V.

II. QUADROTOR'S DYNAMIC WITH SLUG LOAD: PROBLEM STATEMENT

As shown in Fig. 1 a quadrotor slung load system is considered, which consists of a quadrotor UAV, a rigid cable and a suspended payload. The following assumptions are made in order to obtain the dynamic model of the system:

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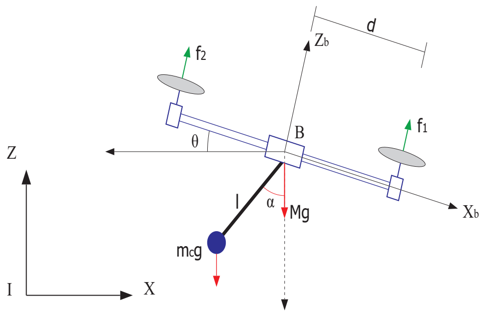

Fig. 1: quadrotor with pay load

1) The quadrotor is symmetrical.
2) The cable attachment point is at the UAV center of mass.
3) The cable is mass-less, inelastic and rigid.
4) The mass of the suspended load is punctual.
5) The suspension point coincides with the quadrotor's center of gravity.

Let $I = \{x, y\}$ be the inertial frame and $B = \{x_b, y_b\}$ a set of coordinates fixed in the vehicle. The generalized variables for the system are $q = [x\ z\ \theta\ \alpha]$, with $q \in \mathbb{R}^4$, where x, z represent the position around the horizontal and vertical axis in the inertia frame, respectively, θ is the pitch angle of the quadrotor and α is the payload angle with respect to the z axis. M and m_c are the mass of the quadrotor and the payload, respectively. I_θ is the inertia of the quadrotor, l is the length of the cable, d is the distance between the motors and the centre of mass of the quadrotor, while f_1 and f_2 are the thrust force of each motor. Finally, the control input is defined as $u = [u_1, \tau]^T$, with $u \in \mathbb{R}^2$, where $u_1 = f_1 + f_2$ is the magnitude of the total thrust force and $\tau = (f_1 - f_2)d$ is the torque relative to the body.

The dynamical model is obtained via the Euler-Lagrange formalism. Thus, the kinetic energy terms associated with the moving quadrotor and the payload (K_{Quad} and K_p, respectively) are found to be

$$K_{Quad} = \frac{1}{2}M(\dot{x}^2 + \dot{z}^2) + \frac{1}{2}I_\theta\dot{\theta}^2, \tag{1}$$

$$K_p = \frac{1}{2}m_c(\dot{x}_p^2 + \dot{z}_p^2) + \frac{1}{2}I_\theta\dot{\alpha}^2, \tag{2}$$

where the position coordinates of the payload are given by $x_p = x - l\sin\alpha$, and $z_p = z - l\cos\alpha$. The potential energy of the system is given by

$$U = (M + m_c)gz - m_c g l\cos\alpha, \tag{3}$$

where g is the constant of gravity. The Lagrangian function of the system, $L = K_{Quad} + K_p - U$, is hence given by

$$\begin{aligned} L = &\frac{1}{2}M(\dot{x}^2 + \dot{z}^2) + \frac{1}{2}I_\theta\dot{\theta}^2 + \frac{1}{2}m_c(\dot{x}_p^2 + \dot{z}_p^2) \\ &+ \frac{1}{2}I_\theta\dot{\alpha}^2 - (M + m_c)gz - m_c g l\cos\alpha. \end{aligned} \tag{4}$$

From this equation and the Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = u, \tag{5}$$

the nonlinear model of the quadrotor with suspended payload is given by [3]

$$(M + m_c)\ddot{x} - m_c l\ddot{\alpha}\cos\alpha + m_c l\dot{\alpha}^2\sin\alpha = u_1\sin\theta, \tag{6}$$

$$(M + mc)(\ddot{z} - g) + m_c l\ddot{\alpha}\sin\alpha + m_c l\dot{\alpha}^2\cos\alpha = u_1\cos\theta, \tag{7}$$

$$I_\theta\ddot{\theta} = \tau, \tag{8}$$

$$-m_c l\ddot{x}\cos\alpha + m_c l\ddot{z}\sin\alpha + m_c l^2\ddot{\alpha} + m_c g l\sin\alpha = 0, \tag{9}$$

After some computations, this system can also be written as

$$\ddot{x} = g_x(\theta, \alpha)u_1 + f_x(\theta, \alpha), \tag{10}$$

$$\ddot{z} = g_z(\theta, \alpha)u_1 + f_z(\theta, \alpha), \tag{11}$$

$$\ddot{\theta} = g_\theta(\theta)\tau, \tag{12}$$

$$\ddot{\alpha} = g_\alpha(\theta, \alpha)u_1, \tag{13}$$

where

$$f_x(\alpha, \dot{\alpha}) = -\frac{M m_c l}{M+m_c}\dot{\alpha}^2\sin\alpha, \tag{14}$$

$$g_x(\theta, \alpha) = \frac{m_c\left(\sin\theta - \sin\alpha\cos\alpha\cos\theta - \sin\theta\sin^2\alpha + \frac{M\sin\theta}{m_c}\right)}{M+m_c}, \tag{15}$$

$$f_z(\alpha, \dot{\alpha}) = -\frac{M m_c l}{M+m_c}\dot{\alpha}^2\cos\alpha - \frac{g}{M+m_c}, \tag{16}$$

$$g_z(\theta, \alpha) = \frac{m_c\left(\cos\theta\sin^2\alpha - \sin\alpha\cos\alpha\sin\theta + \frac{M\cos\theta}{m_c}\right)}{M+m_c}, \tag{17}$$

$$g_\theta(\theta) = -I_\theta, \tag{18}$$

$$g_\alpha(\theta, \alpha) = \frac{1}{Ml}\left(\cos\alpha\sin\theta - \sin\alpha\cos\theta\right). \tag{19}$$

The control problem addressed is stated follows.

Problem Statement Let us consider the quadrotor with payload whose dynamics is given by equations (6)-(9) or, equivalently, equations (10)-(13). It is required that the position x and z of the quadrotor with suspended load tracks a given smooth trajectory with the minimum swing of the suspended load. Namely, the maneuver of the quadrotor must be carried out while the pay load evolves closely around its equilibrium point.

The approach followed in this work in order to solve the above problem is based on a linear approximation of system (10)-(13) together with some differential flatness properties, as it is described in the following.

A. A linear approximation

Since $(x(t), z(t), \theta(t), \alpha(t)) = (\bar{x}, \bar{z}, 0, 0)$ is an equilibrium point of system (10)-(13) for $u_1(t) = \bar{u}_1 = (M + m_c)g$ and $\tau = 0$, with $\bar{x}$ and $\bar{z}$ being positive real constants, one can obtain the following linear approximation of the system at that point in a state space form, more precisely

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{20}$$

where the state vector $\mathbf{x}$ is given by

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{bmatrix}^T, \quad (21)$$

with components $x_1 = x - \bar{x}$, $x_2 = \dot{x}$, $x_3 = z - \bar{z}$, $x_4 = \dot{z}$, $x_5 = \theta$, $x_6 = \dot{\theta}$, $x_7 = \alpha$, $x_8 = \dot{\alpha}$ and input vector $\mathbf{u}$ given by

$$\mathbf{u} = \begin{bmatrix} u_\delta = u_1 - \bar{u}_1 \\ \tau_\delta = \tau \end{bmatrix}. \quad (22)$$

The matrices $\mathbf{A}$ and $\mathbf{B}$ are expressed as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{g(M+m_c)}{M} & 0 & -\frac{gm_c}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{g(M+m_c)}{Ml} & 0 & -\frac{g(M+m_c)}{Ml} & 0 \end{bmatrix}, \quad (23)$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{M+m_c} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I_\theta & 0 & 0 \end{bmatrix}^T. \quad (24)$$

B. Differential flatness of the linear approximation

A system is differentially flat if there exists a set of flat outputs, such that the state and inputs of the system can be expressed as smooth functions of the flat outputs and their higher-order derivative [1].

In order to define the flat output of the linearized system, one first obtains to the controllability matrix. $\mathcal{C} = [b_1 \ Ab_1 ... \ A^7 b_1 \ b_2 \ Ab_2 ... \ A^7 b_2]$, where b_1 and b_2 represent the columns of the matrix $\mathbf{B}$, this is $\mathbf{B} = [b_1 \ b_2]$. The rank of the controlability matrix $\mathcal{C}$ is eight, thus system (20) is controllable and, hence, flat [2].

The Kronecker indices of the incremental input $u_{1\delta}$ and τ_δ, corresponding to the chosen controlability matrix $\mathcal{C}$, are then, respectively, given by $\gamma_1 = 2$ and $\gamma_2 = 6$. The inverse matrix of $\mathcal{C}$ determines the flat outputs which are given by

$$\mathcal{F} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathcal{C}^{-1} \mathbf{x}, \quad (25)$$

this is

$$\mathcal{F}_1 = (M + m_c) x_3, \quad (26)$$

$$\mathcal{F}_2 = \frac{Ml}{I_\theta g^2 (M + m_c)} x_1 - \frac{Ml^2}{I_\theta g^2 (M + m_c)} x_7. \quad (27)$$

One can define a proportional factor $\gamma_1 = (M + m_c)$ and $\gamma_2 = \frac{Ml}{I_\theta g^2 (M+m_c)}$ in such a way that

$$\mathcal{F}_1 = \gamma_1 x_3, \quad (28)$$

$$\mathcal{F}_2 = \gamma_2 (x_1 - l x_7), \quad (29)$$

so that one can choose the flat outputs as

$$\mathcal{F}_{1\gamma} = x_3, \quad (30)$$

$$\mathcal{F}_{2\gamma} = x_1 - l x_7. \quad (31)$$

Indeed, all system variables may be parameterized in terms of $\mathcal{F}_{1\gamma}$ and $\mathcal{F}_{2\gamma}$, and a finite number of its time derivatives. Notice that the flat output $\mathcal{F}_{2\gamma}$ can be interpreted as the projection of the suspended load mass in the horizontal plane.

Since the control input $u_{1\delta}$ only has effect in the dynamics of x_3, we are only interested in the parameterization of the horizontal dynamics x_1 and the angular dynamics x_7, more precisely

$$x_1 = \mathcal{F}_{2\gamma} - \frac{l}{g} \ddot{\mathcal{F}}_{2\gamma}, \quad \dot{x}_1 = \ddot{\mathcal{F}}_{2\gamma} - \frac{l}{g} \mathcal{F}_{2\gamma}^{(3)}, \quad x_7 = \frac{1}{g} \ddot{\mathcal{F}}_{2\gamma},$$

$$\dot{x}_7 = \frac{1}{g} \mathcal{F}_{2\gamma}^{(3)}, \quad x_5 = \frac{Ml}{g^2 (M + m_c)} \mathcal{F}_{2\gamma}^{(5)} + \frac{1}{g} \mathcal{F}_{2\gamma}^{(4)},$$

$$\tau_\delta = \frac{Ml}{g^2 I_\theta (M + m_c)} \mathcal{F}_{2\gamma}^{(6)} - \frac{1}{g} \mathcal{F}_{2\gamma}^{(4)}$$

The output (31) can be completely parameterized in terms of the system variables and its time derivatives as:

$$\mathcal{F}_{2\gamma} = x_1 - l x_7, \quad (32)$$

$$\dot{\mathcal{F}}_{2\gamma} = \dot{x}_1 - l \dot{x}_7, \quad (33)$$

$$\ddot{\mathcal{F}}_{2\gamma} = g x_7, \quad (34)$$

$$\mathcal{F}_{2\gamma}^{(3)} = g \dot{x}_7, \quad (35)$$

$$\mathcal{F}_{2\gamma}^{(4)} = \frac{g^2 (M + m_c)}{Ml} (x_5 - x_7), \quad (36)$$

$$\mathcal{F}_{2\gamma}^{(5)} = \frac{g^2 (M + m_c)}{Ml} (\dot{x}_5 - \dot{x}_7), \quad (37)$$

$$\mathcal{F}_{2\gamma}^{(6)} = \frac{g^2 I_\theta (M + m_c)}{Ml} \tau_\delta - \frac{g(M + m_c)}{Ml} \mathcal{F}_{2\gamma}^{(4)}. \quad (38)$$

III. Control law strategy

In this section a control law for the attitude and the altitude of the quadrotor based on the sliding mode technique is presented. The goal of the control law is that the flat output $\mathcal{F}_{2\gamma}$ tracks a smooth reference that changes over time both, in the horizontal axis and in the vertical axis.

A. Altitude Robust Sliding Mode Control

Since the altitude control concerns only the displacement in the z axis, one considers the nonlinear model given by the equation (11); such a consideration is based on the fact that the altitude dynamics are decoupled in the linear approximation model (20). Endogenous and exogenous disturbances are introduced in the dynamics as

$$\ddot{z} = g_z(\theta, \alpha) u_1 + \xi_z(t), \quad (39)$$

where $\xi_z(t) = f_z(\alpha, \dot{\alpha}) + d_z(t)$, and $d_z(t)$ contains the uncertainty disturbances such as wind gust and other perturbations which may affect the altitude dynamics. It is assumed that the term $|\xi_z(t)|$ is bounded, this is $\xi_z(t) \leq L$, with $L > 0$. Defining the tracking error as $e_z = z - z_d$, where z_d is a smooth reference trajectory, $u_s = \ddot{z}_d$ and $e_u = g_z(\theta, \alpha)u_1 - u_s$, system (39) can be written in terms of tracking error as

$$\ddot{e}_z = e_u + \xi_z(t). \tag{40}$$

The sliding mode control scheme introduces *"sliding surfaces"* along which the sliding motion is to take place. Thus, a sliding surface is defined by means of the switching function

$$s = \dot{e}_z + \lambda e_z, \tag{41}$$

where $\lambda > 0$ is the slope of the sliding line. If the trajectories of the dynamics (39) are enforced to the sliding surface $s = 0$ then e_z asymptotically converges to the origin. For this purpose, a Lyapunov function is proposed as

$$V = \frac{1}{2}s^2, \tag{42}$$

whose time derivative (40) is given by

$$\dot{V} = s(e_u + \xi_z(t) + \lambda\dot{e}_z). \tag{43}$$

If $\dot{V} < 0$, the trajectories of the system (40) are driven to $s = 0$. This can be achieved by choosing the feedback

$$e_u = -\lambda\dot{e}_z - \rho\text{sign}(s), \tag{44}$$

where $\rho > 0$ and $\text{sign}(s) = \frac{s}{|s|}$. When the feedback (44) is substituted into (43), $\dot{V}$ takes the form

$$\dot{V} = -\rho|s| + s\xi_z(t), \tag{45}$$

where the fact that $s \cdot \text{sign}(s) = |s|$ has been used. Majoring this last expression and taking into account the bound on the term $\xi_z(t)$ one has that

$$\dot{V} \leq -|s|(\rho - L). \tag{46}$$

Thus, if ρ is chosen such that $\rho > L$, $\dot{V} < 0$ and the surface $s = 0$ is attained. The sliding mode control that achieves altitude trajectory tracking is then given by

$$u_1 = \frac{-\lambda\dot{e}_z - \rho\text{sign}(s) + \ddot{z}_d}{g_z(\theta, \alpha)} \tag{47}$$

B. Attitude Robust Sliding mode control

A sliding mode control is also implemented for the dynamics (38) which can be written as

$$\mathcal{F}_{2\gamma}^{(6)} = u_2 + \xi_x(t), \tag{48}$$

where $u_2 = \frac{g^2 I_\theta (M+m_c)}{Ml}\tau_\delta$, $\xi_x(t) = -\frac{g(M+m_c)}{Ml}\mathcal{F}_{2\gamma}^{(4)} + d_x(t)$. $d_x(t)$ contains uncertainty such as a high order terms neglected in the linear approximations together with external disturbances, it is also assumend that the term $\xi_x(t)$ is bounded, this is $|\xi_x(t)| \leq P$ with $|P| > 0$. A flat output tracking error is defined as $e_\mathcal{F} = \mathcal{F}_{2\gamma} - \mathcal{F}_d$, where $\mathcal{F}_{2\gamma}$ is the flat output and $\mathcal{F}_d$ is a smooth trajectory reference, thgether with $e_{ux} = u_2 - u_{sx}$ where $u_{sx} = \mathcal{F}_d^{(6)}$. Then, the dynamic of the tracking error $e_\mathcal{F}$ can be expressed as

$$e_\mathcal{F}^{(6)} = e_{ux} + \xi_x(t). \tag{49}$$

A switching function is then defined as

$$\sigma = e_\mathcal{F}^{(5)} + \gamma_4 e_\mathcal{F}^{(4)} + \gamma_3 e_\mathcal{F}^{(3)} + \gamma_2 \ddot{e}_\mathcal{F} + \gamma_1 \dot{e}_\mathcal{F} + \gamma_0 e_\mathcal{F}. \tag{50}$$

where γ_4, γ_3, γ_2, γ_1 and γ_0 are positive real coefficent different from zero.

In this way, when the dynamics of system (48) is restricted to the sliding surface $\sigma = 0$, the flat output tracking error $e_\mathcal{F}$ tends to zero asymptotically. In order to attack the movement of the dynamics (49) to $\sigma = 0$, the following Lyapunov function is proposed

$$V = \frac{1}{2}\sigma^2, \tag{51}$$

for which

$$\begin{aligned}\dot{V} = \sigma(&e_{ux} + \xi_x(t) + \gamma_4 e_\mathcal{F}^{(5)} + \gamma_3 e_\mathcal{F}^{(4)} + \gamma_2 e_\mathcal{F}^{(3)} \\ &+ \gamma_1 \ddot{e}_\mathcal{F} + \gamma_0 \dot{e}_\mathcal{F}).\end{aligned} \tag{52}$$

Choosing the feedback

$$\begin{aligned}e_{ux} = &-\gamma_4 e_\mathcal{F}^{(5)} - \gamma_3 e_\mathcal{F}^{(4)} - \gamma_2 e_\mathcal{F}^{(3)} - \gamma_1 \ddot{e}_\mathcal{F} \\ &- \gamma_0 \dot{e}_\mathcal{F} - \rho_1 \text{sign}(\sigma),\end{aligned} \tag{53}$$

allows to writte $\dot{V}$ as

$$\dot{V} \leq -\rho_1|\sigma| + \sigma\xi_x(t). \tag{54}$$

Taking into account the bounded on the $\xi_x(t)$ and majoring (54) one obtains

$$\dot{V} \leq -|\sigma|(\rho_1 - P). \tag{55}$$

And, if ρ_1 is such that $\rho_1 > P$, $\dot{V} < 0$ and the trajectories of the system (49) attain the surface $\sigma = 0$. From (53) one obtains the sliding mode control that allows to have attitude trajectory tracking , this is

$$\begin{aligned}u_2 = \mathcal{F}_d^{(6)} &- \gamma_4 e_\mathcal{F}^{(5)} - \gamma_3 e_\mathcal{F}^{(4)} - \gamma_2 e_\mathcal{F}^{(3)} - \gamma_1 \ddot{e}_\mathcal{F} \\ &- \gamma_0 \dot{e}_\mathcal{F} - \rho_1 \text{sign}(\sigma)\end{aligned} \tag{56}$$

IV. Simulation Result

Some numerical simulations were carried using the nonlinear model (6)-(9) and the sliding mode controllers (47) and (56). The model parameters used in the simulations correspond to those of a real aerial platform and are listed in Table I. The reference trajectory to be followed by the flat output $\mathcal{F}_{2\gamma}$ and the altitude variable z were sinusoidal signals, shown in figures 2 and 5. Also external disturbances $d_x(t)$ and $\xi_z(t)$ were introduces al $t = 50$ sec as alreatory signals as shown in Fig. 11. The controllers parameters were selected as Hurwitz polynomials that guarantee the asymptotic stability of the tracking errors e_z and $e_{\mathcal{F}}$ in the sliding surfaces $s = 0$ and $\sigma = 0$, respectively. The gains ρ and ρ_1 were selected using a trial procedure leading to the values $\rho = 50$ and $\rho_1 = 1$.

Figures 3, 4 y 6 show the behavior of the quadrotor horizontal position, the flat output trajectory tracking error $e_{\mathcal{F}}$ and the trajectory tracking error e_z. The behavior of the quadrotor pitch angle, θ, and the angular position of the suspended load are shown in figures 7 and 8. Figures 9 y 10 show the torque control input τ and the thrust force control input u_1, respectively. Figure 12 shows the trajectory of both, the quadrotor and the suspended load in the space using an animation algorithm.

From the validation results, it can be observed that trajectory tracking is performed with small and bounded tracking errors while keeping the swinging of the payload within small bounds (close to 0.04 rad). The control signals were kept bounded although their amplitude is high and the chattering effect has a high frequency. Actual research is carried out to diminish the amplitude of the control signals and reduce the chattering.

Parameter	Value [unit]
M	0.5 $[kg]$
m_c	0.2 $[kg]$
l	0.3 $[m]$
g	9.8 $[m/s^2]$
I_θ	0.1 $[kg.m^2]$

TABLE I: Model parameters

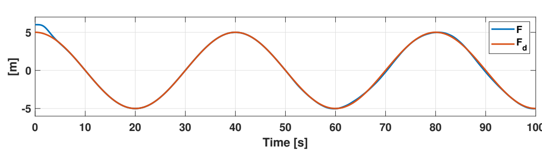

Fig. 2: Flat output $\mathcal{F}_{2\gamma}$ and reference flat output $\mathcal{F}_d$.

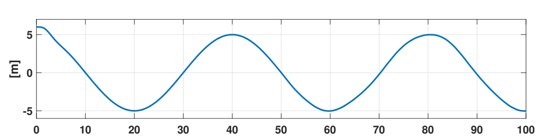

Fig. 3: x variable (horizontal position).

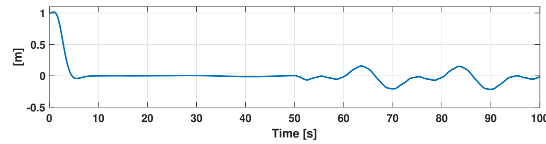

Fig. 4: Flat output trajectory tracking error.

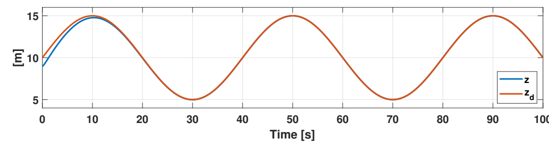

Fig. 5: z variable (altitude)

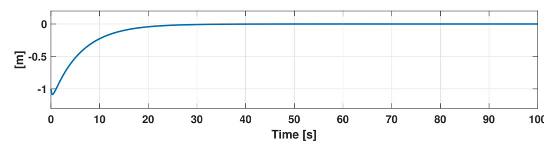

Fig. 6: Trajectory tracking error of the z variable.

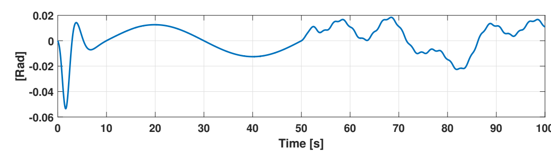

Fig. 7: Attitude angle of the quadrotor θ.

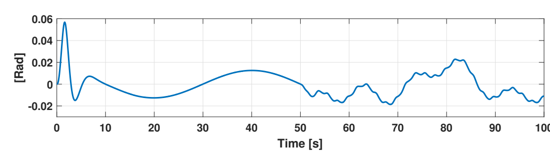

Fig. 8: Angular position of the suspended load α.

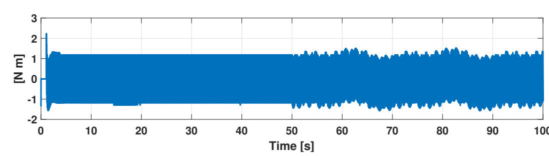

Fig. 9: Control torque force input τ.

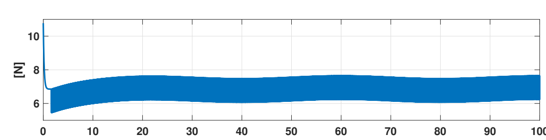

Fig. 10: Control thrust force input u_1.

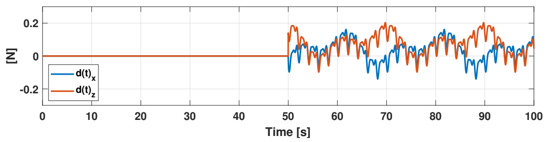

Fig. 11: External disturbances $d_x(t)$ and $d_z(t)$.

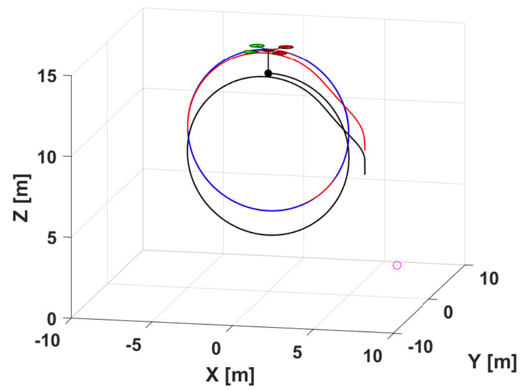

Fig. 12: Trajectory tracking of the cable payload quadrotor in the space.

V. CONCLUSIONS

This work addressed the problem of trajectory tracking of a quadrotor transporting a cable suspended load. The approach followed for the trajectory control problem was based on a nonlinear model for which a sliding mode controller was designed to control the quadrotor altitude. The horizontal position controller of the quadrotor was designed using a linear approximation of the nonlinear model together with the notion of differential flatness and sliding mode techniques; such an approach is different from other techniques that make use of these concepts and also lead to its easy implementation. From the numerical simulations, one can conclude that the controller achieved trajectory tracking of the quadrotor in the presence of exogenous disturbances with a minimum swing of the payload.

The control signals obtained in the numerical simulations introduce frequency components (chattering) which is an important problem in real-time experiments. This problem will be addressed in the implementation of real-time experiments by means of attenuation and elimination techniques of chattering. Also, research is in progress in order to find an optimum algorithm for the selection of the controller parameter.

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