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Particles Swarm Optimization for Minimal Energy Consumption in Complex Networks Node Search

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***Abstract*—In this paper, a new optimization scheme, for minimal energy consumption, is proposed based on a heuristic approach to achieve synchronization in complex networks; the main objective is to determine nodes and gains for the pinning control technique by means of a particles swarm algorithm. A stability equation is used to determine the pinned nodes. The particles swarm seek for a set of nodes and gains, which guarantees asymptotic stability with the minimum number of nodes and simultaneously minimizing energy consumption. The proposed heuristic algorithm is a modification of the particle swarm optimization one. Applicability of the proposed scheme is tested via simulations.**

I. Introduction

In the last decade, the study of complex networks properties has rapidly increased. The interest in this topic is due to fact that such networks appears everywhere in nature and society; they are part of our daily life and are presented in different kinds of systems [1]–[3]. Control of large-sized complex networks has become a hot topic, in order to achieve adequate collective behaviors. For networks consisting of a large number of nodes, this task is complicated, either because of the mathematical difficulty involved or the required computational cost.

The pinning control technique is used to determinate a reduced number of controlled nodes to achieve desired objectives in complex networks [4], [5]. An open research topic is to determinate how many controllers are needed, and which ones to select so as to achieve the best performance? [6]; for this goal, heuristic algorithms can be used [7], to calculate the necessary required energy, and to determine key nodes to apply control actions. There are different heuristic methods available: parasitic algorithms, others based in the hunting method of some animal species, algorithms with search hierarchies depending on the social role of each agent in the population, and some others based on the symbiotic relationships of more than one species in the same population [8]–[10].

The main contribution of this paper is the proposal of a new optimization criteria scheme which guarantees synchronization in the complex networks and minimal controller energy consumption by means of PSO (particle swarm optimization). The evaluation criterium is selected such as to determine a solution, which ensures synchronization based on the Lyapunov stability theory, using the V-stability tool [11]. This new approach works regardless of the network size, the nodes dynamics, network topologies and the weight of their connections. To illustrate the pertinence of the proposed optimizer, simulations are included.

A. Formulation

Consider a network (1) of N nodes with linear diffusive couplings, where each node is an n-dimensional dynamical system [12].

$$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i) + \sum_{j=1, j\neq i}^{N_n} c_{ij} a_{ij} \Gamma(\mathbf{x}_j - \mathbf{x}_i) \tag{1}$$

where $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in \mathbb{R}^n$ for $i = 1, 2, \ldots, N_n$ represents the states of the i-th node, $f_i : \mathbb{R}^n \to \mathbb{R}^n$ represents the self-dynamics of node i, N_n is the number of nodes in the complex network, constants c_{ij} are the coupling strengths between node i and node j, $\Gamma \in \mathbb{R}^{n\times n}$ describes the way of linking the components in each pair of connected node vectors $(\mathbf{x}_j, \mathbf{x}_i)$, the coupling matrix $A = [a_{ij}] \in \mathbb{R}^{N_n \times N_n}$ is the topological structure of the network.

If there is a connection between node i and node j, then $a_{ij} = a_{ji} = 1$; else $a_{ij} = a_{ji} = 0$. If the node degree k_i is defined to be the number of edges connected to node i

$$k_{ij} = \sum_{j=1, j\neq i}^{N_n} a_{ji}, \quad i = 1, 2, \ldots, N_n. \tag{2}$$

Let the diagonal element of matrix A : $a_{ii} = -k_i$, $i = 1, 2, \ldots, N_n$, which means the linear diffusive coupling, and

$$c_{ii} = \frac{1}{k_i} \sum_{j=1, j\neq i}^{N_n} a_{ij} c_{ji}. \quad (3)$$

In this article, the heuristic algorithm goals are: (i) Synthesize a controller that stabilizes the network (1) in a homogeneous steady state $\overline{x}$, such that,

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_{N_n}(t) \to \overline{\mathbf{x}} \text{ as } t \to \infty.$$

(ii) Minimize an objective function

$$E_{\mathbf{u}}(\mathbf{x}) = \sum_{i=1}^{N_p} \|\mathbf{u}_i(\mathbf{x}_i)\|^2, \quad \mathbf{x} = (\mathbf{x}_1^T, \cdots, \mathbf{x}_{N_p}^T)^T, \quad (4)$$

where $E_{\mathbf{u}}(\mathbf{x})$ is the energy consumption of the controlled nodes.

The first goal is achieved by applying local feedback injections to a small fraction of the nodes in the network, which is named pinning nodes [13]. Thus, the controlled network can be written as

$$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i) + \sum_{j=1}^{N_n} c_{ij} a_{ij} \Gamma \mathbf{x}_j + \mathbf{u}_i, \quad i = 1, 2, \cdots, N_n, \quad (5)$$

where $\mathbf{u}_i \in \mathbb{R}^n$ is the control input, defined as

$$\mathbf{u}_i = \begin{cases} -k_i \mathbf{x}_i, & i = i_1, i_2, \cdots, i_{N_p}, \\ 0, & otherwise, \end{cases} \quad (6)$$

$k_i \in \mathbb{R}$ is the control gain of the i-th pinning node, considering that the first N_p nodes be selected to be pinned, where $1 \leq N_p \leq N_n$, and N_p can be as small as one [4]. Thus, the self-dynamics of the controlled nodes becomes

$$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i) - k_i \mathbf{x}_i, \quad i = 1, 2, \ldots, N_p, \quad (7)$$

The following assumption is needed to demonstrate network stability, where

$$D_i = \{\mathbf{x}_i : \|\mathbf{x}_i - \overline{\mathbf{x}}_i\| < \alpha\}, \quad \alpha > 0, \quad D = \bigcup_{i=1}^{N_n} D_i.$$

Assumption 1: There is a continuously differentiable Lyapunov function $V(\mathbf{x}_i) : D \subseteq \mathbb{R}^n \mapsto \mathbb{R}_+$, satisfying $V(\overline{\mathbf{x}}) = 0$ with $\overline{\mathbf{x}} \in D$, such that for each self-dynamics (7) there is a scalar θ_i guaranteeing

$$\frac{\partial V(\mathbf{x}_i)}{\partial \mathbf{x}_i} (f_i(\mathbf{x}_i) + \mathbf{u}_i(\mathbf{x}_i) + (\theta_i + \psi_i)\Gamma \mathbf{x}_i) < 0, \quad (8)$$

$\forall \mathbf{x}_i \in D_i \subseteq D$, $\mathbf{x}_i \neq 0$, with constants $\psi_i \geq 0$ and passivity degree θ_i [11].

To determine if network (5) is synchronized to the equilibrium point $\overline{\mathbf{x}}$, consider the following theorem

Theorem 1: Suppose that there exists a function

$$V_N(X) = \sum_{i=1}^{N_n} \mathbf{x}_i^T \mathbf{P} \mathbf{x}_i, \quad \mathbf{X} = (\mathbf{x}_1^T, \cdots, \mathbf{x}_{N_n}^T)^T \quad (9)$$

with $\mathbf{P}$ being a symmetric and positive definite matrix, satisfying Assumption 1, with passivity degree values θ_i, such that the following inequality holds:

$$\mathbf{P}\Gamma + \Gamma^T \mathbf{P} \geq 0. \quad (10)$$

Then, the network (5) is synchronized to the equilibrium point $\overline{\mathbf{x}}$ if the following condition

$$C = -\Theta + G - \Psi \leq 0 \quad (11)$$

is satisfied, where $\Psi \in \mathbb{R}^{N_n \times N_n}$ is an diagonal matrix with N_p elements ψ_i, $\quad i = 1, 2, \ldots, N_p$, and its remaining $N_n - N_p$ elements are all zero, $\Theta \in \mathbb{R}^{N_n \times N_n}$ is an diagonal matrix with N_n passivity degree values θ_i, $\quad i = 1, 2, \ldots, N_p$, and $G = g_{ij} = c_{ij} a_{ij} \in \mathbb{R}^{N_n \times N_n}$.

Proof: Consider the Lyapunov Function

$$V_N(\mathbf{X}) = \sum_{i=1}^{N_n} \frac{1}{2} \mathbf{x}_i^T \mathbf{P} \mathbf{x}_i, \quad \mathbf{X} = (\mathbf{x}_1^T, \cdots, \mathbf{x}_N^T)^T \quad (12)$$

for the controlled network (5), such that,

$$\begin{aligned} \dot{V}_N(\mathbf{X}) &= \sum_{i=1}^{N_n} \mathbf{x}_i^T \mathbf{P} \left(f_i(x_i) + \sum_{j=1}^{N_n} c_{ij} a_{ij} \Gamma x_j + \mathbf{u}_i \right) \\ &< \sum_{i=1}^{N_n} \mathbf{x}_i^T \mathbf{P} \left(\sum_{j=1}^{N_n} c_{ij} a_{ij} \Gamma x_j + (\theta_i + \psi_i)\Gamma x_i \right) \end{aligned} \quad (13)$$

or, in the Kronecker product form,

$$\dot{V}_N(\mathbf{X}) < \mathbf{X}^T(-\Theta + G - \Psi) \otimes \mathbf{P}\Gamma\mathbf{X}. \quad (14)$$

By (10), one can easily show that the matrix $(-\Theta + G - \Psi) \otimes \mathbf{P}\Gamma$ is negative definite [14]. The proof is thus completed. If the characteristic matrix $-\Theta + G$ has N_p non-negative eigenvalues, and if the closed-loop characteristic matrix C is negative definite, then the number of nodes to be selected for control cannot be less than N_p [11].

For the second heuristic algorithm goal proposed in this paper, consider the following theorem:

Theorem 2: Suppose that (4) is a candidate nonlinear Lyapunov function $V(\mathbf{x})$ and there is a diagonal matrix $K \in \mathbb{R}^{N_n \times N_n}$ with N_p elements k_i, $\quad i = 1, 2, \ldots, N_p$, and its remaining $N_n - N_p$ elements are all zero, that satisfy (11), such that the equilibirum point $\overline{\mathbf{X}} = 0$ is exponentially stable, then

$$\mathbf{E_u} = \sum_{i=1}^{N_p} \|k_i \mathbf{x}_i(t_0)\|^2 \mid E_{\mathbf{u}}(t) \leq E_{\mathbf{u}}(t_0), \quad t \geq 0 \quad (15)$$

where $\mathbf{E_u}$ is the maximum energy consumption of a pinning node-set in a synchronized complex network. it depends only on a gain k and the initial conditions of the pinning nodes.

Proof: Suppose that $\overline{\mathbf{X}} = 0$ is an exponentially stable equilibrium point of network (5), furthermore, that K guarantees the criterion of the V-stability tool. Let $E : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function, such that,

$$e_1 \|\mathbf{x}\|^2 \leq E_u(\mathbf{x}) \leq e_2 \|\mathbf{x}\|^2 \quad (16)$$

$$\sum_{i=1}^{N_p} -\frac{\partial E_u(\mathbf{x}_i)}{\partial \mathbf{x}_i} k_i \mathbf{x}_i \leq -e_3 \|\mathbf{x}\|^2 \tag{17}$$

$\forall\, t \geq 0$ and $\forall\, \mathbf{x}_i \in \mathbb{R}^n$, where e_1, e_2 and e_3 are positive constants. Note that (17) and the exponential stability of the zero solution $\overline{\mathbf{x}} = 0$ implies that

$$E_u(\mathbf{x}(t)) \leq E_u(\mathbf{x}(t_0)) e^{-(e_3/e_2)(t-t_0)}, \quad t \geq 0 \tag{18}$$

hence,

$$\lim_{t\to 0} \left\{ E_u(\mathbf{x}(t_0)) e^{-(e_3/e_2)(t-t_0)} \right\} = \sum_{i=1}^{N_p} \|k_i \mathbf{x}_i(t_0)\|^2, \tag{19}$$

the proof is this completed.

II. Modified PSO

The main contribution of this paper is to propose a scheme under an optimization criterium, which guarantees synchronization in complex networks and minimal energy consumption, therefore, by the V-stability criterium the first N_p dimensions of each agent correspond to the minimum number of pinning nodes necessary to synchronize the network, and based on theorem 2 the last N_p dimensions are the control gains. The proposed algorithm is explained below.

A. Agents initialization

At this stage, it is necessary to generate a healthy population; that is, a "good initial population" guaranteeing a correct reference for the new generations of particles [15]. The following equation describes population initialization.

$$I_{i,j} = \alpha_1 N_n, \tag{20}$$

$$L_{i,j} = l_j + \alpha_2(u_j - l_j), \tag{21}$$

$$P_i^t = \begin{bmatrix} I_{i,j} & L_{i,j} \end{bmatrix}^T, \tag{22}$$

such that,

$$I = \{I_{i,j} \in \mathbb{R}^{N_p} \mid i = 1, 2, \cdots, N_a,\, j = 1, 2, \cdots, N_p\},$$
$$L = \{L_{i,j} \in \mathbb{R}^{N_p} \mid i = 1, 2, \cdots, N_a,\, j = 1, 2, \cdots, N_p\},$$
$$P^t = \{P_i^t \in \mathbb{R}^{2N_p} \mid i = 1, 2, \cdots, N_a\},$$

where, P_i^t is the position of the $i-$th particle at iteration t, N_a is the number of agents, N_p is the minimum number of pinning nodes for stabilizing the complex network, N_n is the number of nodes in the complex network, α_1 and α_2 are random numbers evenly distributed in the $[0, 1]$ interval, $I_{i,j}$ is the $j-$th dimension of the $i-$th particle, this dimension corresponding a candidate node to be pinning node, $L_{i,j}$ represents the $j-$th pinning control gains k proposed by the $i-$th particle for equation (6), this dimension is bounded in $[l_j, u_j]$ interval.

Note that I belongs to the real numbers, however, the index of the nodes must be discrete, thus to calculate the fitness of each particle in the optimization criteria without arbitrarily modifying the movement of the agents, a ceiling function $\lceil\cdot\rceil$ (also known as the least integer function) is applied only at the evaluation stage of part I of agents P, such that, $\lceil I_{i,j} \rceil : \mathbb{R}^{N_p} \to \mathbb{N}^{N_p} \setminus \{0\}$. The algorithm 1 summarizes the agent initialization stage with the optimization criterium (15).

Algorithm 1 Agents Initialization

Data: N_p : Number of pinning nodes, N_a : Number of agents, (u^L, l^L) : Limits upper and lower of L respectively, (u^I, l^I) : Limits upper and lower of I respectively.
Result: P^t : population position, F_g : better global fitness, F_l : better local fitness, B_g : best global position, B_l : best local position.

$e \leftarrow 0$: stop criterion
while $e = 0$ **do**
  $F_l \leftarrow (\overbrace{[0, 0, \cdots, 0]}^{N_a})^T$: better local fitness
  for all $i \in [1, N_a]$ **do**
    for all $j \in [1, N_p]$ **do**
      $I_{i,j} \leftarrow$ by the Eq.(20)
      $L_{i,j} \leftarrow$ by the Eq.(21)
    end for
    $P_i \leftarrow$ by the Eq.(22)
    % Index of the gain matrix (Eq.(6))
    $K_{n,m} \leftarrow 0 \,\forall\, n,\, m \in [1, N_n]$
    $k_j \leftarrow \lceil I_{i,j} \rceil \mid j = 1, 2, \cdots, N_p$
    $K_{k_j,k_j} \leftarrow L_{i,j} \mid j = 1, 2, \cdots, N_p$
    $C \leftarrow$ by the Eq.(11)
    if $max\,\{R_e\,\{\lambda(C)\}\} \leq 0$ **then**
      $F_{l_i} \leftarrow$ by the Eq.(15)
      $e \leftarrow 1$
    else
      $F_{l_i} \leftarrow \infty$
    end if
  end for
  $F_g \leftarrow min\,\{F_l\}$
  $B_g \leftarrow P_i \mid P_i \Rightarrow min\,\{F_l\}$
  $B_l \leftarrow P$
end while

B. PSO iteration cycle

For the iterative cycle by means of equations (23) and (24), position P of the agents is updated for each iteration t, Fig. 1 portrays new position P^{t+1} and increment ΔP^t. Alg. 2 is the pseudocode of the iterative cycle. The new increment displacement for the $i-$th particle is defined as

$$\Delta P_i^{t+1} = \Delta P_i^t + \alpha_3(B_{l_i}^t - P_i^t) + \alpha_2(B_g^t - P_i^t), \tag{23}$$

$$P_i^{t+1} = P_i^t + \beta \Delta P_i^{t+1}, \tag{24}$$

where ΔP_i^t is the increment displacement of the $i-$th particle, influenced by the best global value $B_g^t \in \mathbb{R}^{1\times 2N_p}$ and the best local value $B_{l_i}^t \in B_l^t \in \mathbb{R}^{N_a \times 2N_p}$ of the i-nth particle, α_3 and α_4 are random numbers uniformly distributed in the $[0, 1]$ interval.

Once the increment displacements are calculated, particles move to a new position P^{t+1}, equation (24) operates by combining the increment displacement ΔP_i^t with the previous positions P^t of the population and a influence coefficient β. For different some versions of this algorithm, influence coefficients are added in equation (23) , such that, the global or local influence on each agent increases or decreases. The results may vary according to the optimization criteria or the system to optimize.

Algorithm 2 Iteration Cycle

Data: N_p : Number of pinning nodes, N_a : Number of agents, (u^L, l^L) : Limits upper and lower of L respectively, (u^I, l^I) : Limits upper and lower of I respectively, P^t : population position, F_g : better global fitness, F_l : better local fitness, B_g : best global position, B_l : best local position, N_t : number of iterations.

Result: F_g : better global fitness, B_g : best global position.

$t \leftarrow 1$
$D \leftarrow \begin{bmatrix} N_n & u \\ 1 & l \end{bmatrix} \mid D \in \mathbb{R}^{2\times 2N_p}$
$F \leftarrow (\overbrace{[0,0,\cdots,0]}^{N_a})^T$: actual fitness
$\Delta P_i^t \leftarrow \overbrace{[0,0,\cdots,0]}^{2N_p} \mid i = 1,2,\cdots,N_a$
while $t \leq N_t$ **do**
  for all $i \in [1, N_a]$ **do**
    $\Delta P_i^{t+1} \leftarrow$ by the Eq.(23)
    $P_i^{t+1} \leftarrow$ by the Eq.(24)
    for all $j \in [1, N_p]$ **do**
      if $P_{i,j}^t > D_{1j}$ **then**
        $P_{i,j}^t \leftarrow D_{1,j}$
      else if $P_{i,j}^t < D_{2,j}$ **then**
        $P_{i,j}^t \leftarrow D_{2,j}$
      end if
    end for
    % Index of the gain matrix (Eq.(6))
    $K_{n,m} \leftarrow 0 \, \forall \, n, \, m \in [1, N_n]$
    $k_j \leftarrow \lceil I_{i,j} \rceil \mid j = 1,2,\cdots,N_p$
    $K_{k_j,k_j} \leftarrow L_{i,j} \mid j = 1,2,\cdots,N_p$
    $C \leftarrow$ by the Eq.(11)
    if $max\,\{R_e\,\{\lambda(C)\}\} \leq 0$ **then**
      $F_i \leftarrow$ by the Eq.(15)
    else
      $F_i \leftarrow \infty$
    end if
    if $F_i < F_{l_i}$ **then**
      $F_{l_i} \leftarrow F_i$
      $B_{l_i} \leftarrow P_i$
    end if
  end for
  if $min\,\{F_l\} < F_g$ **then**
    $F_g \leftarrow min\,\{F_l\}$
    $B_g \leftarrow P_i \mid P_i \Rightarrow min\,\{F_l\}$
  end if
  $t \leftarrow t + 1$
end while

III. Simulations

To illustrate the proposed optimization scheme applicability, consider a network with 50 different nodes 622 non-uniform

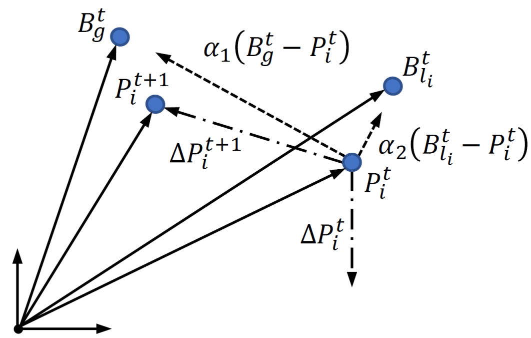

Fig. 1. Graphic representation of the swarm displacement

edges (Fig. 3). Fig. 2 represents represents the topology of the proposed network.

As mentioned, the first goal is to determine the minimal number of pinning nodes that guarantees asymptotic stability in the complex network by means of the V-stability tool. The second goal is to determinate a set of the control gains that guarantee the minimal energy consumption in the pinning nodes, in equation (15), it is observed that this last goal depends directly on the control gain K and the initial conditions $x(0)$. Therefore, Fig. 2 displays the squared norm of the initial conditions for each node.

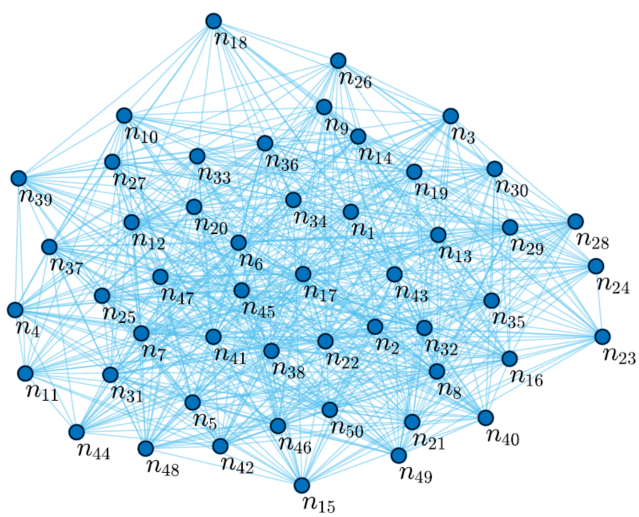

Fig. 3. Topology of a complex network with 50 nodes and 582 non-uniform edges.

A. Simulation parameters and results

The simulation parameters: 50 samples, 300 particles, 500 iterations, a $\beta = 0.5$ influence coefficient, by the V-stability tool 2 positive eigenvalues are identified in matrix C, such values are $\lambda_1 = 9.2964$ and $\lambda_2 = 17.2337$, therefore, the minimum number of pinning nodes is $N_p = 2$, such that each particle in the population have 4 dimensions, the bounds of the search spaces are: $[1, N_n = 50]$ for I and $[l = -10000, u = 0]$ for L.

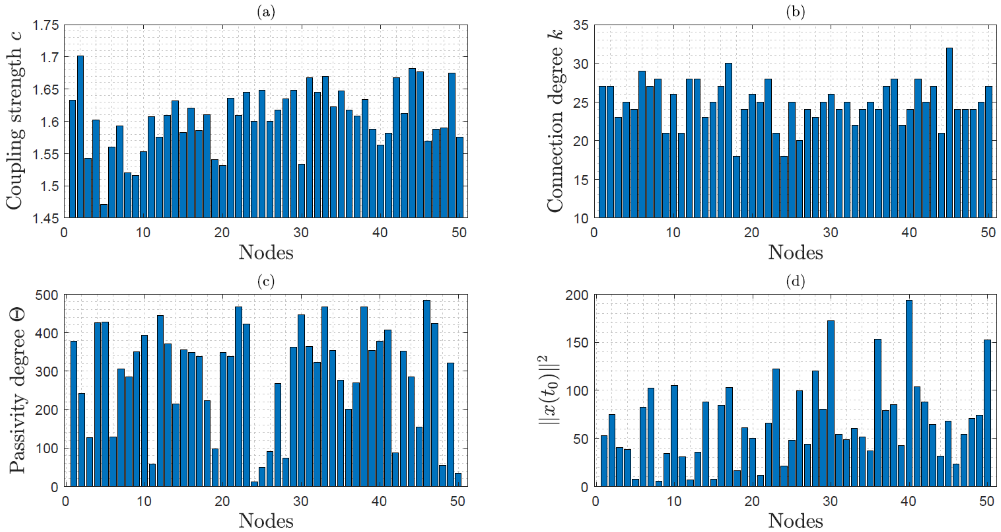

Fig. 2. Complex network parameters: (a) Coupling strength c, (b) Connection degrees k, (c) Passivity degrees θ and (d) Norm squared of the initial conditions for each system in the network nodes.

The average time of each iteration cycle is 11,458 sec, the average convergence curve is portrayed in Fig. 4. Note that in approximately 100 iterations the convergence of the algorithm decreases, which is mainly caused by the inertial decrease of the particles, such that, once the best pinning nodes are defined, the probability that other nodes will generate better performance with neighboring gains, decreases.

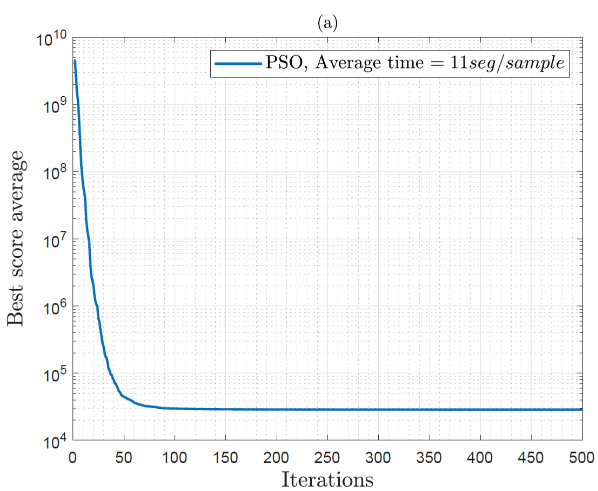

Fig. 4. Best fitness convergence curve.

The first N_p dimensions of each particle correspond to the indices of the pinning nodes. In Fig. 5, the convergence nodes alternate between n_{50} and n_{23} during the 50 samples, that is, the order does not matter, which can be corroborated in the pseudocodes 1 and 2, the average convergence nodes are n_{35} and n_{39}.

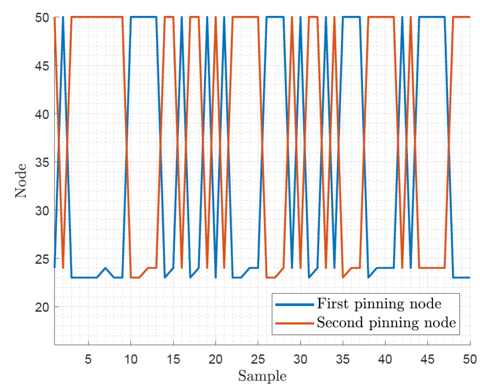

Fig. 5. Oscillation of convergence values for pinning nodes

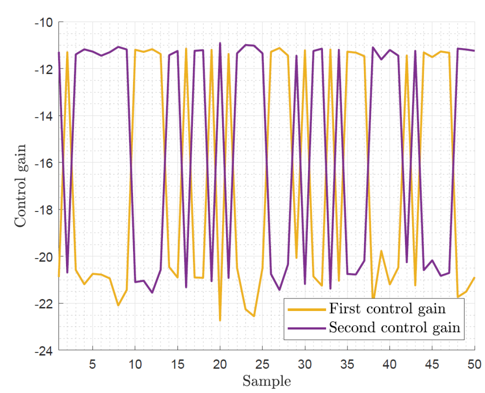

Fig. 6. Oscillation of the convergence values for control gains, with the presence of local minimus.

Finally, Fig. 6 represents the last N_p dimensions of the convergence value, corresponding to the control gains. Note that, as in Fig. 5, the values oscillate for each sample.

In contrast with the convergence values for the pinning nodes, the algorithm in this part of the search is vulnerable to "local minima", it is observed in the peak values of Fig. 6, . The optimal control gains average for the minimal energy consumption are: $K_{39,39} = -16.9920$ and $K_{35,35} = -15.2717$ with an fitness attitude of 2.8563×10^4.

IV. Conclusions

The proposed optimization scheme for minimal energy consumption is a useful tool in pinning control application for complex networks; especially when their size is considerably large. The respective algorithm determines the minimal global energy consumption of the network based on the Lyapunov theory, seeking for the best set of pinned nodes. The proposed approach can be applied regardless of the network size, the nodes dynamics, the network topology, and the weight of its connections. For future work, it is planned to implement optimization criteria to minimize convergence , synchronization times simultaneously and to develop a high-performance algorithm for optimization of pinning control [16] [17].

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