

A Study by Finite Elements of the Transport of Magnetic Nanoparticles in a Straight Microchannel under the Influence of a Magnetic Field Generated by a Current Line

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Abstract — Some microfluidic devices for the analysis of biological samples use magnetic nanoparticles as markers of bioactive agents. When they circulate through the microchannels, they diffuse depending on the forces acting in the MNP. In this article a model of the distribution of magnetic nanoparticle concentration in a straight microchannel is present. The distribution of MNP is affected by the influence of a magnetic field generated by a current line and the dynamics of the fluid that transports them. To model the dynamic behavior of the nanoparticles, the Navier-Stokes equation and the convection-diffusion equation were combined with the magnetic field equation.

The concentration of MNP was distributed evenly in the region near the current line. The simulations performed are used as a tool for the design of a microfluidic system for the detection and quantification of cancer cells.

Keywords — Microfluidics, magnetic nanoparticles, concentration, convection-diffusion, gradient magnetics.

I. INTRODUCTION

CURRENTLY it is possible to detect some biological entities, such as cancer cells by labeling with magnetic nanoparticles (MNP) functionalized with antibodies [1-2]. Detection is performed with the small magnetic signal emitted by a certain concentration of MNP that binds to a cell. Due to the magnetic nature of MNP, they can be manipulated by magnetic gradients in so-called microfluidic devices. A microfluidic based sample analyzer can offer multiple advantages over a macroscale system. For example, small sample volume, portability and low manufacturing costs compared to the costs of commercial equipment such as the flow cytometer [1].

Nowadays, microfluidic chips have been developed for

different applications in biosensors and for the separation of samples using MNP [1]. In order to verify the adequate operation of the microfluidic devices, theoretical foundations of the behavior of the fluids and the transport of MNP in confined spaces of micrometric dimensions are required. The magnetic particles have magnetic susceptibility so they can interact with external magnetic fields [3]. Computational models can be used as a tool for the design of microfluidic devices, which can improve the efficiency and the functions for which they are manufactured.

Microfluidics is studied in the macroscopic sense with the equations of fluid mechanics; however, the fluid that crosses the microchannels transports matter. Therefore, it is necessary to make theoretical models that allow us to know the distribution of the concentration of MNP in a moving fluid. This distribution depends on two main forces: the fluidic force and the magnetic force with which the MNP are excited. The magnetic source can be: A permanent magnet or current lines to direct the MNP to a certain region [4].

The distribution model of the concentration of MNP was analyzed and calculated theoretically by the finite element method [4-5]. The flow conditions and the magnetic field generated by a current line determine the behavior of the MNP transport in the microchannel.

This model is inspired by a microfluidic system that is currently being developed for the detection and quantification of cancer cells [5], to detect the magnetic field emitted by a concentration of MNP, GMR sensors [5] are used. This model works as a reference for the behavior of the concentration of MNP in the sensor region.

II. METHODOLOGY

The flow and magnetic field conditions generated by the current line determine the behavior of the distribution of MNP along the microchannel. There are two forces that act on the

magnetic particle: the fluidic force (Fs) and the magnetic force (Fm); forces such as gravity were not taken into account in the model.

The fluidic force with which a spherical particle is transported by a fluid is obtained with Stokes's law [6]

$$\vec{F}_s = -6\pi\eta r_p (\vec{v}_p - \vec{u}), \quad (1)$$

where r_p is the radius of the particle [m], η it is the viscosity [Ns/m²], and \vec{u} is the speed of the fluid [m/s].

The magnetic force on a particle with magnetic susceptibility χ is given by the following expression [6]

$$\vec{F}_m = \mu_0 V_m \frac{3\chi}{\chi+3} (\nabla \cdot \vec{H}_a) \vec{H}_a, \quad (2)$$

where V_m is the particle volume [m³] and \vec{H}_a is the magnetic field strength.

A. Magnetic field equation

The distribution of the magnetic field \vec{B} [T] is obtained by solving the following equation [6]

$$\nabla \times \left(\frac{1}{\mu_0 \mu_r} (\nabla \times \vec{A} - \vec{B}_r) \right) = \vec{j} \quad (3)$$

$$\vec{B} = \nabla \times \vec{A} \quad (4)$$

where \vec{j} is the current density of the line [A / m²]; and Br, the remaining flux density that is equal to zero, \vec{A} is the vector potential magnetic and μ_r is the relative permeability ($\mu_r = 1$) [6].

B. Fluid flow equation

To describe the behavior of the fluid velocity field within the microchannel, the continuity equation and the modified Navier-Stokes equation are solved [6]

$$\nabla \cdot \vec{u} = 0 \quad (5)$$

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + (\nabla \cdot \vec{u}) \vec{u} \right) = \nabla P + \eta \nabla^2 \vec{u} + \vec{F}_{vol} \quad (6)$$

where P is the pressure [Pa], \vec{u} is the fluid velocity field [m/s], ρ is the fluid density [kg/m³], and η is the viscosity of the fluid [Ns/m²].

\vec{F}_{vol} It is the force of the volume. it is related to the magnetic force of a single particle with the following relation [6]

$$\begin{aligned} \vec{F}_{vol} &= n \vec{F}_m = \mu_0 \eta V_m \frac{3\chi}{\chi+3} (\nabla \cdot \vec{H}_a) \vec{H}_a \\ &= C_v \mu_0 \frac{3\chi}{\chi+3} (\nabla \cdot \vec{H}_a) \vec{H}_a \end{aligned} \quad (7)$$

where C_v is the volume concentration [mol/m³].

C. Convection–diffusion equation

To the concentration of the dimensionless volume c is given by [6]

$$C_v = c C_{v0} \quad (8)$$

where C_{v0} is the initial volume concentration [mol / m³]

To describe the transport of MNP within the microfluidic channel, the diffusion convection equation, which is given by [6]

$$\frac{\partial c}{\partial t} + (\nabla c) \cdot \vec{v}_p = D \nabla^2 c \quad (9)$$

where D is the diffusion coefficient [m²/s]

\vec{v}_p is the velocity of the particle. It is given by the following relationship [6]

$$\vec{v}_p = \vec{u} + \frac{\vec{F}_m}{6\pi\eta r_p} \quad (10)$$

where \vec{u} is the fluid velocity field from Eqs. (5) y (6) and \vec{F}_m it is the magnetic force of the particle.

D. Implementation and domains

The microchannel was modeled with a two-dimensional rectangle of 20 mm x 500 μ m. To numerically solve the partial differential equations in two dimensions described earlier in this geometry, the COMSOL Multiphysics software was used.

The magnetic field was produced by a current line, which is 20 mm long and 0.3 mm wide. Between the current line and the channel, an aerial region was defined. External limits were defined as magnetic insulation. The mesh element size for the current line and the air region was 0.1 mm. The mesh size in the microchannel region was 0.005 mm.

The complete simulation consisted of three parts, the magnetic model, the modeling of the flow through the microchannel and the convection-diffusion of the MNP through the channel. They were resolved sequentially with a time-dependent study.

The magnetic model was calculated in the microfluidic channel and in the air region.

The modeling of the flow through the microchannel depends on the volumetric force, which is well defined with the intensity of the magnetic field obtained from the magnetic

model. At the entrance of the channel, the speed was set as $u_0 = 0.004 \text{ m/s}$. At the exit of the channel, the pressure is zero. The boundary conditions are zero speed on the walls of the channel. To know the distribution of the MNP concentration, the convection and diffusion model was used, it was calculated only in the microfluidic channel.

The speed of the particle depends on the flow velocity profile and the magnetic force of the particle according to equation (10). The parameters used during the simulation are presented in Table I.

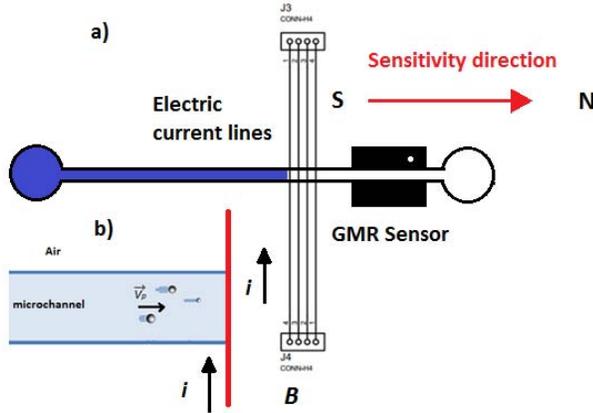


Fig. 1. a) Representation of the microfluidic channel implemented, before the detection region, four current lines were implemented to manipulate the MNP. However, the model is made with a single current line. b) Expansion of the microfluidic channel before the current line. The liquid flows in laminar conditions [6].

TABLE I
SIMULATION CONSTANTS.

| Symbol | Parameter | Value |
|--------|-------------------------|---|
| r_p | Radius of the particle | $150 \times 10^{-8} \text{ m}$ |
| χ | susceptibility magnetic | 1 |
| η | Viscosity | 0.001 Ns/m^2 |
| D | Diffusion coefficient | $1 \times 10^{-9} \text{ m}^2/\text{s}$ |
| u_0 | Entry speed | $1 \times 10^{-4} \text{ m/s}$ |
| Cv_0 | Initial concentration | 0.001 |
| ρ | Fluid density | 1000 kg/m^3 |

III. RESULTS AND DISCUSSIONS

In the region of 10 mm to 20 mm, the magnetic field density is more intense, the results of the concentration distribution for different instants of time ($t = 0.5 \text{ s}$, $t = 2 \text{ s}$ and $t = 3 \text{ s}$) are explained below.

In Fig. 2 it is shown (the diffusion of the MNP for $t = 0.5 \text{ s}$). The concentration region is highly distributed in the channel, where the magnetic field is relatively large, that is, 0.35 mT to 13 mm of the channel; On the other hand, the speeds are small. In this case, we can consider a situation in which the magnetic force is greater than the drag force of the fluid.

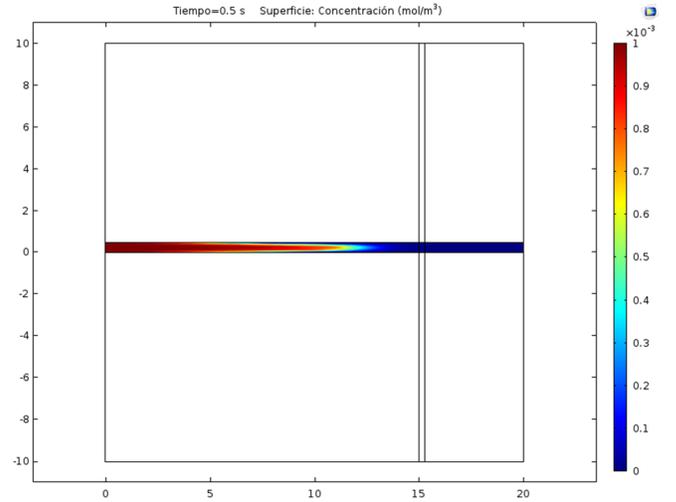


Fig. 2. Distribution of MNP concentration at $t = 0.5 \text{ s}$. In this case, we can consider a situation in which the magnetic force is greater than the drag force of the fluid.

In Fig. 3 it is shown (the diffusion of the MNP for $t = 2 \text{ s}$). A region of high concentration is observed in the current line where the magnetic field is more intense and uniform, in this case the MNP have been concentrated uniformly in the region of the current line.

In Fig. 4 is shown (the diffusion of the MNP for $t = 3 \text{ s}$). The distribution of the concentration of MNP has moved almost outside the current line, the particles have some resistance to the flow of the fluid due to the magnetic field given the elapsed time.

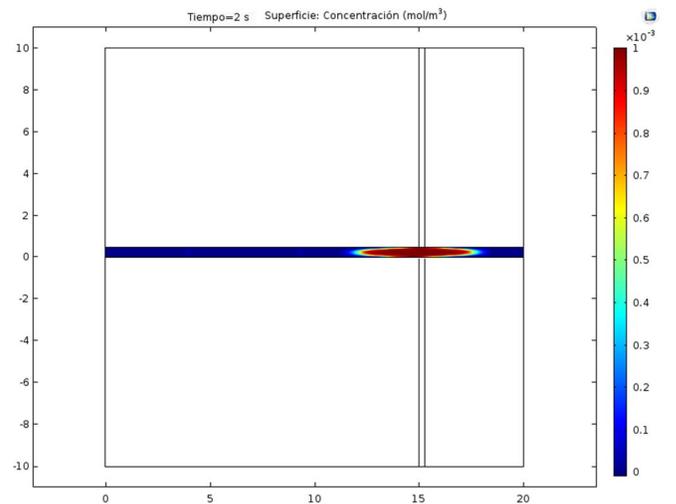


Fig. 3. Distribution of the MNP concentration at $t = 2 \text{ s}$, the concentration is evenly distributed in the region of the current line where the magnetic field is constant.

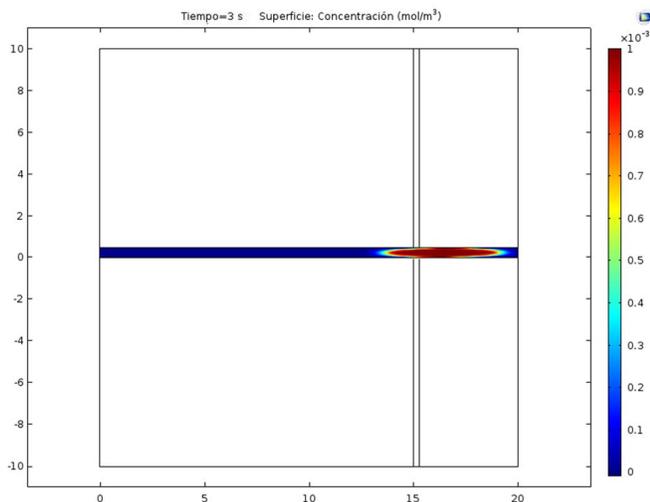


Fig. 4. Distribution of MNP concentration at $t = 3$ s, the particles have some resistance to fluid flow due to the magnetic field.

This model under ideal conditions is consistent with what would intuitively be expected from the distribution of magnetic nanoparticle concentration. However, the model can be improved by varying the current of the line and observing the distribution for different magnetic fields. On the other hand, a parametric study can be implemented by varying the flow velocity to observe the concentration of MNP. What can be useful to have a reference to the minimum excitation field of the MNP and the optimal flow velocity.

IV. CONCLUSION

This model was made using the finite element method, combining Navier-Stokes nonlinear equations and convection diffusion with the magnetic field equation to obtain the distribution of the concentration of MNP in a frontal microchannel under the influence of the two forces, namely, the drag force of the fluid and the magnetic force generated by a current line. The results appear to be intuitively consistent with the concentration distribution in the region near the current line. On the contrary, to validate the model, it is necessary to carry out experiments that allow the visualization of the MNP in a specific region of the channel. However, the model serves as a reference to design microfluidic systems.

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