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Decentralized sliding-mode control of robotic manipulator with constraint workspace: a finite-convergent barrier Lyapunov approach

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***Abstract*—The aim of this manuscript is to develop a robust output-based controller for a robot manipulator (RM), under the presence of state constraints. A novel methodology is proposed to solve the tracking trajectory problem between the joints position of a RM and desired reference trajectories, using a barrier sliding mode (BSM) control. The controller design guarantees the tracking trajectory and the fulfillment of the state constraints considering the presence of perturbations. In this study, it is considered that only the angular position of each RM articulation is available. A decentralized adaptive super-twisting algorithm estimates the angular velocity of the corresponding RM. A class of barrier Lyapunov function (BLF) is used to analyze the finite-time stability of the sliding surface in a closed-loop with the proposed controller. An example of a numerical evaluation considering a two-link RM illustrates the advantages obtained with the proposed methodology compared to the non-barrier control design.**

***Index Terms*—Sliding Modes, Constrained Systems, Robot Manipulator**

I. INTRODUCTION

Robot manipulators (RM) are electro-mechanical devices used in research and industrial sectors [1]. Nowadays, there are diverse mechanical designs, manufacturing materials and configurations of RM that are designed taking into account the complexity of the task to be realized. Also, the environment where the robot has to exert their trajectories plays an important role for the mechanical design, the electronic instrumentation and the controller proposal [2]. Usually, the RM movements depend on a predefined set of reference trajectories (closed-loop) [3]. In other cases, these trajectories can be provided by a human operator (open-loop) [4].

From the mechanical point of view, the RM is a classic type of systems with position constraints, due to the diverse mechanisms used to integrate their joints [5]. Then, it is reasonable to assume that the desired trajectories must be bounded, in order to avoid damage to the robotic device.

Notice that the mechanical limits are not the only reason that induce a constraint for the RM. There exist many examples corresponding to RM tasks, where the device must exert the movements of each link with a constraint joint space, that implies indirectly a class of constrained working space. The surgical robots may be a clear example of such working scenario [6], [7], under the assumption that each joint of the surgical robot must avoid the collision with surrounding anatomical structures. Figure 1, evidences the possible conditions inducing a position constraints in the RM.

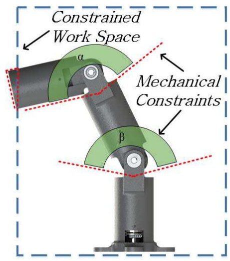

Fig. 1. Constraints example.

The RM operation realizing their movements in a constrained joint space has motivated the developing of new control schemes, which can guarantee the tracking of a reference trajectory and the simultaneous fulfillment of predefined position constraints. From the automatic control point of view, the trajectory tracking problem, can be solved by diverse robust and adaptive control schemes. Being of the most notable options, the linear feedback controllers, linear quadratic regulators (LQR) [8], sliding mode controllers (SMC) [9] appeared as natural options to solve the trajectory tracking problem in the presence of uncertainties, measurement noises and some other challenging operative scenarios. Nevertheless, these approaches may not satisfy the state constraints all the time, at least theoretically.

Among all the aforementioned controllers, the continuous forms only guarantee the convergence of the tracking error to the origin (theoretically) if the system is known everywhere. However, due to the parametric uncertainties and the non-modeled dynamics, the tracking error only converges to an invariant zone close to the origin. Due to the simplicity to implement in robotic systems and the high efficiency showed to control systems with matching disturbances [10], the sliding mode controller is one of the most developed control technique with the aim of solving the tracking trajectory problem in RM. Nevertheless, the classic sliding mode controllers do not include in its mathematical structure, the technical elements to guarantee the fulfillment of the known state constraints.

Recently, some control schemes such as adaptive sliding modes [11], model predictive control [12], reference governors [13] and barrier Lyapunov functions (BLF) appeared as potential approaches to solve the tracking trajectory problem for constraint dynamic systems. In literature, one may find some studies that improve the sliding mode controller performance using BLFs. However, they are not justifying

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the finite-time convergence.

This study offers a first order barrier sliding mode (BSM) control design which is proposed to solve the tracking trajectory problem for a RM considering the full-state constraints. The main contributions of this work are:

-The control scheme guarantees that the tracking error between the joints position of a RM and a set of desired reference trajectories converges in a predefined zone close to the origin considering the presence of external disturbances.

-The characteristic overshoot effect of the first order sliding mode can be constrained in a predefined set. Then, the fulfillment of the joints position constraints is ensured while the trajectory tracking is realized.

II. Problem statement

A. Preliminaries

This subsection introduces some basic preliminaries regarding the concept and applications of the BLF.

Definition 1: (Barrier Lyapunov Function) [14] Let the set $\Psi \subset \mathbb{R}^n$ be an open set with the boundary $\partial\Psi$ and $V : \Psi \to \mathbb{R}_+$ be a continuous function in $\mathbb{R}^n$.

V is a BLF if it is a positive definite, continuously differentiable in Ψ,

$$\lim_{x \to \partial\Psi} V(x) \to +\infty \tag{1}$$

and $V(x) \leq c, \quad \forall t \geq 0$ for some $c \in \mathbb{R}_+$ and for any $x(0) \in \Psi$.

If $\dot{V}(x) < 0$ and $x(0) \in \Psi$, it is clear that $c = V(x(0))$, any future trajectory is bounded inside of Ψ.

The next lemmas are useful in the stability analysis presented in this manuscript to obtain the time derivative of the BLF.

Lemma 1: Let λ a positive constant. Then, the following inequality holds:

$$-\log(1-\lambda) < \frac{\lambda}{1-\lambda}, \quad \forall|\lambda| \leq 1 \tag{2}$$

Lemma 2: Let λ a positive constant, the following inequality holds:

$$\frac{-\lambda^2}{1-\lambda^2} \leq -\log\left(\frac{1}{1-\lambda^2}\right), \quad \forall\lambda < 1 \tag{3}$$

B. Plant description

Let consider the following Euler-Lagrange equation describing the motion of a RM system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + P(q) + \xi(q,u) = u \tag{4}$$

where $\dot{q} \in \mathbb{R}^n$ and $q \in \mathbb{R}^n$ describe the velocity and the angular position of the RM, respectively. The matrix $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the corresponding positive definite inertia matrix. The matrix $C : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ corresponds to the Coriolis matrix, $P : \mathbb{R}^n \to \mathbb{R}^n$ is the vector of gravitational torques. The control input of the manipulator is represented by $u \in \mathbb{R}^n$. The nonlinear function $\xi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ represents the effect of internal uncertainties, external perturbations like dry friction and so on.

The inertia and Coriolis matrices are assumed to satisfy the following properties [15]:

P1. There exist positive constants m^+ and m^- such that $m^- \leq \|M\| \leq m^-$.

P2. The matrix $M - 2C$ are skew symmetric.

P3. There exists a positive constant C^+ such that $\|C\dot{q}\| \leq C^+\|\dot{q}\|^2$.

Let define $x_1 = q$ and $x_2 = \dot{q}$. Using the state variables theory, the equation (4) can be transformed to:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= H(x) + G(x_1)u + \Xi(x,u) \end{aligned} \tag{5}$$

where $x = \begin{bmatrix} x_1^\top & x_2^\top \end{bmatrix}^\top$ is the vector of generalized coordinates for the manipulator, with $x_1 \in \mathbb{R}^n$ and $x_2 \in \mathbb{R}^n$. The terms associated with the Coriolis matrix C and the gravitational vector torques P are gather in $H : \mathbb{R}^{2n} \to \mathbb{R}^n$ that is a quasi-Lipschitz function described as follows:

$$H(x) = M^{-1}(x_1)(-C(x)x_2 - P(x_1)) \tag{6}$$

Considering that the matrix M and C fulfill the properties **P1** and **P3**. Then, the function H satisfies the following assumption.

Assumption 1: The nonlinear function H is unknown but satisfy the next inequality:

$$\|H(x)\| \leq h_0 + h_1\|x\|^2 \tag{7}$$

In equation (5), $G : \mathbb{R}^n \to \mathbb{R}^n$ is the state dependent matrix defined as $G = M^{-1}$ that characterizes the effect of the input toque described by u. The nonlinear function $\Xi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is defined as $\Xi = M^{-1}\xi$.

Throughout the paper, the following assumptions are assumed to be fulfilled.

Assumption 2: The function G is known, invertible and bounded according to:

$$g^- \leq \|G(x_1)\| \leq g^+ \quad \forall x_1 \geq 0 \tag{8}$$

with $g^- \in \mathbb{R}^+$ and $g^+ \in \mathbb{R}^+$.

Assumption 3: The control input u belongs to the admissible set defined as:

$$U_{adm} \triangleq \{u : |u| \leq u_1\|x\|^2 + u_2\} \tag{9}$$

where $u_1 \in \mathbb{R}^+$ and $u_2 \in \mathbb{R}^+$.

In *Assumption 3*, the control set U_{adm} includes discontinuous controllers or state feedback controllers. Moreover, the control input u enforces that the equilibrium point of the system (5) is stable (with bounded evolution) at least.

Notice that a RM is a clear example of the class of systems with physical constraints. In consequence, it is realistic to consider that all states of the system are bounded. Then, the robotic system satisfies the next assumption:

Assumption 4: The states of the RM are bounded as follows:

$$\begin{aligned} \|x_1\|^2 &\leq x_1^+, \quad x_1^+ \in \mathbb{R}^+ \\ \|x_2\|^2 &\leq x_2^+, \quad x_2^+ \in \mathbb{R}^+ \end{aligned} \tag{10}$$

then, it is valid assuming that $\|x\|^2 \leq x^+$.

Assumption 5: The nonlinear function $\Xi(x)$ is unknown but satisfies:

$$\|\Xi(x)\| \leq \Xi_0 + \Xi_1\|x\|^2 \tag{11}$$

with $\Xi_0 \in \mathbb{R}^+$ and $\Xi_1 \in \mathbb{R}^+$.

III. PROBLEM STATEMENT

The issue tackle in this study is: to solve the trajectory tracking between the states associated to the system (5) and a set of desired reference trajectories $x^* = \begin{bmatrix} x_1^{*\top} & \dot{x}_1^{*\top} \end{bmatrix}^\top$ with $x_1^* \in \mathbb{R}^n$, $\dot{x}_1^* \in \mathbb{R}^n$ subject to the state constraints given by $K_b = \begin{bmatrix} K_{b1}^\top & K_{b2}^\top \end{bmatrix}^\top$ with $K_{b1} \in \mathbb{R}^+$ and $K_{b2} \in \mathbb{R}^+$ that represents a desire constraints for the tracking error.

The RM is a typical system with mechanical constraints. Therefore, it is reasonable assuming that their trajectories are bounded. Based on the previous fact, the desired trajectories fulfill the following assumption.

Assumption 6: The reference trajectory x^* and its derivative are know and satisfies:

$$\begin{aligned} \|x_1^*\| &\leq r_1^+ < x_1^+ + K_{b1} \\ \|\dot{x}_1^*\| &\leq r_2^+ < x_2^+ + K_{b2} \end{aligned} \tag{12}$$

where $r_1^+ \in \mathbb{R}^+$ and $r_2^+ \in \mathbb{R}^+$ represent the upper bound for the desired reference trajectory and its derivative.

The *Assumption 6* evidences that all the desired trajectories for the robotic system are bounded and never must violate the constraint defined by $x_1^+ + K_{b1}$. The previous fact matches with the mechanical nature of the robotic system propose in this study.

Let us introduce the tracking error $\Delta \in \mathbb{R}^{2n}$ defined as follows:

$$\Delta = x - x^* \tag{13}$$

Then, the problem statement could be rephrased as follows: to propose a feedback controller for the system (5) to satisfy:

$$\overline{\lim}_{t\to\infty}\|\Delta\| \leq \beta \quad \forall t \geq 0, \quad \beta \in \mathbb{R}^+ \tag{14}$$

subject to the following constraints:

$$\|\Delta_1\| \leq K_{b1}, \quad \|\Delta_2\| \leq K_{b2} \tag{15}$$

In equation (14), $\Delta = \begin{bmatrix} \Delta_1^\top & \Delta_2^\top \end{bmatrix}^\top$ represents the tracking trajectory error between the states of the RM described by x and the desired reference trajectory x^*. Each component of the vector Δ is defined as:

$$\Delta_1 = x_1 - x_1^*, \quad \Delta_2 = x_2 - \dot{x}_1^* \tag{16}$$

Remark 1: Notice that if the assumptions 4 and 6 (that define the bounds for the states and the desired reference trajectories, respectively) are fulfilled, implies that the quadratic norm of the tracking error Δ must be bounded as follows:

$$\|\Delta\|^2 \leq \|K_b\|^2 + x^+ \tag{17}$$

IV. CONTROL DESIGN

In this section, the controller design based on the sliding mode theory is developed. The proposed controller solves the tracking trajectory between the joints of the RM and the desired reference trajectories subject to the full state constraints.

Usually the proposed controllers for RMs assumed that x_1 and x_2 can be measured on-line and simultaneously. However, the regular electronic instrumentation in the RM considers only position sensors, then only x_1 is available.

The previous problem has been solve using the so-called state observers. Actually, there exist in the literature many options to solve the velocity estimation problem, being one of the most remarkable the finite-time observers.

In particular, this manuscript proposes the use of a class of super-twisting algorithm which possesses many relevant properties that can be useful to solve the tracking trajectory problem.

A. Adaptive sliding mode observer

A finite-time observer based on the sliding mode theory is used to estimate the angular velocity of the robotic device.

Remark 2: Notice that the considered RM is a class of forward-complete system, then the stability analysis is carried out independently of the state observer.

In particular, this study considers an adaptive super-twisting algorithm [16] that satisfies the following equation.

$$\begin{aligned} \dot{\hat{x}}_1 &= -\lambda_1|\Delta_o|^{0.5}\text{sign}(\Delta_o) \\ \dot{\hat{x}}_2 &= -\lambda_2\text{sign}(\Delta_o) \end{aligned} \tag{18}$$

where $\hat{x}_1 \in \mathbb{R}^n$ describes the state estimation of x_1 (position estimation) and $\hat{x}_2 \in \mathbb{R}^n$ provides the velocity estimation. Here, λ_1 and λ_2 are the time-varying gains of the adaptive super-twisting. According with Utkin and Poznyak in [16], the adaptation law of these gains, satisfies the following equation:

$$\dot{\lambda}_i = \gamma_i\lambda_i\text{sign}(\overline{\Delta}_o) - \Lambda[\lambda_i - \lambda_i^+]_+ + \Lambda[\nu - \lambda_i]_+ \tag{19}$$

where $\overline{\Delta}_o$ is defined as:

$$\overline{\Delta}_o = |[\text{sign}(x_1)]_{eq}| - \zeta, \quad \zeta \in (0,1) \tag{20}$$

Here, the function $[\text{sign}(x_1)]_{eq}$ is an averaged value of the function $\text{sign}(x_1)$ that can be easily obtained by a low-pass filter. For more details see ([10]).

In the adaptation law (19), the operator $[\mu]_+$ is described as follows:

$$[\mu]_+ = \begin{cases} 1 & if\mu \geq 0 \\ 0 & if\mu < 0 \end{cases} \tag{21}$$

with the constants $\Lambda_i > \gamma_i\lambda_i^+$ and $\gamma_i > 0$, where the subscript $i = 1, 2$. The upper value λ_i^+ for the gain λ_i must satisfy $\lambda_i^+ > \varrho^+$ with ϱ^+ a positive constant such that $(h_0 + \xi_0) + (h_1 + \Xi_1\|x\|^2) < \varrho^+$.

Notice that the gain λ_i can vary in the range $[\nu \quad \lambda_i^+]$. The term $\nu > 0$ defines a pre-selected minimal value of λ_i.

B. System decomposition

Consider the system (5) that describes a fully actuated RM with n degrees of freedom. The dynamic model of RM can be represented as a set of second order systems, then the system (5) satisfies the next representation:

$$\begin{aligned} \dot{x}_{1j} &= x_{2j} \\ \dot{x}_{2j} &= H_j(x) + G_j(x_1)u_j + \tilde{\Xi}_j(x,u) \end{aligned} \tag{22}$$

where x_{1j} and x_{2j} describe the $j - th$ and $(j+1) - th$ states of the robotic system, with $j = 1, ..., n$. In (22), the functions $H_j(\cdot)$ and $G_j(\cdot)$ describe the dynamics of the states x_{1j} and x_{2j}.

In equation (22), the function $\bar{\Xi}_j$ represents the disturbance that affect the $j-th$ subsystem of the RM. The case study presented in this manuscript considers that each state x_{2j} is affected by all components of the vector input u. This fact can be rewritten as follows:

$$\bar{\Xi}_j = \sum_{p=1,p\neq j}^{n} G_{j,p}(x_1)u_j + \Xi_j(x) \tag{23}$$

Notice that in equation (23), each product $G_{j,p}(x_1)u_j$ can be handled using the following inequality:

$$|G_{j,p}(x_1)u_j| \leq |G_{j,p}||u_j| \tag{24}$$

Then, by the assumptions 2 and 3, it is clear that:

$$|G_{j,p}(x_1)u_j| \leq g^+(u_1 + u_2\|x\|^2) \tag{25}$$

Under the *Assumption 5*, the term $\overline{\Xi}_j$ is bounded as:

$$\|\overline{\Xi}_j\|^2 \leq \overline{\Xi}_0 + \overline{\Xi}_1\|x\|^2 \tag{26}$$

where $\overline{\Xi}_0 = \Xi_0 + g^+u_2$ and $\overline{\Xi}_1 = \Xi_1 + g^+u_1$.

C. Barrier Sliding Mode Control

Consider the system decomposition proposed in section IV A. The main goal of this section is to design a controller for each one of the second order subsystems that describe the MR.

By considering the subsystem (22), the corresponding tracking error Δ_j can be represented as

$$\Delta_j = x_j - x_j^* \tag{27}$$

The full-time derivative of Δ_j satisfies:

$$\begin{aligned}\dot{\Delta}_{1,j} &= \Delta_{2,j} \\ \dot{\Delta}_{2,j} &= H_j(x) + G_j(x_1)u_j + \overline{\Xi}_j(x) - \dot{x}^*_{1,j}\end{aligned} \tag{28}$$

To solve the tracking trajectory problem of the RM, let us introduce the sliding variable s_j for each articulation defined as:

$$s_j = \rho_j\Delta_{1,j} + \Delta_{2,j} \tag{29}$$

where ρ_j is a positive constant. Notice that the constraints proposed in equation (15) implies that the sliding variable is bounded by a constant $K_{s,j}$, that satisfy:

$$K_{s,j} = \rho_j K_{b1,j} + K_{b2,j} \tag{30}$$

Consider the following control scheme:

$$u_j = G_j^{-1}(u_{1,j} + u_{2,j}) \tag{31}$$

with $u_{1,j} \in \mathbb{R}^n$ and $u_{2,j} \in \mathbb{R}^n$ satisfy the following equations:

$$\begin{aligned}u_{1,j} &= -(h_0 + h_1\|x\|^2 + \rho_j\Delta_{2,j} - \dot{x}^*_{1,j}) \\ u_{2,j} &= -k_j\text{sign}(s_j)\end{aligned} \tag{32}$$

In equation (32) the control gain k_j satisfy the next structure:

$$k_j = \frac{\epsilon_j}{2(1-\zeta_j)}\left(\Xi_0 + \Xi_1\|x\|^2 + \frac{0.5^{0.5}s_j}{\zeta_j}\right) \tag{33}$$

with the auxiliary variable $\zeta_j = \dfrac{s_j}{K_{s,j}}$.

Theorem 1:

Let us consider the system defined in equation (22) in closed-loop with the control u_j described by (31) adjusted with the control gain proposed in (33). If there exist some constants $\epsilon_j \in \mathbb{R}^+$ and $K_{s,j} \in \mathbb{R}^+$ such that $|\zeta_j| < 1$, then the sliding surface converges in finite-time to the origin. Furthermore, the states of system (22) converges asymptotically to the desired reference trajectory fulfilling the full state constraints defined by K_b.

Proof 1: Let introduce the next candidate BLF:

$$V_j(s_j) = \frac{1}{2}\log\left(\frac{1}{1-\zeta_j^2}\right) \tag{34}$$

The full-time derivative of the BLF V_j is given by:

$$\dot{V}_j(s_j) = \frac{\zeta_j\dot{\zeta}_j}{(1-\zeta_j^2)} \tag{35}$$

Substituting the full-time derivative of ζ_j in (35), one gets:

$$\dot{V}_j(s_j) = \frac{\zeta_j\left(H_j + G_ju_j + \overline{\Xi}_j - \dot{x}_1^* + \rho_j\Delta_{2,j}\right)}{K_{s,j}(1-\zeta_j^2)} \tag{36}$$

with the application of the *Assumption 1* for the function H_j and by taking into account that $\overline{\Xi}_j$ is bounded as in (26). Then, the left hand-side of equation (36) satisfies the following inequality after substituting the control u_j defined in (31).

$$\dot{V}_j(s_j) \leq -\frac{\zeta_j}{K_{s,j}(1-\zeta_j^2)}\left((\overline{\Xi}_0 + \overline{\Xi}_1\|x\|^2) + \frac{k_js_j}{|K_{b,j}||\zeta_j|}\right) \tag{37}$$

Considering that $|\zeta_j| < 1$, the Lemma 1 can be applied, then the inequality (37) can be expressed as follows (after substituting the control gain k_j):

$$\dot{V}_j(s_j) \leq -\rho V_j(s_j)\left(\frac{0.5^{0.5}}{\zeta_j}\right) \tag{38}$$

In view of the term $-\left(\dfrac{0.5^{0.5}}{\zeta_j}\right)$ satisfies the following inequality:

$$-\left(\frac{0.5^{0.5}}{\zeta_j}\right) \leq -\frac{1}{\left(\log\left(\frac{1}{1-\zeta^2}\right)\right)^{0.5}} \tag{39}$$

Then, the function $\dot{V}_j(s_j)$ satisfies:

$$\dot{V}_j(s_j) \leq -\rho V_j(s_j)\left(\frac{0.5^{0.5}}{V_j(s_j)^{0.5}}\right) \tag{40}$$

The previous result evidences the stability of the closed-loop system (28) with the control (31).

V. Numerical Results

The numerical evaluation of the proposed controller considered a two-link RM as testing system (Figure 1). The proposed model of the RM was designed using SolidWorks™. To implement the proposed controller, the RM model was exported to the Simulink environment using the Simscape Toolbox.

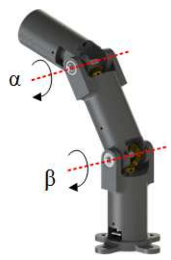


Fig. 2. Two-link manipulator

The dynamic model of this device satisfies the structure presented in (4), where $q \in \mathbb{R}^2$, is defined as $q = \begin{bmatrix}\alpha & \beta\end{bmatrix}^\top$, characterizes the generalized position of the robotic system.

The inertia matrix M is:

$$M(q) = \begin{bmatrix} \upsilon_1 + \upsilon_2 + 2\upsilon_3\cos(\beta) & \upsilon_2 + 2\upsilon_3\cos(\beta) \\ \upsilon_2 + 2\upsilon_3\cos(\beta) & \upsilon_2 \end{bmatrix} \quad (41)$$

where $\upsilon_1 = (m_1 + m_2)l_1^2$, $\upsilon_2 = m_2 l_2^2$, $\upsilon_3 = m_2 l_1 l_2$.

The matrix C is:

$$C(q, \dot{q}) = \upsilon_3 \cos(\beta) \begin{bmatrix} -\dot{\beta} & -\dot{\alpha} - \dot{\beta} \\ \dot{\alpha} & 0 \end{bmatrix} \quad (42)$$

The vector P is given by

$$P(q) = \begin{bmatrix} \upsilon_4 \cos(\alpha) + \upsilon_5 \cos(\alpha + \beta) \\ \upsilon_5 \cos(\alpha + \beta) \end{bmatrix} \quad (43)$$

with $\upsilon_4 = (m_1 + m_2)gl_1$ and $\upsilon_5 = m_2 g l_1$.

The term ξ was selected as follows:

$$\xi(q, u) = \begin{bmatrix} 0.5\sin(30t) & 0.8\sin(15t) \end{bmatrix}^\top \quad (44)$$

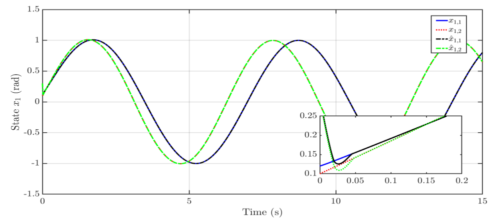
The RM system considers the following parameters in the simulation: $m_1 = 0.2kg$, $m_2 = 0.6kg$, $l_1 = 0.6m$, $l_2 = 0.3m$, $g = 9.8\frac{m}{s^2}$. The simulation results considered $x_1^* = \begin{bmatrix}\sin(0.9t) & \sin(t)\end{bmatrix}^\top$ as desired reference trajectories.

Here, it is assumed that the RM is fully actuated, then, the dynamic model admits the representation as in (5) by selecting $x_1 = q$ and $x_2 = \dot{q}$. Moreover the RM model satisfies the system decomposition proposed in section IV. Therefore, the control strategy proposed here can be used.

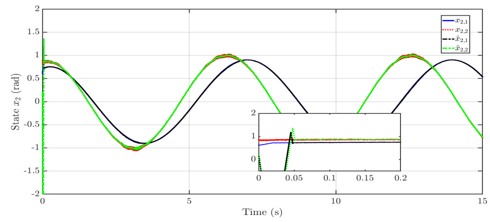
In order to apply the suggested controller, it is necessary to obtain the velocity estimation of the joints included in the RM structure. Then, the adaptive super-twisting algorithm was implemented in the Simulink environment. Notice that the application of the adaptive super-twisting requires the representation of the system (5) as a distributed representation of n second-order subsystems like the system decomposition proposed in (22) (For more details see [17]).

Figure 3a compares the state x_1 and its estimation obtained with the adaptive super-twisting. It is important to mention, that the Simscape Toolbox, provides a virtual sensor to measure the state x_2 online. Then, the sensor measurements of x_2 are compared whit the corresponding estimation obtained using the adaptive super-twisting algorithm in Figure 3b.

The constraint for each tracking error is selected as $K_{b_1} = \begin{bmatrix}0.25 & 0.15\end{bmatrix}^\top$ and $K_{b_2} = \begin{bmatrix}0.5 & 0.18\end{bmatrix}^\top$, this implies that $K_{s,j} = \rho_j K_{b_{1,j}} + K_{b_{2,j}}$. The constraint for the sliding surface s is given by $K_s = \begin{bmatrix}0.875 & 0.405\end{bmatrix}^\top$, with $\rho = \begin{bmatrix}1.5 & 1.5\end{bmatrix}^\top$ and $\epsilon = \begin{bmatrix}13 & 6\end{bmatrix}^\top$.



(a) Velocity estimation of the state $x_{1,1}$

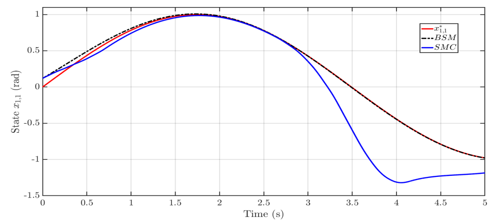


(b) Velocity estimation of the state $x_{1,2}$

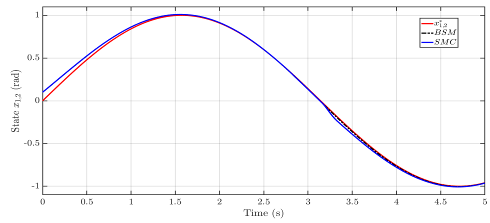
Fig. 3. Velocity estimation of the RM obtained with the adaptive super -twisting

Figure 4 compares the tracking trajectory obtained with the proposed barrier sliding mode (BSM) controller and the performance obtained with a classic first order sliding mode controller (SMC). That is the control $u_{2,j}$ becomes into $u_{2,j} = -K_j \text{sign}(s_j)$ where $K_j \in \mathbb{R}^+$. Here, the gain for the SMC is $K = \begin{bmatrix}5 & 3\end{bmatrix}^\top$.

Notice that in the case of the state $x_{1,1}$, the SMC cannot perform the trajectory tracking.



(a) Tracking trajectory performance for the state $x_{1,1}$

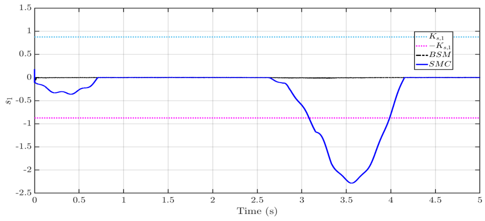


(b) Tracking trajectory performance for the state $x_{1,2}$

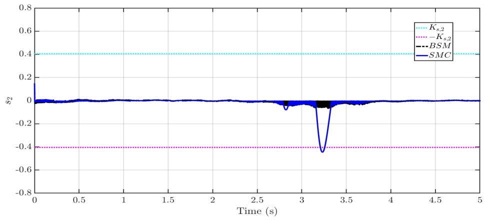
Fig. 4. Comparison between the BSM and SMC for the tracking trajectory.

Figure 5, evidences the fulfillment of the constraints for the sliding surfaces. Moreover, the finite-time convergence has been evidenced.

For the simulation case, it was possible to select the constraints for the tracking error, because the movement constraints are known and the upper bound for the joints velocity is also known. Therefore, the estimated constraints for each state are given by $|x_{1,j}| < |x_{1,j}^*| + |K_{b_{1,i}}|$ and $|x_{2,j}| < |\dot{x}_{1,j}^*| + |K_{b_{2,i}}|$. Under the given technical results, the proposed control guarantees the state constraints

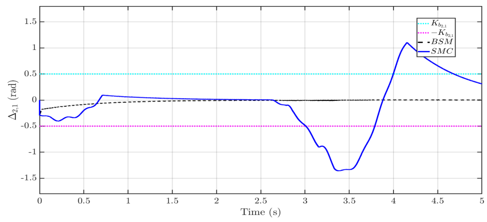


(a) Sliding surface s_1

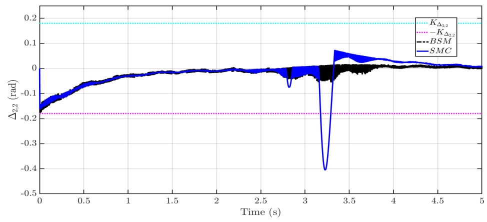


(b) Sliding surface s_2

Fig. 5. Sliding surface performances using SMC and BSM.



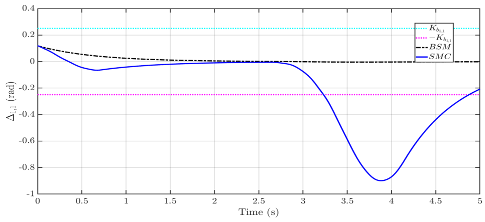
(a) Tracking error $\Delta_{2,1}$



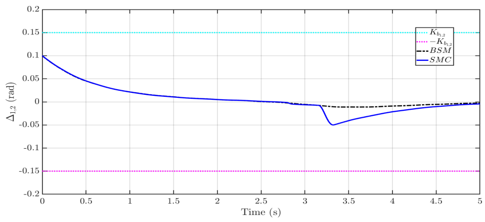
(b) Tracking error $\Delta_{2,2}$

Fig. 7. Tracking error Δ_2 performances with the SMC and BSM.

but also, the satisfaction of the state constraints for the sliding surface and the tracking error. The fulfillment of this conditions is evidenced in Figures 6 and 7.



(a) Tracking error $\Delta_{1,1}$



(b) Tracking error $\Delta_{1,2}$

Fig. 6. Tracking error Δ_1 performances with the SMC and BSM.

VI. Conclusions

In this paper, a novel design of output feedback controller based in the sliding-mode theory is introduced. This control scheme includes the state constraints in the control structure. The effectiveness of the control introduced in this paper to solve the trajectory tracking problem for a class of RM with full state constraints was demonstrated. Moreover the finite-time convergence of the sliding surface to the origin is proven. The application of the sliding mode control over a barrier Lyapunov stability analysis and the justification of the finite-time convergence offers a starting point for developing new discontinuous controllers for state constraint systems.

References

[1] C. Groth and D. Henrich, "Multi-tasking of competing behaviors on a robot manipulator," in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2013, pp. 3057–3064.

[2] A. L. Orekhov, C. B. Black, J. Till, S. Chung, and D. C. Rucker, "Analysis and validation of a teleoperated surgical parallel continuum manipulator," *IEEE Robotics and Automation Letters*, vol. 1, no. 2, pp. 828–835, 2016.

[3] A. Gaytán, M. Sanchez-Magos, D. Cruz-Ortiz, M. Ballesteros-Escamilla, I. Salgado, and I. Chairez, "Adaptive proportional derivative controller of cooperative manipulators," *IFAC-PapersOnLine*, vol. 51, no. 22, pp. 232–237, 2018.

[4] H. M. La, R. S. Lim, B. B. Basily, N. Gucunski, J. Yi, A. Maher, F. A. Romero, and H. Parvardeh, "Mechatronic systems design for an autonomous robotic system for high-efficiency bridge deck inspection and evaluation," *IEEE/ASME Transactions on Mechatronics*, vol. 18, no. 6, pp. 1655–1664, 2013.

[5] D. Liu and A. N. Michel, *Dynamical systems with saturation nonlinearities: analysis and design*. Springer-Verlag, 1994.

[6] P. Marayong, M. Li, A. M. Okamura, and G. D. Hager, "Spatial motion constraints: Theory and demonstrations for robot guidance using virtual fixtures," in *2003 IEEE International Conference on Robotics and Automation (Cat. No. 03CH37422)*, vol. 2. IEEE, 2003, pp. 1954–1959.

[7] A. Kapoor, M. Li, and R. H. Taylor, "Constrained control for surgical assistant robots." in *ICRA*, 2006, pp. 231–236.

[8] M. Baroudi, M. Saad, and W. Ghie, "State-feedback and linear quadratic regulator applied to a single-link flexible manipulator," in *2009 IEEE International Conference on Robotics and Biomimetics (ROBIO)*. IEEE, 2009, pp. 1381–1386.

[9] A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time and fixed-time stabilization: Implicit lyapunov function approach," *Automatica*, vol. 51, pp. 332–340, 2015.

[10] V. I. Utkin, *Sliding modes in control and optimization*. Springer Science & Business Media, 2013.

[11] D. Cruz-Ortiz, I. Chairez, and A. Poznyak, "Adaptive sliding-mode control with integral compensation for robotic devices with state constraints," *IFAC-PapersOnLine*, vol. 51, no. 22, pp. 506–511, 2018.

[12] F. Allgöwer and A. Zheng, *Nonlinear model predictive control*. Birkhäuser, 2012, vol. 26.

[13] E. Gilbert and I. Kolmanovsky, "Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor," *Automatica*, vol. 38, no. 12, pp. 2063–2073, 2002.

[14] J. Nocedal and S. J. Wright, *Numerical Optimization {Springer Series in Operations Research}*. Springer-Verlag New York Incorporated, 1999.

[15] M. W. Spong, S. Hutchinson, M. Vidyasagar, *et al.*, *Robot modeling and control*, 2006.

[16] V. I. Utkin and A. S. Poznyak, "Adaptive sliding mode control with application to super-twist algorithm: Equivalent control method," *Automatica*, vol. 49, no. 1, pp. 39–47, 2013.

[17] D. Cruz, A. Luviano-Juarez, and I. Chairez, "Output sliding mode controller to regulate the gait of gecko-inspired robot," in *Memorias del XVI Congreso Latinoamericano de Control Automatico, CLCA*, 2014, pp. 558–563.