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Non-Linear Control for PVTOL Without Algebraic Restrictions

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***Abstract*—This paper presents a Proportional-Derivative Control strategy for the non-linear PVTOL system without algebraic restrictions. The total thrust is computed using a non-linear feedback compensation so that the altitude reaches the desired reference. The horizontal position x is then controlled by choosing the orientation angle θ as a smooth saturation function of x and $\dot{x}$. A proof of convergence is presented using a Lyapunov approach. The proposed control strategy is successfully tested in numerical simulations.**

***Index Terms*—PVTOL, non-linear systems, saturation functions, helicopter control.**

I. Introduction

The PVTOL (Planar Vertical Take-Off and Landing) system has attracted a lot of attention during the last decades. The reason for such an interest stems from the fact that it is an important theoretical problem in the automatic control community which is clearly related to a practical application. It is an underactuated system, because it has three degrees of freedom and just two control inputs, also it is a suitable candidate for the development of control laws that can be translated into controllers for more complex flying devices. The existing design methodologies for the flight control of the PVTOL aircraft model are numerous. The dynamic model is very simple but contains two trigonometric functions that make the stabilization problem quite challenging.

One of the first papers to deal with the stabilization of the PVTOL was Aguilar-Ibanez et al. [1]. This paper motivated the search for control algorithms to stabilize the PVTOL. Garcia et al. [2] proposed the use of a nested saturation control algorithm to stabilize the PVTOL. The nested saturations control structure was introduced by Teel et al. [3] to control a series of integrators in cascade with a control input having an arbitrary bound. In Castillo et al. [4] the stabilization was proved only for the linearized model of the PVTOL in the vecinity of the origin. A globally stable control algorithm for the PVTOL was proposed in Tran et al. [5]. They used the concept of virtual or ideal forces such that the PVTOL model becomes fully actuated. The orientation angle and the thrust required to generate such virtual forces are then computed. The first and second order derivatives of the orientation are used in the control algorithm and therefore the virtual forces have to be smooth functions.

Lin et al. [6] presents a robust hovering control for a V/STOL (Vertical Short Take-Off and Landing) modelling as PVTOL. In Consolini et al. [7] presents a solution to the path following problem for the PVTOL which is applicable to a class of smooth Jordan curves. Carrillo L. et al. [9], Castillo et al. [10], Hauser J. et al. [11], Lozano. L. et al [13], these authors usually use linearization around a stability point of the PVTOL system. In this way, the mathematical model and the control strategy are simplified. However, there are characteristics that are not considered or eliminated when operating in a specific region. In systems where derivatives are used, nonlinear functions operators are important. Such is the case of the trigonometric functions sine, cosine, tangent, etc. This work considers non-linear functions that are important in the mathematical model performance and control strategy design.

The contribution of the present paper is to obtain a non-linear control algorithm for the PVTOL without algebraic restrictions. The proposed control algorithm gives priority to the altitude stabilization independently of the horizontal dynamics x. In practice this avoids loosing altitude while reaching a desired horizontal set point. We propose a non-linear control using a smooth saturation function of x and $\dot{x}$ using trigonometric functions and prove stability using a Lyapunov approach. The algorithm has been simplified since we avoid including the hyperbolic tangent and sine functions in the computation of the required orientation θ.

The controller has been tested in numerical simulations. We applied it to control the vertical displacement (altitude), the horizontal displacement and the orientation angle (pitch) of the PVTOL mathematical model. Contrary to other approaches presented in the literature, the controller proposed in this document does not contemplate algebraic restrictions. The stability analysis is carried out in a simple way and a satisfactory performance in the simulation is shown.

The paper is organized as follows: Section 2 presents the PVTOL model. Section 3 describes the altitude or vertical displacement control law. Section 4 is devoted to the controller for the attitude θ. Section 5 presents the proof of stability based on the Lyapunov approach. The orientation control algorithm is given in Section 6. The numerical simulations of the proposed control strategy are shown in Section 7.

Subsection *A* describes how to implement the proposed control to a quadrotor. Final remarks are given in the conclusions.

II. PVTOL MATHEMATICAL MODEL

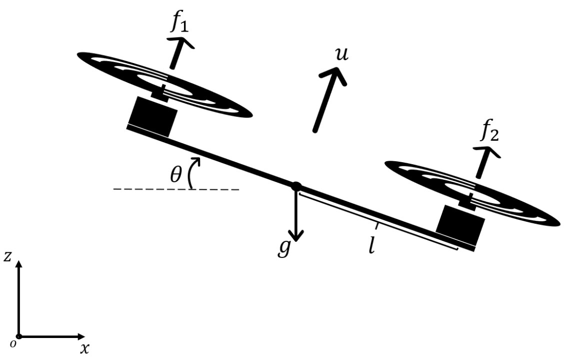

Fig. 1. PVTOL diagram. Where f_1 and f_2 are the control forces, l is the distance, m is the mass, θ is the angle of the aircraft with respect to the horizontal line, g is the gravitational acceleration, x is the horizontal displacement and z is the vertical displacement.

Figure 1 presents the classic representation of PVTOL. Let us define the total thrust u and the torque τ respectively as follows in Lozano et al. [8].

$$\begin{aligned} u &= f_1 + f_2 \\ \tau &= (f_1 - f_2)l \end{aligned}$$

Therefore the PVTOL model can be written as

$$\begin{aligned} \ddot{x} &= u \sin\theta \\ \ddot{z} &= u\cos\theta - mg \\ \ddot{\theta} &= \tau \end{aligned} \tag{1}$$

This model is the most used because it is a reduced helicopter dynamic's that is related to a practical application. In general, the PVTOL system is described as a system of 3 degrees of freedom, which is studied in 2 dimensions like Hauser J. et al. [11].

III. ALTITUDE CONTROL (z)

The vertical displacement z will be controlled by the altitude feedback as a non-linear system. We propose the following u input

$$u = mg - \dot{z} - (z - z_d) \tag{2}$$

then

$$\ddot{z} = (\cos\theta)(mg - \dot{z} - (z - z_d)) - mg \tag{3}$$

when $\theta \to 0$ then $z \to z_d$, $u \to mg$.

From (1)

$$\ddot{x} = mg\sin\theta \tag{4}$$

We propose the state feedback control input for θ as

$$\tau = \ddot{\theta}_d - k_\alpha(\dot{\theta} - \dot{\theta}_d) - k_\beta(\theta - \theta_d) \tag{5}$$

where $k_\alpha, k_\beta > 0$, from (1) after a time the x dynamics remains

$$\ddot{x} = mg\sin\theta_d \tag{6}$$

IV. HORIZONTAL CONTROL (x)

In order to stabilized the horizontal displacement x we proposed the following θ_d.

$$\theta_d = k_1 \tanh(-\dot{x} + k_2\tanh(-\dot{x} - k_1 x)) \tag{7}$$

we can rewritten (6) as

$$\ddot{x} = \sin[k_1\tanh(-\dot{x} + k_2\tanh(-y))] \tag{8}$$

where $k_1, k_2 > 0$. And

$$y \triangleq \dot{x} + k_1 x, mg = 1 \tag{9}$$

V. STABILITY PROOF

We propose the following positive function

$$V_1 = \frac{1}{2}\dot{x}^2 \tag{10}$$

Differentiating V_1 we obtain

$$\begin{aligned} \dot{V}_1 &= \dot{x}\ddot{x} \\ \dot{V}_1 &= \dot{x}\sin[k_1\tanh(-\dot{x} + k_2\tanh(-y))] \end{aligned} \tag{11}$$

If $|\dot{x}| > k_2|\tanh(-y)| > k_2$ then $\dot{V}_1 \leq 0$.

after a time $t_1 > t_0$ then $|\dot{x}| \leq k_2|\tanh(-y)| \leq k_2$.

By choosing $k_2 < \frac{1}{2}$.

$$\ddot{x} = \sin[k_1(-\dot{x} + k_2\tanh(-y))] \tag{12}$$

and selecting $k_1 < 1$, then we have

$$\dot{y} = k_1 k_2 \tanh(-y) \tag{13}$$

We propose the following positive function

$$V_2 = \frac{1}{2}y^2 \tag{14}$$

Differentiating V_2 we obtain

$$\begin{aligned} \dot{V}_2 &= y\dot{y} \\ \dot{V}_2 &= y(k_1 k_2 \tanh(-y)) \end{aligned} \tag{15}$$

$$\dot{V}_2 = y(k_1 k_2\tanh(-y)) \leq 0 \tag{16}$$

therefore $y \to 0$.

From (9) and (13)

$$\ddot{x} + k_1\dot{x} = k_1 k_2 \tanh(-y) \tag{17}$$

Then after a time $\dot{x} \to 0$ and from (9) $x \to 0$.

1) Barbalat's Lemma: Being a non-autonomous system it is necessary to verify the following assumptions.

- $V(t,x)$ is lower bounded.
- $\dot{V}(t,x)$ is negative semi-definite along the trajectories.
- $\dot{V}(t,x)$ is uniformly continuous in time.

then $\dot{V}(t,x) \to 0$, as $t \to \infty$.
Differentiating $\dot{V}_1$ and $\dot{V}_2$ we obtain

$$\begin{aligned} \ddot{V}_1 &= \dot{x}\ddot{x} = u[\dot{u}\sin\theta_d + u\dot{\theta}_d\cos\theta_d]\sin\theta_d \\ \ddot{V}_1 &\leq 0 \\ \ddot{V}_2 &= \dot{y}\ddot{y} = -\dot{y}k_1k_2 sech^2(-y)[2\tanh(-y)sech^2(-y)] \\ \dot{V}_2 &\leq 2k_1^2k_2^2 \end{aligned} \tag{18}$$

then from (16) and that $\dot{V}_1$ and $\dot{V}_2$ are bounded, it is possible to establish asymptotic stability like Khalil H. [14].

VI. Attitude Control (θ)

In order to obtain τ

$$\theta_d = k_1\tanh(-\dot{x} + k_2\tanh(-\dot{x} - k_1x)) \tag{19}$$

Differentiating θ_d we obtain

$$\begin{aligned} \dot{\theta}_d = &- k_1(k_2(k_1\dot{x} + \ddot{x})sech^2(k_1x + \dot{x})\ddot{x})* \\ &* sech^2(k_2\tanh(k_1x + \dot{x}) + \dot{x}) \end{aligned} \tag{20}$$

$$\begin{aligned} \ddot{\theta}_d =& 2k_1\tanh(k_2\tanh(k_1x + \dot{x}) + \dot{x})* \\ &* (k_2(k_1\dot{x} + \ddot{x})sech^2(k_1x + \dot{x}) + \ddot{x})^2* \\ &* sech^2(k_2\tanh(k_1x + \dot{x}) + \dot{x})- \\ &- k_1 sech^2(k_2\tanh(k_1x + \dot{x}) + \dot{x})* \\ &* (-2k_2(k1\dot{x} + \ddot{x})^2\tanh(k_1x + \dot{x})* \\ &* sech^2(k_1x + \dot{x}) + k_2(k_1\ddot{x} + \dddot{x}) \\ &* sech^2(k_1x + \dot{x}) + \dddot{x}) \end{aligned} \tag{21}$$

then from (18), (19) and (20) can be rewritten τ as

$$\begin{aligned} \tau =& 2k_1\tanh(k_2\tanh(k_1x + \dot{x}) + \dot{x})* \\ &* (k_2(k_1\dot{x} + \ddot{x})sech^2(k_1x + \dot{x}) + \ddot{x})^2* \\ &* sech^2(k_2\tanh(k_1x + \dot{x}) + \dot{x})- \\ &- k_1 sech^2(k_2\tanh(k_1x + \dot{x}) + \dot{x})* \\ &* (-2k_2(k1\dot{x} + \ddot{x})^2\tanh(k_1x + \dot{x})* \\ &* sech^2(k_1x + \dot{x}) + k_2(k_1\ddot{x} + \dddot{x})* \\ &* sech^2(k_1x + \dot{x}) + \dddot{x}) - k_\alpha(\dot{\theta} - k_1(k_2(k_1\dot{x} + \ddot{x})* \\ &* sech^2(k_1x + \dot{x})\ddot{x})sech^2(k_2\tanh(k1x + \dot{x}) + \dot{x}))- \\ &- k_\beta(\theta - k_1\tanh(-\dot{x} + k_2\tanh(-\dot{x} - k_1x))) \end{aligned} \tag{22}$$

where

$$\dddot{x} = \dot{u}\sin\theta + u\dot{\theta}\cos\theta \tag{23}$$

considering z_d as constant

$$\dot{u} = -\ddot{z} - \dot{z} \tag{24}$$

VII. Simulation Results

The PVTOL model and the non-linear control proposed have been tested in numerical simulations using MatLab-Simulink. Where $m = 1$, $g = 9.81$, $d = 1$, $k_a = 50$, $k_b = 100$, $k_1 = 0.8$, $k_2 = 0.4$ and $z_d = 1$. These values were chosen to ensure a sequential convergence in the dynamics, ensuring that the first convergence is θ then z and finally x.

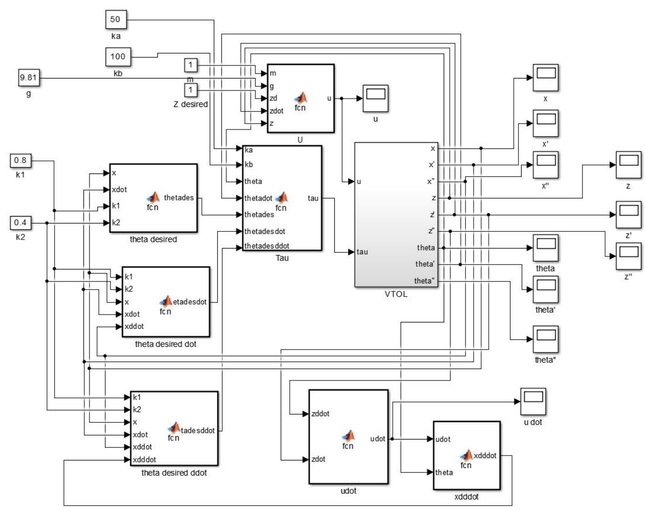

Fig. 2. Simulink diagram. The blocks that contain: θ values and their derivatives (calculated in the previous section), PVTOL model (corresponding to section 2), values of selected gains and the input control u and its derivative.

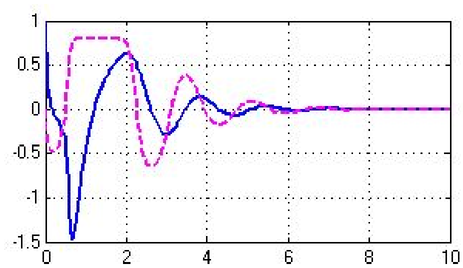

Fig. 3. Attitude θ in continuous line and attitude desired θ_d in dashed line. The vertical axis corresponds to the PVTOL attitude orientation angle (rad) and the horizontal axis corresponds to the time (t) in seconds.

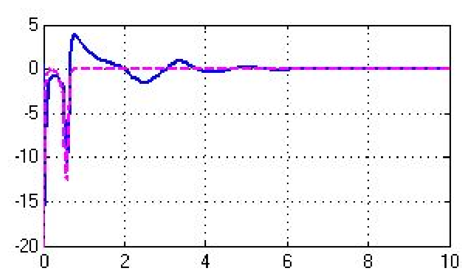

Fig. 4. Attitude first derivative $\dot{\theta}$ in continuous line and attitude desired $\dot{\theta}_d$ first derivative in dashed line. The vertical axis corresponds to the PVTOL attitude orientation angle first derivative (rad/s) and the horizontal axis corresponds to the time (t) in seconds.

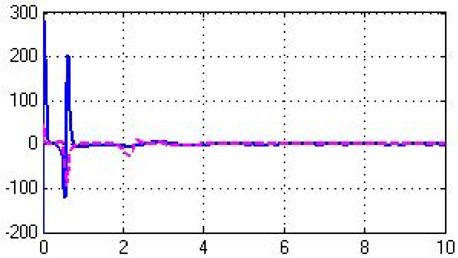

Fig. 5. Attitude second derivative $\ddot{\theta}$ in continuous line and attitude desired $\ddot{\theta}_d$ second derivative in dashed line. The vertical axis corresponds to the PVTOL attitude orientation first derivative (rad/s^2) and the horizontal axis corresponds to the time (t) in seconds.

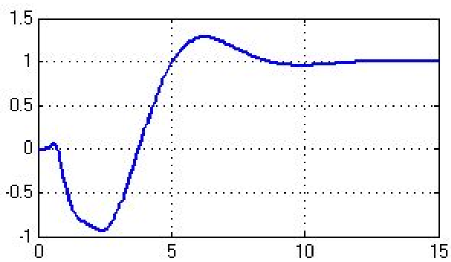

Fig. 6. Altitude z. The vertical axis corresponds to the PVTOL altitude z in meters (m) and the horizontal axis corresponds to the time (t) in seconds.

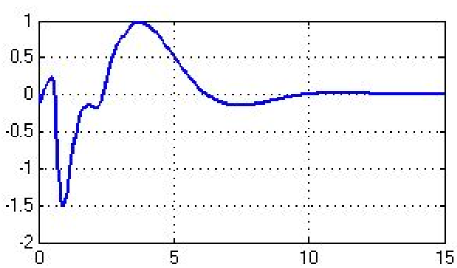

Fig. 7. Altitude first derivative $\dot{z}$. The vertical axis corresponds to the PVTOL altitude first derivative (m/s) and the horizontal axis corresponds to the time (t) in seconds.

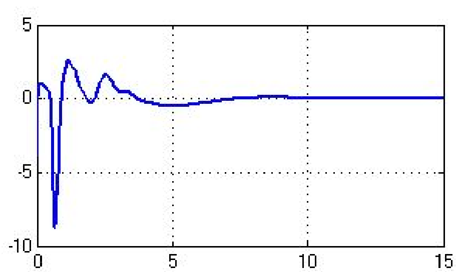

Fig. 8. Altitude second derivative $\ddot{z}$ The vertical axis corresponds to the PVTOL altitude second derivative (m/s^2) and the horizontal axis corresponds to the time (t) in seconds.

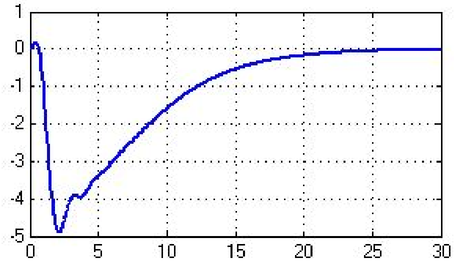

Fig. 9. Horizontal displacement x. The vertical axis corresponds to the PVTOL horizontal displacement (x) in meters (m) and the horizontal axis corresponds to the time (t) in seconds.

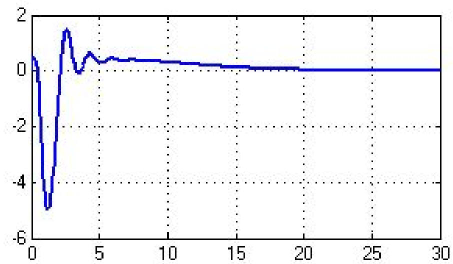

Fig. 10. Horizontal displacement first derivative $\dot{x}$. The vertical axis corresponds to the PVTOL horizontal displacement first derivative $\dot{x}$ (m/s) and the horizontal axis corresponds to the time (t) in seconds.

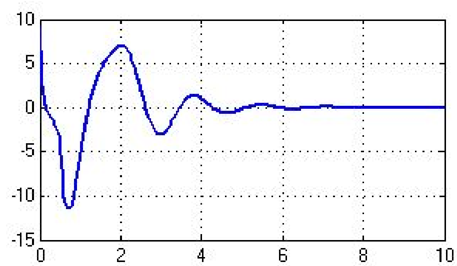

Fig. 11. Horizontal displacement second derivative $\ddot{x}$. The vertical axis corresponds to the PVTOL horizontal displacement second derivative $\ddot{x}$ (m/s^2) and the horizontal axis corresponds to the time (t) in seconds.

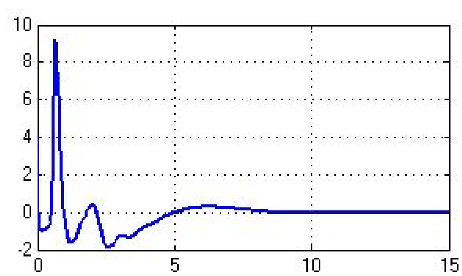

Fig. 12. Control input u. The vertical axis corresponds to the PVTOL control input u in newtons (N) and the horizontal axis corresponds to the time (t) in seconds.

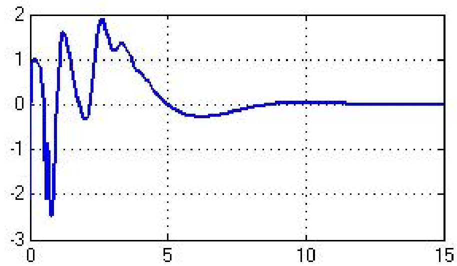

Fig. 13. Control input first derivative $\dot{u}$.

A. Quadrotor mathematical model

Indeed, the PVTOL is a non-linear dynamical system that represents the simplified longitudinal model of a quadrotor Figure 14. The controller has been tested in numerical simulations, but also it is possible to apply it to control the altitude, the pitch angle and the horizontal displacement of a quad-rotor helicopter. Therefore the quadrotor model can be written as Lozano et al. [8].

$$\begin{aligned} m\ddot{x} &= u\sin\theta \\ m\ddot{y} &= -u\sin\phi\cos\theta \\ m\ddot{z} &= u\cos\phi\cos\theta - mg \\ \ddot{\eta} &= \tau \end{aligned} \tag{25}$$

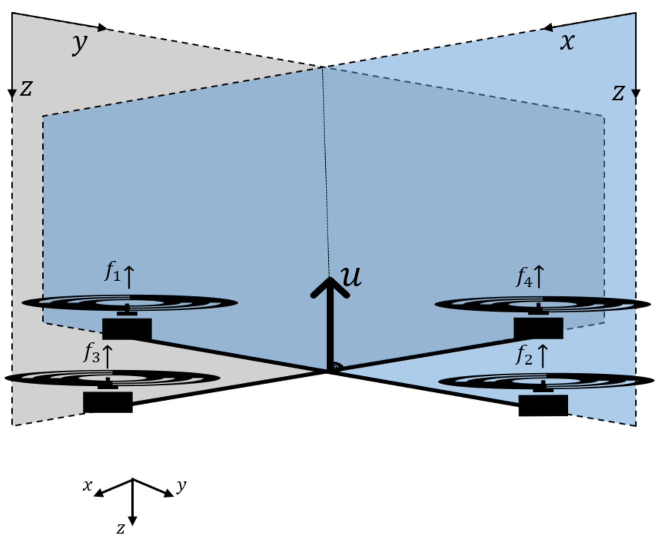

Fig. 14. Quadrotor diagram.

- x-axis. When $\phi\to 0$

$$\begin{aligned} m\ddot{x} &= u\sin\theta \\ m\ddot{z} &= u\cos\theta - mg \\ \ddot{\theta} &= \tau_\theta \end{aligned} \tag{26}$$

- y-axis. When $\theta\to 0$

$$\begin{aligned} m\ddot{y} &= -u\sin\phi \\ m\ddot{z} &= u\cos\phi - mg \\ \ddot{\phi} &= \tau_\phi \end{aligned} \tag{27}$$

The equations (25) and (26) could be represent the PVTOL dynamics like (1) only in the case of y axis we have the following definition:

$$\phi_d = -k_{1\phi}\tanh(-\dot{y} + k_{2\phi}\tanh(-\dot{y} - k_{1\phi}y)) \tag{28}$$

we follows the same procedure describe before.

VIII. Conclusions

This paper has presented a new non-linear control strategy for the well known problem of the PVTOL without algebraic constraint considerations. The altitude control has been obtained by non-linear compensation which guarantees that the altitude will reach the desired altitude in a short period of time. A desired orientation has been proposed as a function of smooth saturation function of x and $\dot{x}$. It has been proved that when the orientation reaches the desired orientation then the closed loop for the horizontal displacement is Lyapunov stable. The proposed control strategy has been tested in numerical simulations. In fact, we present in this article the approach to apply the control design methodology proposed in a quadrotor helicopter where the applicability of the method has been validated in section *A*.

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