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Modelling and Analysis of Flow Rate and Pressure Head in Pipelines

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***Abstract*— Currently, various approaches with several utilities are proposed to identify damage in the pipeline. The pipeline system is modeled in the form of a distributed parameter system, such that the state space related to the distributed parameter system contains infinite dimension. In this paper, a novel technique is proposed to analyze and model the flow in the pipeline. Important theorems are proposed for testing the observability as well as controllability of the proposed model.**

Keywords— State space, Modeling, Analysis, hyperbolic equation.

I. INTRODUCTION

By increasing the industrial needs, a great number of long-distance pipelines are constructed and delivered. Pipelines are considered as one of the secured transport tools. However, this does not signify that they are riskless. Transiting through the rough environment may break or block the pipeline which may result in an enormous economic loss [1] [2] [3] [4]. Hence ensuring the assurance of the pipeline is highly essential for the energy sector.

Since some problems such as leakage or blockage in the pipeline may cause damage to the environment, so identifying the damage in the pipeline is important. In [5] some techniques for gas leak identification are studied. In [6] and [7] novel algorithms are proposed for identifying leakage in the pipeline. In [8] an observation approach is introduced for directly observing the leakage in the pipeline. In [9] the instantaneous frequency analysis methods are studied for detecting leaks in the pipelines. In [10] decentralized scheme for leak location in a branched pipeline is studied. In [11] a new technique is proposed for multi-leak identification in the pipeline.

Current research works are mainly focused on modeling the observer, controller or fault detection methods [12] [13] [14] [15]. However, studying the observability and controllability of the pipeline is remarkably essential [16] [17] [18] [19]. In this paper, a novel technique is proposed to analyze and model the flow in the pipeline. Pressure and flow are kept stable by utilizing control valves based on the various pressure and flow rate of the transferring pipe. This article remaining sections are organized into four Sections. Section II presents the proposed model of the pipeline. In Section III the observability, as well as controllability of the proposed model of the pipeline, is analyzed. In section IV the example is introduced. The Conclusion is presented in Section V.

II. PIPELINE MODELING

Here, the convective alteration in velocity and the compactness in the line of length (L), are ignored. The values which can be measured are liquid density (ρ), flow rate (Φ), and pressure (p) at the entry and exit of the pipeline. It has been assumed that the cross-sectional area (A) along the pipeline is stable. Figure 1 demonstrates the proposed pipeline.

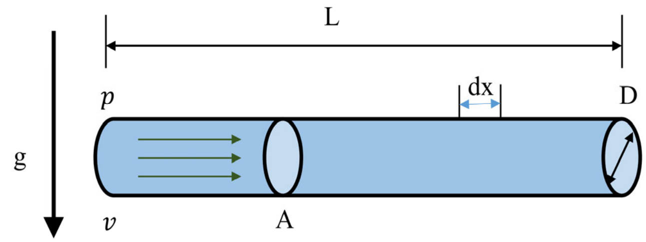

Fig. 1. The designed pipeline

The dynamics of the fluid inside the pipeline is stated based on the momentum equation as well as the continuity equation. The momentum equation is defined as [20],

$$\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{f}{2D}v^2=0 \tag{1}$$

By replacing $v=\frac{\Phi}{A}$ as well as $p=\rho g\Lambda$ in (1) we have,

$$\frac{\partial \Phi}{\partial t}+Ag\frac{\partial}{\partial x}\Lambda+\frac{f\Lambda^2}{2DA}=0 \tag{2}$$

such that Λ is taken to be the pressure head (m), Φ is taken to be the flow rate (m^3/s), x is considered as the length coordinate (m), t is taken to be the time coordinate (s), g is considered as the gravity(m/s^2), A is taken to be the section area (m^2), D is considered as the diameter (m), also f is taken to be the friction coefficient.

The continuity equation is defined as [21],

$$\frac{\partial p}{\partial t}+\rho a^2\frac{\partial v}{\partial x}=0 \tag{3}$$

By replacing the pressure head as well as the flow rate in (3) we have,

$$\frac{\partial \Lambda}{\partial t}+\frac{a^2}{gA}\frac{\partial \Phi}{\partial x}=0 \tag{4}$$

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in which a is the velocity of the wave (m/s) in an elastic conduit filled with a slightly compressible fluid. Λ and Φ are functions of position and time such that $\Lambda(x,t)$ and $\Phi(x,t)$, where $x \in [0, L]$, L is the length of the pipeline.

If the flow rate, in a pipeline system has slight alterations then the momentum equation linearized from the nonlinear pipeline system as follows

$$\frac{\partial \Phi}{\partial t} + Ag\frac{\partial}{\partial x}\Lambda + \frac{f\Phi}{DA} = 0 \quad (5)$$

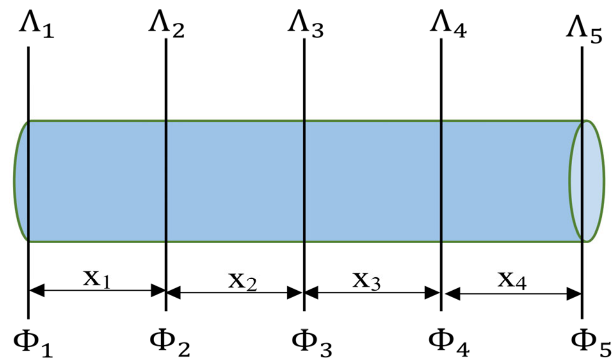

Fig. 2. A pipeline with four sections

The system of the pipeline is modeled using (4) and (5). Since it is difficult to find the solutions for these equations, so in this paper, we use the finite difference approach which is easy to apply. The finite difference approach partitions the whole pipeline into N number of sections [22] [23]. The pressure head, as well as flow rate, are obtained as below,

$$\begin{aligned} \dot{\Lambda}_i &= \frac{-a^2}{gA\Delta x}(\Delta\Phi_i) \\ \dot{\Phi}_i &= \frac{-Ag}{\Delta x}(\Delta\Lambda_i) - \frac{f}{DA}\Phi_i \end{aligned} \quad (6)$$

Figure 2 demonstrates a pipeline with four sections. In this figure, $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5$ are pressure head and $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5$ are flow rates. The common linearized model is defined as below

$$\frac{dx}{dt} = Ax + Bu \quad (7)$$

$$y = Cx \quad (8)$$

such that x is taken to be the state vector having the i unknown flow perturbation quantities at each point. Control parameters in various cases may be represented with the pressures as well as flow rates at the starting point and ending point of the pipeline system. The selection of boundary conditions alters the design of the model. Figure 3 shows the different boundary conditions in a discretized pipeline system. In this figure, six cases are taken into consideration.

Case 1: The initial and boundary conditions that can be controlled and measured are the pressure heads in the beginning and end of the pipeline. The inputs are

$$\begin{cases} \Lambda(0,t) = \Lambda_{in}(t) \\ \Lambda(L,t) = \Lambda_{out}(t) \end{cases} \quad (9)$$

and outputs are

$$\begin{cases} \Phi(0,t) = \Phi_{in}(t) \\ \Phi(L,t) = \Phi_{out}(t) \end{cases} \quad (10)$$

The proposed model, inputs as well as outputs are $x = (\Phi_1 \;\; \Phi_2 \;\; \Phi_3 \;\; \Phi_4 \;\; \Phi_5 \;\; \Lambda_2 \;\; \Lambda_3 \;\; \Lambda_4)^T$, $u \coloneqq (\Lambda_1 \;\; \Lambda_5)^T$ and $y = (\Phi_1 \;\; \Phi_5)^T$, respectively.

$$\begin{aligned} \dot{\Phi}_1 &= \frac{-Ag}{\Delta x}(\Lambda_2 - \Lambda_1) - \frac{f}{DA}\Phi_1 \\ \dot{\Phi}_2 &= \frac{-Ag}{2\Delta x}(\Lambda_3 - \Lambda_1) - \frac{f}{DA}\Phi_2 \\ \dot{\Phi}_3 &= \frac{-Ag}{2\Delta x}(\Lambda_4 - \Lambda_2) - \frac{f}{DA}\Phi_3 \\ \dot{\Phi}_4 &= \frac{-Ag}{2\Delta x}(\Lambda_5 - \Lambda_3) - \frac{f}{DA}\Phi_4 \\ \dot{\Phi}_5 &= \frac{-Ag}{\Delta x}(\Lambda_5 - \Lambda_4) - \frac{f}{DA}\Phi_4 \\ \dot{\Lambda}_2 &= \frac{-a^2}{2gA\Delta x}(\Phi_3 - \Phi_1) \\ \dot{\Lambda}_3 &= \frac{-a^2}{2gA\Delta x}(\Phi_4 - \Phi_2) \\ \dot{\Lambda}_4 &= \frac{-a^2}{2gA\Delta x}(\Phi_5 - \Phi_3) \end{aligned} \quad (11)$$

In (7) and (8), A, B, as well as C are represented by the block matrices as follows

$$x = (\Phi_1 \;\; \Phi_2 \;\; \Phi_3 \;\; \Phi_4 \;\; \Phi_5 \;\; \Lambda_2 \;\; \Lambda_3 \;\; \Lambda_4)^T$$

$$A = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 2\beta & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & \beta \\ 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & -2\beta \\ -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma & 0 & \gamma & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} -2\beta & 0 \\ -\beta & 0 \\ 0 & 0 \\ 0 & \beta \\ 0 & 2\beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \quad (12)$$

$$c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

where α, β, γ are defined as follows

$$\begin{aligned} \gamma &= \frac{-a^2}{2gA\Delta x}, \\ \alpha &= \frac{-f}{DA}, \\ \beta &= \frac{-Ag}{2\Delta x}. \end{aligned} \quad (13)$$

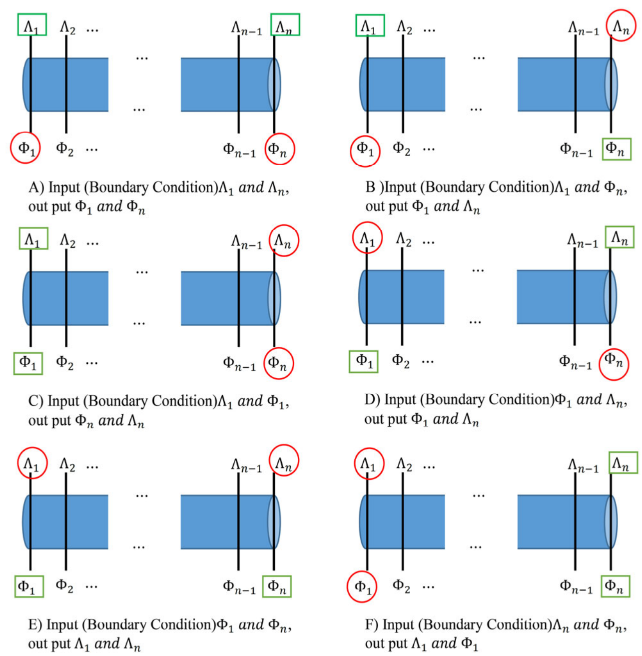

Fig. 3. boundary conditions

Case 2: The initial and boundary conditions that can be controlled and measured are pressure head in the beginning and the flow rate in end of the pipeline. The inputs are

$$\begin{cases}\Phi(L,t) = \Phi_{out}(t) \\ \Lambda(0,t) = \Lambda_{in}(t)\end{cases} \tag{14}$$

and outputs are

$$\begin{cases}\Phi(0,t) = \Phi_{in}(t) \\ \Lambda(L,t) = \Lambda_{out}(t)\end{cases} \tag{15}$$

The proposed model, inputs as well as outputs are $x = (\Phi_1\ \Phi_2\ \ \Phi_3\ \Phi_4\ \Lambda_2\ \Lambda_3\ \Lambda_4\ \Lambda_5)^T$, $u \coloneqq (\Phi_5\ \Lambda_1)^T$ and $y = (\ \Phi_1\ \Lambda_5)^T$, respectively.

$$\begin{aligned}
\dot{\Phi}_1 &= \frac{-Ag}{\Delta x}(H_2 - H_1) - \frac{f}{DA}\Phi_1 \\
\dot{\Phi}_2 &= \frac{-Ag}{2\Delta x}(H_3 - H_1) - \frac{f}{DA}\Phi_2 \\
\dot{\Phi}_3 &= \frac{-Ag}{2\Delta x}(H_4 - H_2) - \frac{f}{DA}\Phi_3 \\
\dot{\Phi}_4 &= \frac{-Ag}{2\Delta x}(H_5 - H_3) - \frac{f}{DA}\Phi_4 \\
\dot{\Lambda}_2 &= \frac{-a^2}{2gA\Delta x}(\Phi_3 - \Phi_1) \\
\dot{\Lambda}_3 &= \frac{-a^2}{2gA\Delta x}(\Phi_4 - \Phi_2) \\
\dot{\Lambda}_4 &= \frac{-a^2}{2gA\Delta x}(\Phi_5 - \Phi_3) \\
\dot{\Lambda}_5 &= \frac{-a^2}{gA\Delta x}(\Phi_5 - \Phi_4)
\end{aligned} \tag{16}$$

In (7) and (8), A, B, as well as C are represented by the block matrices as follows

$$x = (\Phi_1\ \ \Phi_2\ \ \Phi_3\ \Phi_4\ \ \Phi_5\ \Lambda_2\ \Lambda_3\ \Lambda_4)^T$$

$$\tilde{A} = \begin{bmatrix} \alpha & 0 & 0 & 0 & 2\beta & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & \alpha & 0 & -\beta & 0 & \beta & 0 \\ 0 & 0 & 0 & \alpha & 0 & -\beta & 0 & \beta \\ -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 \end{bmatrix} \tilde{B} = \begin{bmatrix} 0 & 2\beta \\ 0 & \beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \gamma & 0 \\ 2\gamma & 0 \end{bmatrix} \text{ and} \tag{17}$$

$$\tilde{c} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Case 3: The boundary conditions that can be measured are the initial flow rate and pressure head in the beginning of the pipeline. The inputs are

$$\begin{cases}\Phi(0,t) = \Phi_{in}(t) \\ \Lambda(0,t) = \Lambda_{in}(t)\end{cases} \tag{18}$$

and outputs are

$$\begin{cases}\Phi(L,t) = \Phi_{out}(t) \\ \Lambda(L,t) = \Lambda_{out}(t)\end{cases} \tag{19}$$

The proposed model, inputs as well as outputs are $x = (\Phi_2\ \Phi_3\ \Phi_4\ \Phi_5\ \Lambda_2\Lambda_3\ \Lambda_4\ \Lambda_5)^T$, $u \coloneqq (\Phi_1\ \ \Lambda_1\ )^T$ and $y = (Q_5\ \ \Lambda_5\ )^T$, respectively.

$$\begin{aligned}
\dot{\Phi}_2 &= \frac{-Ag}{2\Delta x}(\Lambda_3 - \Lambda_1) - \frac{f}{DA}\Phi_2 \\
\dot{\Phi}_3 &= \frac{-Ag}{2\Delta x}(\Lambda_4 - \Lambda_2) - \frac{f}{DA}\Phi_3 \\
\dot{\Phi}_4 &= \frac{-Ag}{2\Delta x}(\Lambda_5 - \Lambda_3) - \frac{f}{DA}\Phi_4 \\
\dot{\Phi}_5 &= \frac{-Ag}{\Delta x}(\Lambda_5 - \Lambda_4) - \frac{f}{DA}\Phi_5 \\
\dot{\Lambda}_2 &= \frac{-a^2}{2gA\Delta x}(\Phi_3 - \Phi_1) \\
\dot{\Lambda}_3 &= \frac{-a^2}{2gA\Delta x}(\Phi_4 - \Phi_2) \\
\dot{\Lambda}_4 &= \frac{-a^2}{2gA\Delta x}(\Phi_5 - \Phi_3) \\
\dot{\Lambda}_5 &= \frac{-a^2}{gA\Delta x}(\Phi_5 - \Phi_4)
\end{aligned} \tag{20}$$

In (7) and (8), A, B, as well as C are represented by the block matrices as follows

$$x = (\Phi_1\ \ \Phi_2\ \ \Phi_3\ \Phi_4\ \ \Phi_5\ \Lambda_2\ \Lambda_3\ \Lambda_4)^T$$

$$A' = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & -\beta & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & \alpha & 0 & \beta & 0 & 0 & -\beta \\ 0 & 0 & 0 & \alpha & 0 & \beta & 0 & -\beta \\ 0 & -\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & -\gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \end{bmatrix} B' = \begin{bmatrix} 0 & \beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \gamma & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and} \tag{21}$$

$$C' = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Case 4: The boundary conditions that can be controlled and measured are the initial flow rate in the beginning and pressure head in the end of the pipeline. The inputs are

$$\begin{cases}\Phi(0,t) = \Phi_{in}(t) \\ \Lambda(L,t) = \Lambda_{out}(t)\end{cases} \tag{22}$$

and outputs are

$$\begin{cases}\Lambda(0,t) = \Lambda_{in}(t) \\ \Phi(L,t) = \Phi_{out}(t)\end{cases} \tag{23}$$

The proposed model, inputs as well as outputs are $x = (\Phi_2\ \ \Phi_3\ \Phi_4\ \Phi_5\ \Lambda_1\ \Lambda_2\ \ \Lambda_3\ \ \Lambda_4)^T$, $u \coloneqq (\Phi_1\ \ \Lambda_5\ )^T$ and $y = (\Phi_5\ \ \Lambda_1\ )^T$, respectively.

$$\begin{aligned}
\dot{\Phi}_2 &= \frac{-Ag}{2\Delta x}(H_3 - H_1) - \frac{f}{DA}\Phi_2 \\
\dot{\Phi}_3 &= \frac{-Ag}{2\Delta x}(H_4 - H_2) - \frac{f}{DA}\Phi_3 \\
\dot{\Phi}_4 &= \frac{-Ag}{2\Delta x}(H_5 - H_3) - \frac{f}{DA}\Phi_4 \\
\dot{\Phi}_5 &= \frac{-Ag}{\Delta x}(H_5 - H_4) - \frac{f}{DA}\Phi_5 \\
\dot{H}_1 &= \frac{-a^2}{2gA\Delta x}(\Phi_2 - \Phi_1) \\
\dot{H}_2 &= \frac{-a^2}{2gA\Delta x}(\Phi_3 - \Phi_1) \\
\dot{H}_3 &= \frac{-a^2}{2gA\Delta x}(\Phi_4 - \Phi_2) \\
\dot{H}_4 &= \frac{-a^2}{2gA\Delta x}(\Phi_5 - \Phi_3)
\end{aligned} \tag{24}$$

In (7) and (8), A, B, as well as C are represented by the block matrices as follows

$$x = (\Phi_1\ \Phi_2\ \ \Phi_3\ \ \Phi_4\ \Phi_5\ \Lambda_2\ \Lambda_3\ \Lambda_4)^T$$

$$A' = \begin{bmatrix} \alpha & 0 & 0 & 0 & -\beta & 0 & \beta & 0 \\ 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta \\ 2\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 \end{bmatrix} B' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \beta \\ 0 & -2\beta \\ -2\gamma & 0 \\ \gamma & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and} \tag{25}$$

$$C' = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Case 5: The boundary conditions that can be controlled and measured are the flow rate in the beginning and pressure head in the end of the pipeline. The inputs are

$$\begin{cases} \Phi(0,t) = \Phi_{in}(t) \\ \Phi(L,t) = \Phi_{out}(t) \end{cases} \tag{26}$$

and outputs are

$$\begin{cases} \Lambda(0,t) = \Lambda_{in}(t) \\ \Lambda(L,t) = \Lambda_{out}(t) \end{cases} \tag{27}$$

The proposed model, inputs as well as outputs are $x = (\Phi_2 \ \ \Phi_3 \ \ \Phi_4 \ \Phi_5 \ \Lambda_2 \ \ \Lambda_3 \ \ \Lambda_4 \ \ \Lambda_5)^T$, $u \coloneqq (\Phi_1 \ \ \Phi_5)^T$ and $y = (\Lambda_1 \ \ \Lambda_5)^T$, respectively.

$$\begin{aligned}
\dot{\Phi}_2 &= \frac{-Ag}{2\Delta x}(\Lambda_3 - \Lambda_1) - \frac{f}{DA}\Phi_2 \\
\dot{\Phi}_3 &= \frac{-Ag}{2\Delta x}(\Lambda_4 - \Lambda_2) - \frac{f}{DA}\Phi_3 \\
\dot{\Phi}_4 &= \frac{-Ag}{2\Delta x}(\Lambda_5 - \Lambda_3) - \frac{f}{DA}\Phi_4 \\
\dot{H}_1 &= \frac{-a^2}{2gA\Delta x}(\Phi_2 - \Phi_1) \\
\dot{H}_2 &= \frac{-a^2}{2gA\Delta x}(\Phi_3 - \Phi_1) \\
\dot{H}_3 &= \frac{-a^2}{2gA\Delta x}(\Phi_4 - \Phi_2) \\
\dot{H}_4 &= \frac{-a^2}{2gA\Delta x}(\Phi_5 - \Phi_3) \\
\dot{H}_5 &= \frac{-a^2}{gA\Delta x}(\Phi_5 - \Phi_4)
\end{aligned} \tag{28}$$

In (7) and (8), A, B, as well as C are represented by the block matrices as follows

$$x = (\Phi_1 \ \ \Phi_2 \ \ \Phi_3 \ \ \Phi_4 \ \ \Phi_5 \ \ \Lambda_2 \ \ \Lambda_3 \ \ \Lambda_4)^T$$

$$A' = \begin{bmatrix} \alpha & 0 & 0 & 0 & -\beta & 0 & \beta & 0 \\ 0 & \alpha & 0 & 0 & 0 & -\beta & 0 & \beta \\ 0 & 0 & \alpha & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & -\beta \\ 2\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 \end{bmatrix} \quad B' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \beta \\ 0 & -2\beta \\ -2\gamma & 0 \\ \gamma & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and} \tag{29}$$

$$C' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Case 6: The boundary conditions that can be measured are the flow rate and pressure head in the end of the pipeline. The inputs are

$$\begin{cases} \Phi(L,t) = \Phi_{out}(t) \\ \Lambda(L,t) = \Lambda_{out}(t) \end{cases} \tag{30}$$

and outputs are

$$\begin{cases} \Phi(0,t) = \Phi_{in}(t) \\ \Lambda(0,t) = \Lambda_{in}(t) \end{cases} \tag{31}$$

The proposed model, inputs as well as outputs are $x = (\Phi_1 \ \Phi_2 \ \Phi_3 \ \Phi_4 \ \Lambda_1 \ \Lambda_2 \ \Lambda_3 \ \Lambda_4)^T$, $u \coloneqq (\Phi_5 \ \ \Lambda_5)^T$ and $y = (\Phi_1 \ \Lambda_1)^T$, respectively.

$$\begin{aligned}
\dot{\Phi}_1 &= \frac{-Ag}{\Delta x}(\Lambda_2 - \Lambda_1) - \frac{f}{DA}\Phi_1 \\
\dot{\Phi}_2 &= \frac{-Ag}{2\Delta x}(\Lambda_3 - \Lambda_1) - \frac{f}{DA}\Phi_2 \\
\dot{\Phi}_3 &= \frac{-Ag}{2\Delta x}(\Lambda_4 - \Lambda_2) - \frac{f}{DA}\Phi_3 \\
\dot{\Phi}_4 &= \frac{-Ag}{2\Delta x}(\Lambda_5 - \Lambda_3) - \frac{f}{DA}\Phi_4 \\
\dot{H}_1 &= \frac{-a^2}{2gA\Delta x}(\Phi_2 - \Phi_1) \\
\dot{H}_2 &= \frac{-a^2}{2gA\Delta x}(\Phi_3 - \Phi_1) \\
\dot{H}_3 &= \frac{-a^2}{2gA\Delta x}(\Phi_4 - \Phi_2) \\
\dot{H}_4 &= \frac{-a^2}{2gA\Delta x}(\Phi_5 - \Phi_3)
\end{aligned} \tag{32}$$

In (7) and (8), A, B, as well as C are represented by the block matrices as follows

$$x = (\Phi_1 \ \Phi_2 \ \Phi_3 \ \Phi_4 \ \ \Lambda_1 \ \Lambda_2 \ \Lambda_3 \ \Lambda_4)^T$$

$$A' = \begin{bmatrix} \alpha & 0 & 0 & 0 & -2\beta & 2\beta & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\beta & 0 & \beta & 0 \\ 0 & 0 & \alpha & 0 & 0 & -\beta & 0 & \beta \\ 0 & 0 & 0 & \alpha & 0 & 0 & -\beta & 0 \\ -2\gamma & 2\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \gamma & 0 \end{bmatrix} \text{ and} \tag{33}$$

$$C' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

III. PIPELINE SYSTEM ANALYSIS

The following lemmas are given to prove the controllability and observability of the system [24] [25].

Lemma. 1. Consider the system described in (7) and (8), where $A_{i\times i}$ is taken to be the state matrix, $B_{i\times 2}$ is taken to be the control matrix, also $C_{2\times i}$ is taken to be the output matrix. The controllability of matrix $\mathbb{C}_{i\times 2i}$ is defined as below [25],

$$\mathbb{C}_{i\times 2i}(A,B) \coloneqq [B \ \ AB \ \ A^2B \ \ \ A^3B \ \ \dots \ \ A^{i-1}B] \tag{34}$$

Matrix $\mathbb{C}$ is called controllable if it has full row rank.

Proof:

For case 1, the matrix $\mathbb{C}_{i\times 2i}$ can be simplified by the elementary column transformation and by using the column Echelon form transformation [26] as

$$\mathbb{C}_{i\times 2i} = \begin{bmatrix} I_{(i)(i)} & G_{(i)(i)} \end{bmatrix} \tag{35}$$

where I is the identity matrix and G is the triangular form matrix with elements of α, β and γ.

If $\gamma \neq 0, \alpha \neq 0, \beta \neq 0$, matrix $\mathbb{C}$ will be full ranked matrix and the controllability is proved.

For other cases using the same method can be proved that the controllability is full rank.

Lemma. 2. For the system described in (7) and (8), the observability of matrix $O_{2i\times i}$ is defined as below [26],

$$O_{2i\times i}(C,A) \coloneqq \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{bmatrix} \tag{36}$$

Matrix O is called observable if it has full columns rank.

Proof:

For case 1, the matrix $O_{2i\times i}$ can be simplifed by the elementary row transformation and by using the row Echelon form transformation [26] as

$$O_{2i\times i} = \begin{bmatrix} I_{(i)(i)} \\ \varphi_{(i)(i)} \end{bmatrix} \tag{37}$$

where I is the identity matrix and φ is the triangular form matrix with elements of α, β and γ.

If $\alpha \neq 0, \beta \neq 0, \gamma \neq 0$, matrix O will be full ranked matrix and the observability is proved.

For other cases using the same method can be proved that the observability is full rank.

IV. EXAMPLE

Consider the model described in Figure 1. Suppose the length of the pipeline is $L = 5.2 \times 10^3\ m$, the diameter of the pipe is $D = 0.5\ m$, the cross-section is $A = 0.1963\ m^2$, density is $\rho = 1000\ kg/m^3$, gravity is $g = 9.806\ m/s^2$, also the wave speed coefficient is $a = 1250\ m/s$. The simulations in this section are carried out for case 1 and case 2.

Control variables according to the case1 model are the pressures at the beginning and end of the pipe (Λ_1 *and* Λ_5) and can be directly measured. Furthermore, the flow rates (Φ_1 *and* Φ_4) are outputs of the system. Implementing Lemma 1 and also Lemma 2, it can be concluded that the rank of discriminant matrices is $rank\ \mathbb{C} = rank\ O\ = 16$, thus, the system is completely observable and controllable. To confirm the theorem further, an observer is designed, see Figure 4.

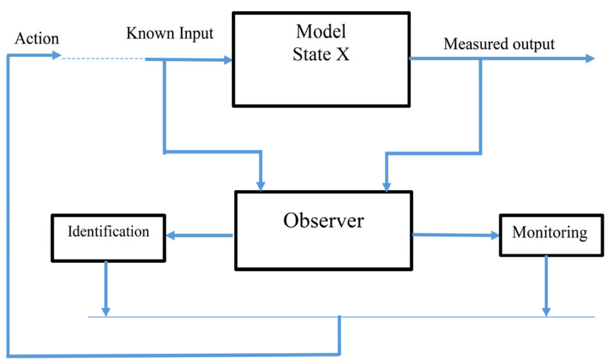

Fig. 4. The designed model with the observer

Figure 5 displays the simulated pressure head at the inlet ($\Lambda\ (in) = \Lambda_1 = 10\ m$) and outlet ($\Lambda(out\ ) = \Lambda_5 = 7\ m$) of the pipeline system (for case 1).

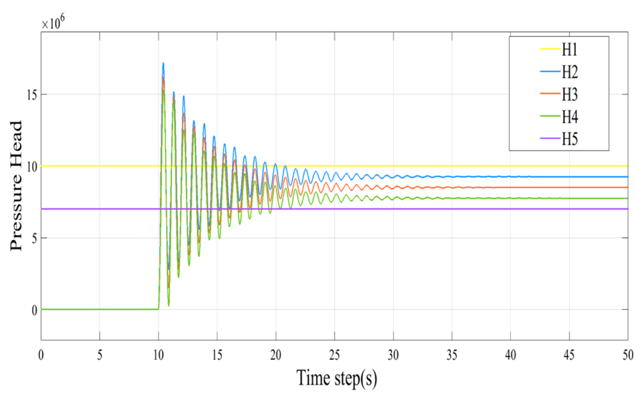

Fig. 5. Pressure head (Input) variation

The input and output flow in the pipeline is shown in Figure 6. In this figure, in just a few seconds the flow rate from the initial amount which is zero approaches into the real value.

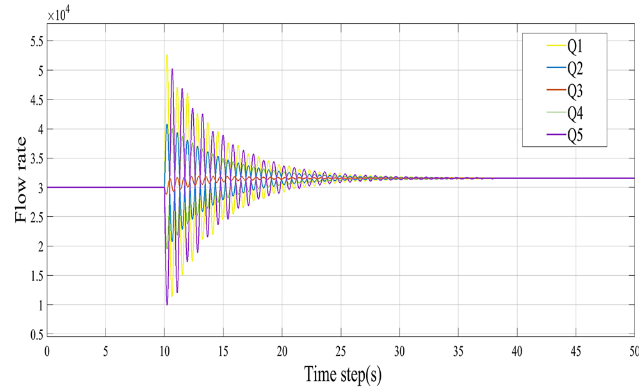

Fig. 6. Output and input flow

For case 2 the model is made of the pressures head at the beginning and flow rate at the end of the pipe (Φ_5 and Λ_1) and can be directly measured. Furthermore, the flow rates and pressures head (Φ_1 and Λ_5) are the output of the system. The pressure head variations are demonstrated in Figure 7, while the flow rate variations are demonstrated in Figure 8.

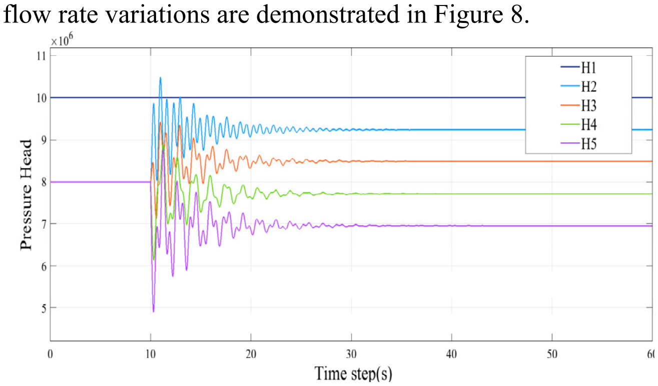

Fig. 7. Pressure head variations

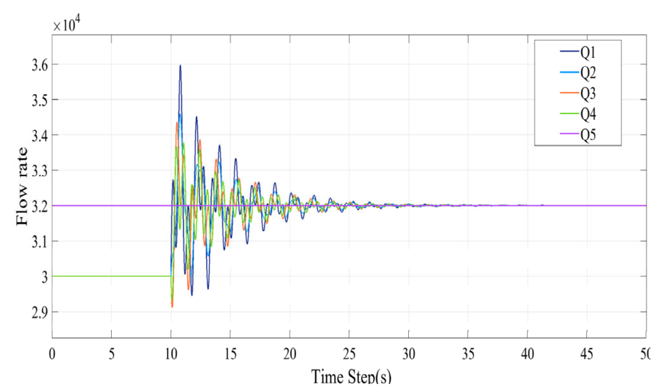

Fig. 8. Flow rate variations

V. CONCLUSION

In this paper, a novel technique is proposed to analyze and model the flow in the pipeline. The system of the pipeline is modeled using hyperbolic partial differential equations. Since it is difficult to find the solutions for these equations, so in this paper, we use the finite difference approach which is easy to apply. Important theorems are proposed for testing the observability as well as controllability of the proposed model. The future work is to study the stability of the pipeline system.

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