Velocity Field Description of the Cartesian Motion Induced by a Sliding Condition in Mechanical Systems

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***Abstract*—Sliding Mode Control is a control design technique based on the induction of an attractive dynamic condition in the phase plane that guides the system to a desired state. It has been broadly used to solve motion problems of regulation and tracking; however, the induced motion in the operational space is not accurately described by the attractive condition and the induced responses of the close loop system intrinsically disturbs the expected performance. In order to improve the knowledge of the Sliding Mode approach to describe control objectives for precision motion control, in this paper the author analyzes the induced motion by a first order Sliding Mode either in the configuration or in the operational space. He describes the sliding condition as an equivalent velocity field to represent the trajectories to reach the motion objective from an initial position, as the streamlines of the field, and determines the equivalent velocity field for regulation and contouring. The convenience of having a precise description of the induced motion in a Sliding Mode is illustrated, first, showing the undesired response of two variables with different convergence speed and then, calculating the induced motion in the operational space when the variables of the system verify the sliding condition in the configuration space using as an example a 2DoF planar manipulator.**

I. Introduction

Sliding Mode Control is a control design technique based on the induction of an attractive dynamic condition to drive the state of a system to a desired position. It is a special mode in Variable Structure Systems where the induced condition is a convergent first order linear equation of the variables describing the state of the system [1] [2]. This equation, or sliding surface, is usually defined as a flat plane describing a manifold of the configuration space induced by forced changes in the dynamic structure of the system [1] [2] [3]. It is represented in the phase plane by a single straight line in the attractive quadrants [4].

The Sliding Mode Control is claimed to be a robust technique that has been broadly used to solve motion problems of regulation and tracking in mechanical systems [5] [6]; in particular, for robot manipulators, designing a sliding condition of an extended error in velocity that induces an attractive

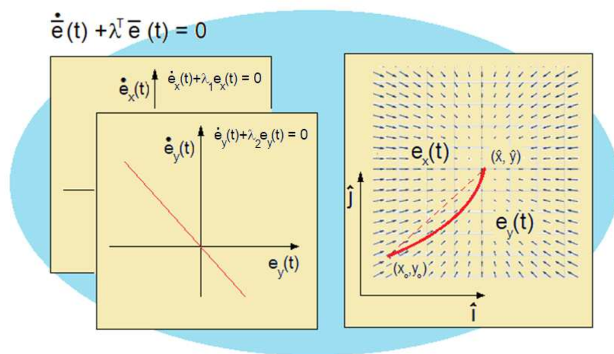

Fig. 1. The Sliding Mode of the position error induces a Cartesian motion in the operational space that can be described by the streamlines of an equivalent velocity field.

behavior on the generalized coordinates where the reference in velocity implicitly include the information about the desired position [7] [8].

It is expected that applying a Sliding Mode Controller in a robot manipulator, it achieves a perfect performance even in the presence of arbitrary parametric inaccuracies, and experimental results show that, besides the problems of chattering caused by the high frequency switching between two dynamic structures, the generalized coordinates convergence to the desired position [9] [10]; even improving the responses obtained with conventional PID controllers [11] [12]. However, some results show only the dynamic response of the generalized coordinates without taking into account the Cartesian motion of the final effector. This motion in the operational space, given the corresponding transformation between the generalized and the Cartesian coordinates, describes curved trajectories instead of the expected trajectories in the direction of the shortest distance to the objective. Even more, if the sliding condition is defined for every coordinate with different rates of convergence, the induced trajectories on the configuration space are curved, increasing the unconsidered deviation from the expected straight-lines trajectories in the operational space (Fig. 1). It is important to remark that the unconsidered deviation of the approaching trajectories is induced by the controller itself as a result of an inconsistency in the designing method creating a conflict between the controller and the expected reaction of the closed loop system.

The Sliding Mode condition is defined as an analytical expression of the state that imposes an attractive dynamic to a desired position. It is represented as a linear surface in the configuration space or as a straight-line in the attractive quadrants of the phase-plane of the generalized coordinates, and describes the attractive condition without accurate information of the followed trajectories once the Sliding Mode is reached. Unfortunately, the connection between the sliding condition and the induced motion of the state, has been ignored letting the description of the final close loop behavior under Sliding Mode Control as an open problem. In this paper, in order to provide a better understanding of the Sliding Mode Control, in particular, concerning the induced Cartesian motion in the operational space, the author analyses the spatial motion of a system when its dynamics reaches a sliding condition and, describes it, either for regulation or for contour tracking, by means of an equivalent velocity field whose streamlines determine the allowed trajectories to reach a motion objective. The author illustrates the convenience of having a precise description of the induced motion in a Sliding Mode, first discussing the case of two variables with different convergence speed and then, calculating the induced Cartesian motion in the operational space when the variables of the system verify the sliding condition in the configuration space, using, as an example of mechanical system, a 2DoF planar manipulator.

II. Connection between Sliding Mode and Velocity Field

The Sliding Mode Control is a robust design technique with a finite-time convergence based on the induction of an attractive dynamic mode to the motion objective [3]. The attractive mode is described as an analytical condition in a reduced-order compensated dynamics, known as Sliding Surface, that, linearly relates the position and its temporal rate of change to simultaneously direct them to the origin. In this sense, the concepts of Sliding Mode and Velocity Field are connected in such a way that the sliding condition can be interpreted as defining a particular convergent velocity field where the allowed trajectories can be described as a Sliding Mode to the motion objective.

A. Sliding Mode as a Velocity Field

A Sliding Surface of a generalized dynamic variable of a second order system is defined in the configuration space as the surface $s(.,t) = 0$, where

$$s(\bar{x}(t)) = \frac{d\bar{x}(t)}{dt} + \lambda \bar{x}(t), \tag{1}$$

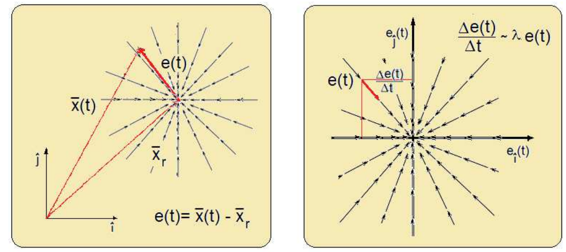

Fig. 2. Graphic representation of the sliding condition as a velocity field describing the spatial distribution of the instantaneous changes in direction of the position error to reach the motion objective.

with $\bar{x}$, a dummy variable used to describe the attractive analytical condition, and λ a strictly positive constant. In the surface $s(\bar{x}(t)) = 0$ the variable verifies the attractive condition $\frac{d\bar{x}(t)}{dt}\bar{x}(t) \leq 0$ implying that $lim_{t\to\infty}\bar{x}(t) = 0$ but imposing an exponential convergence without overshoot nor oscillations [4].

The variable $\bar{x}$ can be defined according to the application of interest as a state variable $\vec{x}(t)$ or as a manifold of the tracking error,

$$e(t) = \vec{x}(t) - \vec{x}_r(t) \tag{2}$$

with $\vec{x}_r(t)$ the reference in position [1] [2]; also, as in the case of mechanical systems, as the extended error in velocity,

$$S_{rx}(t) = \dot{\vec{x}}(t) - \vec{v}(t). \tag{3}$$

where $\vec{v}(t)$ is the reference in velocity which is defined to contain intrinsically information about the desired state of the system either in position or velocity.

If the sliding condition is verified for the position error $e(t)$ with respect to the objective (Eq. 2), the induced dynamic on the Sliding Surface,

$$\dot{e}(t) + \lambda e(t) = 0, \tag{4}$$

implies that the velocity of the error and the error are strongly related. The instantaneous change in direction induced by the Sliding Condition on the error, are determined by the error it-self, i.e.

$$\dot{e}(t) = -\lambda e(t). \tag{5}$$

This relation can be interpreted as a definition of a velocity field in the error that will be used in ulterior sections to describe, graphically, the induced motion by the sliding condition (Fig.2).

B. Trajectories in Velocity Fields as Sliding Modes

Velocity fields in the Cartesian space, $\vec{v}(x) \in \mathbb{R}^3$, are a static vector field of the position describing the instantaneous change in direction experienced by a particle immersed in the field. The field, given the differential relation between position and velocity, $\vec{v} = \frac{dx}{dt}$, intrinsically contains information about the allowed trajectories and their converging positions, x_r, that can be used as a reference for a motion task.

A convergent velocity field, used as a motion reference in velocity by means of the extended error in velocity, $S_{rx}(t)$

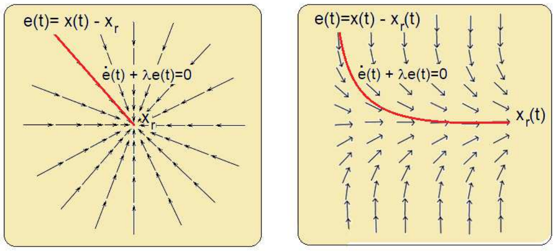

Fig. 3. Perfect tracking of the allowed trajectories in a convergent velocity field verifies a Sliding Mode to the motion objective

(Eq.3), is a dynamic reference $\vec{v}(t) = \vec{v}(x(t))$ from a moving particle immersed in the field with position $x(t)$. The linear approximation of the velocity field in a neighborhood of the motion objective x_r, is

$$\begin{aligned} S_r(t) &= \dot{\vec{x}}(t) - \vec{v}(\vec{x}(t)) \\ &= \dot{\vec{x}}(t) - \vec{v}(\vec{x}_r(t)) + \nabla' v(\vec{x}_r(t))(\vec{x}(t) - \vec{x}_r(t)) + \epsilon^2 \\ &= \dot{e}(t) + \lambda e(t) + \epsilon^2, \end{aligned} \tag{6}$$

where the error, $e(t)$, and its time derivative, $\dot{e}(t)$, are calculated with respect to the position, $\vec{x}_r$, and its corresponding velocity vector $\vec{v}_r$ (see for instance Figure 3). The constant λ is the spatial derivative of the field at the point x_r which is positive in a convergent and continuous flow, and ϵ^2 is a quadratic error such that $\lim_{x \to x_r} \epsilon^2 = 0$. In a perfect tracking of the velocity field, i.e. $S_{rx}(t) = 0$, the condition

$$0 = \dot{e}(t) + \lambda e(t). \tag{7}$$

is verified describing a Sliding Mode that attracts the state of the system to the equilibrium point $(\dot{e}(t), e(t)) = (0, 0)$ as illustrated in Figure 3. The attractive flow in the field converges to its main streamline described by a directed curve $G(.)$, expressed as a function of its arc length s. The references in position and velocity, $(\vec{x}_r(t), v(x_r(t))$, in the linear approximation are replaced by the closest point to the curve and the tangent vector, $(G(s_0), \vec{T}(s_0))$, respectively; thus, the largest invariant set such that $(\dot{e}(t), e(t)) = (0, 0)$ is the uniform speed motion along the curve $G(s)$. If the attractive flow of the field converges soft and continuously to $G(s)$, λ is a positive function of the distance to the curve as described in [14] (Fig. 4).

III. Equivalent Velocity Field of the Induced Motion

The Sliding Mode condition induces a attractive dynamic behavior whose description in terms of spatial trajectories is not always clear. In the case of a scalar variable $\bar{x}(t) \in \mathbb{R}$, the Sliding Mode (Eq. 4) induces a convergent motion described by an exponential function whose constant λ determines its speed of convergence (Fig. 3). However, for vectorial variables describing the motion in the Cartesian space, $\bar{x}(t) \in \mathbb{R}^n|_{n=2,3}$, an intuitive knowledge of the trajectories induced by the sliding condition is not always easy and exposes the engineers to neglect undesired behavior of the closed loop system that

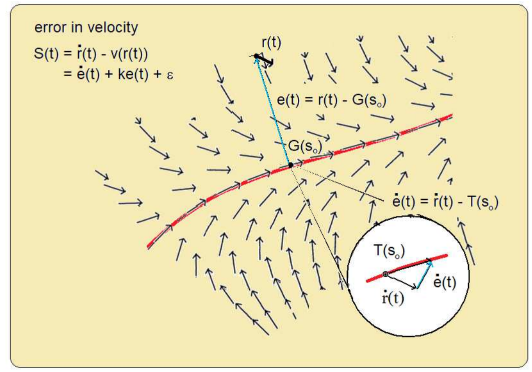

Fig. 4. Errors in Position and Velocity, $(e(t), \dot{e}(t))$, with respect to a uniform speed motion on the curve $G(s)$.

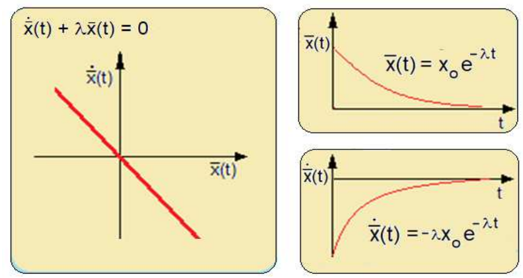

Fig. 5. Sliding Mode condition and the induced motion in a scalar variable $\bar{x}(t) \in \mathbb{R}$.

can affect its whole performance. Recalling that the Sliding Condition is a dynamic constrain strongly relating a variable with its time-derivative, it can be interpreted as a particular case of velocity field (Eq. 5). Then, the induced trajectories when the dynamic of the system stays in the Sliding Mode are the allowed trajectories described by the equivalent velocity field and represented graphically by the streamlines of its flow. An equivalent velocity field for the sliding condition applied to the regulation and tracking problem in motion control, provides a clear and intuitive graphic representation of the possible trajectories induced by the system in the Sliding Mode, allowing a better understanding of the control action.

A. Equivalent Velocity Field for Regulation

Regulation in mechanical systems concerns the motion objective of reaching, from an initial condition, a desired position and remaining there despite incertitude and disturbances. Thus, the position error for regulation, $e_r(t)$, is described as in Equation (2) but considering a static reference $\vec{x}_r$, i.e.

$$e_r(t) := \vec{x}(t) - \vec{x}_r, \tag{8}$$

where $e_r(t) \in \mathbb{R}^n$ with n the number of coordinates in the configuration or in the operational space according to the motion description employed by the control engineer.

The sliding condition (Eq.4)applied to the regulation error can be interpreted as defining a velocity field, $v_r(e)$, where the vectors of velocity are defined to be proportional to the

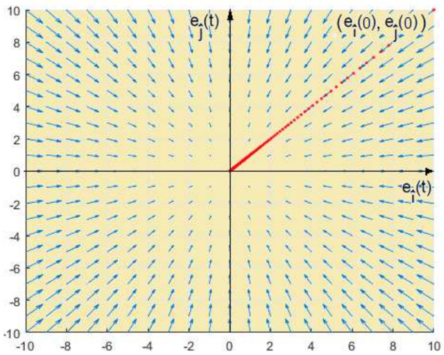

Fig. 6. Equivalent velocity field of the sliding condition applied to the Cartesian error in position for regulation. The red line indicates the solution of the Sliding Mode equation starting from the initial condition $e(0) \in \mathbb{R}^2$.

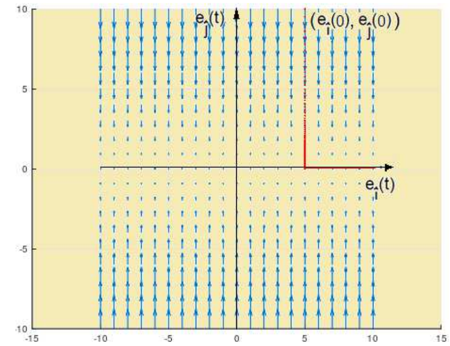

Fig. 7. Equivalent velocity field of the sliding condition applied to the Cartesian error in position for tracking a uniform motion on the axis of abscissae. The red line indicates the followed trajectory starting from the initial condition $e(0) \in \mathbb{R}^2$ to reach the curve before following it.

position error referred to the point of the motion objective (Eq.5), i.e,

$$v(e(t)) := -\lambda e(t). \tag{9}$$

with $\lambda \in \mathbb{R}^n$ a constant vector with positive entries and n the dimension of $e_r(t)$. The velocity field thus defined, is a decreasing attractive radial field pointing to the objective. The induced motion correspond to the allowed radial trajectories described by the streamlines where the largest invariant set when the origin is attain, $e_r(t) = 0$, is the point of reference, $vecx_r$.

In order to illustrate the graphic representation of the induced motion in the Sliding Mode by means of the equivalent velocity field of the Equation (9), in Figure 6, it is introduced an example of a 2D Velocity Field, $n = 2$, with $e_r(t) \in \mathbb{R}^2$ in a Cartesian space where λ is chosen, without loss of generality, as a unitary constant, $\lambda = 1$, to calculated the velocity field as $v_r(e) = e_r(t)$. The streamlines of the velocity flow describe the induced trajectories, starting from a particular initial condition, by the Sliding Condition.

It is verified that the streamlines in the equivalent velocity field describe the trajectories induced by the Sliding Mode including in Figure 6 the trajectory described by the analytical solution of the sliding condition (Eq.4), i.e.

$$(e_i(t), e_j) = (e_i(0)e^{-\lambda t}, e_j(0)e^{-\lambda t}) \tag{10}$$

with $\lambda = 1$, initial condition $e_r(0) = (10, 10)$ and $t = \{1 \dots 10\}$. This discrete trajectory in red, starting from the initial position and ending on the point of reference $e_{r0} = (0, 0)$, follows the attractive radial direction where the speed of the motion, represented by the density of the red dots, continuously decrease as the position gets closer to the objective.

Observe that the position error can be described either in the configuration or in the operational space according to the motion objectives and engineering criteria. Its importance, as well as that of the parameter λ determining the final Cartesian motion in the operational space of the system, will be discussed in the next section.

B. Equivalent Velocity Field for Contour Tracking

The problem of tracking considerer in this work consists of a particular case of reaching and following a time varying objective moving in a continuous trajectory described by a curve $G(.)$ where the close contact to the curve is a priority. Thus, if the tracking problem is solved under performance criteria of contouring instead of a perfect time tracking [13], the position error, $e_t(t)$ is defined with respect to the closest point on the curve $G(s_0)$ from a position $\vec{x}(t)$, i.e. $x_r(t) = G(s_0)$, i.e.

$$e_t(t) := \vec{x}(t) - \vec{x}_r(t) \tag{11}$$

where $e_t(t) \in \mathbb{R}^n$ with n the number of coordinates in the configuration or in the operational space. The equivalent velocity field describing the induced motion by the Sliding Mode for tracking is inspired in the linear relation between position and velocity imposed by the sliding condition (Eq.4) and also, considering the Sliding Mode described by the motion on the streamlines of the field (Eq.7) with the parameter $\lambda = \nabla' v(\vec{x}_r(t))$ (Eq.6) as a constant $\lambda \in \mathbb{R}^n$ with n the dimension of $e_t(t)$, i.e. to assure the allowed trajectories described by the field verify the sliding condition with the motion on the reference curve, $x_r(t) = G(s(t))$, as the largest invariant set when $e_r(t) = 0$ ([14]). The resulting velocity field describes a parallel streamlines flow, perpendicular to the curve of the motion objective, where the velocity vectors $v_t(e)$ are directed to the curve in the direction of the shortest distance with a magnitude proportional to this distance.

An example of the equivalent velocity field for tracking contouring error (Eq.6) showing graphically the induced motion by the sliding condition (Eq.8 is illustrated in Figure 6. The figure shows a 2D Velocity Field, $e_r(t) \in \mathbb{R}^2$, with $\lambda = (1, 1)'$ where the desired trajectory $G(.)$ is an horizontal straight line on the abscissae axis.

As in the previous example, the streamlines of the velocity flow describe the induced trajectories by the sliding condition starting from the initial position $e_r(0) = (e_i(0), e_j(0))$ and ending in a motion along the desired trajectory where the speed is reduced until the curve is attain and then start moving along the curve. This was verified by the straightforward calculation of the dynamic trajectory

$$(e_i(t), e_j) = \begin{cases} (0, -e_j(0)e^{\lambda t}) & if \quad e_j(t) \neq 0 \\ (G(s_0 + t), 0) & if \quad e_j(t) = 0 \end{cases} \qquad (12)$$

with $\lambda = 1$, initial condition $e_r(0) = (10, 10)$ and $t = \{1 \dots 10\}$, and where the objective curve is defined as uniform speed motion along the abscissae axis, $G(t) = (0, t)$. The discrete trajectory in red, starts from the initial position and reach the curve at $G(s_0)$ before continuing its motion on $G(s_0 + t)$. The speed of the motion is discontinuous, first gradually decreasing in the ordinate direction until the curve is reached to suddenly continue with a uniform speed on $G(t)$.

It is important to remark that the velocity field described as a particular case of tracking concerned with contouring, allows a static representation of the velocity field induced by the sliding condition, otherwise, for a perfect time tracking, the induced trajectories to reach the dynamic objective are continuously modified according to the relative position of the moving objective impossible to be described with a static representation.

IV. Example of the induced motion in a 2 DoF manipulator

The graphic representation of the induced motion by a Sliding Mode using an equivalent vector field allows a clear and intuitive understanding of the expected dynamic behavior in a closed loop system. It is possible to appreciate undesired behavior of the induced motion in the operational space to be considered by the control engineers. It is illustrated this possibility with an example based on a 2DoF manipulator showing first, the importance of the parameter λ in the final form of the induced trajectories, and then, the response on the operational space when the Sliding Mode is induced in the configuration space. All the graphics and calculations, as well as those in the previous section, were developed using GNU Octave version 4.2.2 [15].

A. Parameter λ in the form of the attractive trajectories

The parameter λ, associated with the speed of convergence in the exponential response of the Sliding Mode (Eq. 4), is, in general, indifferently selected either as a positive scalar or as a vector with positive entrances, nor even indicating if these entrance are different. In any case, intuitively it is assumed that once on Sliding Mode, the attractive trajectory followed by the system to the point of reference is a straight-line. However, if the speed of convergence is different for any of the variable, the resulting trajectory is curved, disturbing the expected behavior of the closed loop system.

In order to illustrate how the differences in the lambda parameter introduce inherent deviation from an attractive straight-line trajectory to the motion objective once the Sliding Mode is attain, in Figure 8, it is presented the equivalent velocity fields for regulation of a 2DoF system, as introduced in the previous section, but including both cases, either with equal speed of convergence, $\lambda_{e_i} = lambda_{e_j} = 1$, or different speed with twice the speed of convergence in the abscissae with respect to the speed in the ordinates, $\lambda_{e_i} = 2$ and $\lambda_{e_j} = 1$.

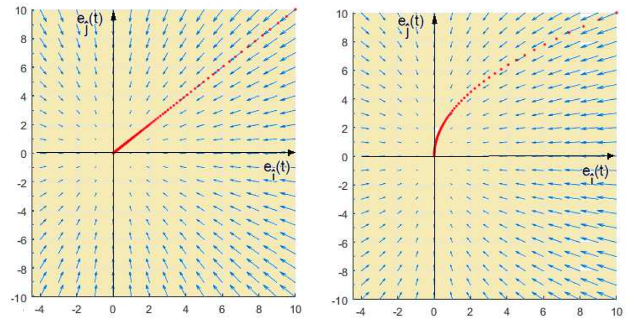

Fig. 8. Deviation from the straight line trajectory induced by the sliding condition when different values of λ are assigned to the different coordinates.

At first sight in both velocity fields the converging flows are similar, almost equal, but observing the induced motion, in red, calculated from the analytical solution of the sliding condition with different values of lambda, initial condition $e_r(0) = (10, 10)$, reference $e_{r0} = (0, 0)$, and $t = \{1 \dots 10\}$, the induced motion describe a convex curve. This behavior must be taken into account in control approaches that use independent joint controllers for each link of the manipulators and especially when they are implied in precision motion tasks to avoid undesired motion that affect the mechanical performance of the system.

B. Trajectories in the Operational Space induced by the Sliding Mode

In order to illustrate the importance for control design of knowing the induced mechanical motion by a Sliding Mode condition. It is analyzed the attractive velocity field described of the sliding condition in the configuration space and its corresponding field in the operational space describing the Cartesian motion. In the velocity field representation it is possible to appreciate how the straight-line trajectories to the objective are transformed into convex streamline curves that deviate the approaching trajectories, even inducing changes in the direction of the motion.

In particular, as an illustrative example, it is calculated the induced velocity field in a Sliding Mode (Eq.1), either in the configuration space or in the operational one for a 2DoF planar manipulator where l_1, l_2 are the length of the links as showed in Figure 9 with $(q_1, q2)$ the angular variables in the configuration space, related to the Cartesian coordinates (x, y) of the operational space by

$$(x(t), y(t)) = (cos(q_1) + cos(q_2), sin(q_1) + sin(q_2)). \quad (13)$$

In Figure 10, it is showed the equivalent velocity field for regulation when the Sliding Mode is attain in the configuration space, with the same values of $\lambda = q_1 = \lambda = q_2 = 1$, describing a radial attractive flow to the desired reference (q_{1r}, q_{2r}), and the corresponding velocity field describing the induced motion in the operational space to the reference (x_r, y_r).

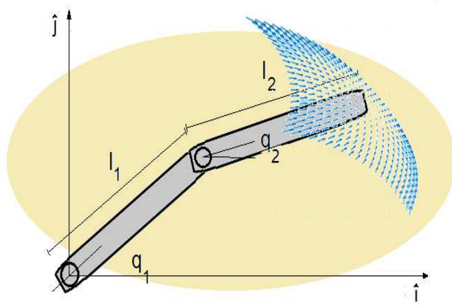

Fig. 9. 2 DoF Manipulator used to illustrate the Cartesian motion in the operational space (x, y), induced by the Sliding Mode condition applied to the generalized coordinates in the configuration space, (q_1, q_2), where $l_1 = l_2 = 1$, $q_r = (0, 77, 0.2)$, and $\Delta q_1 = \Delta q_2 = +/-0.5$.

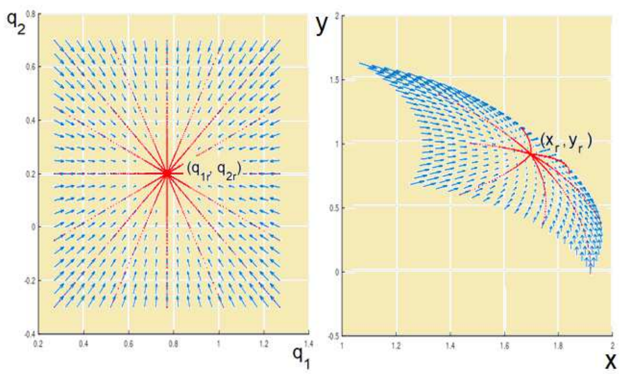

Fig. 10. Approaching trajectories induced in the operational space by the Sliding Condition applied to the configuration space represented by the streamlines of the equivalent velocity field

In order to verify the correspondence between the velocity fields on the configuration and the operational space, it is introduced a series of trajectories calculated from the analytical solution of the Sliding Mode equation. The trajectories starts in different initial conditions uniformly distributed around the motion objective, and the corresponding trajectories are attractive, slowing down, and radial trajectories uniformly distributed. On the other hand, after transforming the field to the operational space, the streamlines in the vector field are curved as in a projection of a curved surface on the plane x-y and the transformed trajectories calculated from the analytical expression are coincident with the allowed trajectories in the Velocity Field (Figure 10). Observe that the deviation from the expected attractive trajectories to the motion objective is even more important if in the Sliding Mode the variables are attracted with different speed of convergence, as explained in the previous section.

V. Conclusion

In this paper it is presented an equivalent representation of the Sliding Mode by velocity fields whose streamlines describe the attractive trajectories induced by the sliding condition, in particular for regulation and contouring, without considering the reaching phase to the sliding surface. This representation provide an intuitive visual image of the induced motion by the Sliding Mode condition that could be very helpful to improve the use of the Sliding Mode as a control design approach. Disregarding the form of the induced motion, assuming a straightforward attraction introduces inherent uncertainties to the control design that affects the performance of the final application, especially in precision motion control, including contouring and touch interaction. This unexpected behaviour could explain why reported results of control design in the configuration space are very sensible to changes in the motion objective, requiring a trial-and-error tuning for every specific solution in the operational space, mainly observed in systems where the state of the system is evaluated in the operational space and the control is applied in the configuration space. The graphic representation of the induced motion by the sliding condition provides a better understanding of the Sliding Mode Control to improve its possibilities as a control design technique for real applications.

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