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Second Adaptive Sliding Mode Control Law for the Non-Inertial Acrobot on a Cart System

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***Abstract*—The work aims to design an adaptive strategy using the methodology Second Adaptive Sliding Mode Control that allows getting an dynamical gain with respect to undesired effects to achieve attitude stabilization of the Acrobot on a cart in a non-inertial framework, with an external acceleration induced by the cart. The implementation scheme of the system by numerical simulation, in the platform MATLAB®/Simulink shows its performance and compares its effectiveness against a Sliding Mode Control.**

***Index Terms*—Non-Inertial System, Acrobot, Adaptive Robust Control.**

I. Introduction

The non-inertial effects involve reaction forces and relative motions which change the performance of the system as perturbations actions where any frame with respect to other differs from point to point and occurs when a physical system is not fixed to a base or surface [1] and are presented as reactions to the non-inertial movement that system experiences. Some pendulums are studied from a non-inertial framework ([2],[3],[4]) with an articular mobile component (cart or link) taken as an accelerated framework (non-inertial) or inertial framework (without an acceleration). Pendulums are used in control and robotics research, also for simulation and the study of some practical systems like robotic arms and the stabilization of the buildings, due to its natural unstable response. The inverted pendulum on a cart is a nonlinear example of an underactuated system [5] in a non-inertial framework.

Some configurations of Pendulums that let to add complexity are the simple pendulum, rotary pendulum, Foucault pendulum, Furuta pendulum, the pendulum on a cart and double pendulum. The Cart Pole [6] consists of a simple inverted pendulum that rotates freely mounted on an actuated cart. The double pendulum on a cart [7] consists of a non-actuated two-link mechanism attached to an actuated mobile platform. In addition, there are a couple of variants for the double pendulum as the double Pendulum mounted on a cart, the Pendubot, the Acrobot; these two last pendulums are planar rotational mechanisms [8]; the first one resembles of a human arm and the second one is similar to a human gymnastic on parallel bars, where the torque is in the second joint while the first one rotate freely [9]. In this case, the system will be considered in a non-inertial framework where reactions forces and relative motions from the cart are included.

The control methods used for Pendulums are made for swing-up and balancing task, considering undesired effects and non-inertial effects in the model to deal with them a control robust is used. One of them is the Sliding Mode Control (SMC) that is a strategy for control of nonlinear uncertain systems ([10],[11]), however, the main drawback of the SMC, is the chattering ([12],[13]). There are several ways to control the chattering, one of them is [10] and another one to decrease this effect is the use of higher-order Sliding Mode (SM) Control ([14],[15]).Also, in [16] an SM strategy was presented for the swing up and stabilization of a Cart-Pole system. Meanwhile, a case for an Acrobot [17] presents a robust controller design based on the Super-Twisting algorithm to stabilize it in the position under a non-inertial framework and reject undesired effects, caused by the cart's motion. However, there are in [18] two approaches based on the adaptive methodology. The first obtained by the direct measurements of the output signals of a first-order low-pass filter and the second one [18] using the method surface-adaptation, which alters the dynamic of a nonlinear system [19] applying a discontinuous control signal that forces the system to slide until reaching the desired stability point, whose the main advantage is reject modeling uncertainties and nonlinear effects ([20],[21]), known as Second Adaptive Sliding Mode Control Law (ASMC). Although the ASMC allows adjusting dynamically the control gain [18] without knowledge of uncertainties and perturbations bounds, it implies improving dynamic characteristics while properties of a controlled plant or environment is variant.

In this paper, the Second ASMC law is a strategy studied to achieve the attitude stabilization of the inverted position in the Acrobot on a cart in a non-inertial framework, including non-inertial effects from the cart. The controller will be compared with an SMC and it will show the robustness to reject the non-inertial undesired effects with the Second ASMC method.

II. MATHEMATICAL MODEL FOR ACROBOT ON A CART.

The Acrobot, which is intended to model the movement of a horizontal bar gymnast [9] has a single actuator (torque). Hence, considering the Acrobot as a model of a human body is possible to simulate motion, and present its dynamic. The Acrobot on a cart is presented in Fig.1, and the parameters in TableI. A constant horizontal acceleration $\bar{w}$ is taken into account due to the dynamic of the Acrobot is affected by the motion of the cart and this means the system is under a non-inertial framework.

The matrix form of Acrobot on a cart [17] is described:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Q \tag{1}$$

with $q \in \mathbb{R}^2$ as the vector of generalized coordinates. Considering as in [17] two generalized coordinates.

The inertia matrix is

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{2}$$

with

$$M_{11} = m_2 L_1^2 + m_1 l_{c1}^2 + m_2 l_{c2}^2 + I_{1,zz} + I_{2,zz} + 2m_2 L_1 l_{c2}\cos(q_2)$$
$$M_{12} = m_{21} = m_2 l_{c2}^2 + I_{2,zz} + m_2 L_1 l_{c2}\cos(q_2)$$
$$M_{22} = m_2 l_{c2}^2 + I_{2,zz}$$

The Coriolis and centripetal forces as:

$$C(q,\dot{q}) = \begin{bmatrix} -2\phi\dot{q_2} & -\phi\dot{q_2} \\ \phi\dot{q_1} & 0 \end{bmatrix} \tag{3}$$

where $\phi = m_2 L_1 l_{c2}\sin q_2$. The gravity vector is

$$G(q) = \begin{bmatrix} (h_1+h_2)g & h_2 g \end{bmatrix}^T \tag{4}$$

taking into account that

$$h_1 = (m_1 l_{c1} + m_2 L_1)\cos(q_1)$$
$$h_2 = m_2 l_{c2}\cos(q_1+q_2)$$

The generalized vector forces follows:

$$Q = \begin{bmatrix} \delta_1 & \tau + \delta_2 \end{bmatrix}^T \tag{5}$$

where $\tau \in \mathbb{R}$ is the torque input, and $\delta_1, \delta_2 \in \mathbb{R}$ are non-inertial effects induced by the displacement of the cart:

$$\delta_1 = [(m_1 l_{c1} + m_2 L_1)\sin(q_1) + m_2 l_{c2}\sin(q_1+q_2)]\bar{w}$$
$$\delta_2 = [m_2 l_{c2}\sin(q_1+q_2)]\bar{w}$$

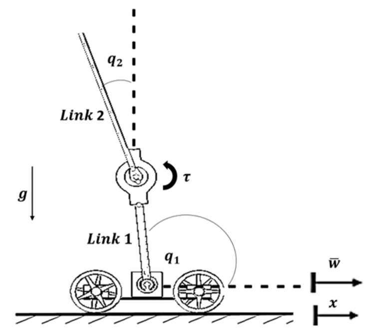

Fig. 1: Acrobot on a cart in a non-inertial framework.

TABLE I: Parameters of the Acrobot on a Cart.

Symbol	Description
$\bar{w}$	Horizontal acceleration of the cart.
x	Horizontal displacement of the cart.
q_1	The angle between link 1 and the horizontal.
q_2	The angle between link 2 and 1.
τ	Torque.
g	Gravitational accleration.
m_1, m_2	Masses of link 1 and 2.
L_1, L_2	Lengths of link 1 and 2.
l_{c1}, l_{c2}	Distances axis center of mass of links 1 and 2.
$I_{1,zz}, I_{2,zz}$	Inertia moments of link 1 and 2.

Although, the Acrobot on a cart can be represented into Space State. Where

$$\det(M) = M_{11}M_{22} - M_{12}^2 > 0, \tag{6}$$

then the Acrobot in [17] is rearranged and multiplied by the inverse inertial matrix $M^{-1}(q)$

$$\ddot{q} = M^{-1}[Q - C(q,\dot{q})\dot{q} - G(q)]$$

$$= \begin{bmatrix} \delta_1 + \mu_1 \\ \tau + \delta_2 + \mu_2 \end{bmatrix}$$

Now, the terms are:

$$\mu_1 = -\phi\dot{q}_2^2 - 2\phi\dot{q}_1\dot{q}_2 + (h_1+h_2)g$$
$$\mu_2 = \phi\dot{q_1}^2 + h_2 g$$

The inverse matrix M^{-1} is:

$$M^{-1} = \begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} \\ \bar{M}_{12} & \bar{M}_{22} \end{bmatrix}$$

with components,

$$\bar{M}_{11} = \frac{M_{22}}{M_{11}M_{22} - M_{12}^2}$$

$$\bar{M}_{12} = \bar{M}_{21} = \frac{-M_{12}}{M_{11}M_{22} - M_{12}^2}$$

$$\bar{M}_{22} = \frac{M_{11}}{M_{11}M_{22} - M_{12}^2}$$

The equations for the accelerations are

$$\begin{aligned}\ddot{q_1} &= \bar{M}_{11}(\delta_1+\mu_1)+\bar{M}_{12}(\tau+\delta_2+\mu_2)\\ \ddot{q_2} &= \bar{M}_{12}(\delta_1+\mu_1)+\bar{M}_{22}(\tau+\delta_2+\mu_2)\end{aligned} \tag{7}$$

Reducing equations (7)

$$\begin{aligned}f_1(t,q(t)) &= \bar{M}_{11}(\delta_1+\mu_1)+\bar{M}_{12}(\delta_2+\mu_2)\\ f_2(t,q(t)) &= \bar{M}_{12}(\delta_1+\mu_1)+\bar{M}_{22}(\delta_2+\mu_2)\end{aligned} \tag{8}$$

These terms represent non desired effects acting on the Acrobot. The non-inertial effects δ_1, δ_2 induced by the cart are included as part of f_1, f_2. Redefine $\tau := u(t)$,

$$\begin{aligned}\ddot{q_1} &= f_1(t,q)+\bar{M}_{12}u(t)\\ \ddot{q_2} &= f_2(t,q)+\bar{M}_{22}u(t)\end{aligned}$$

The Space State are defined as follows:

$$x := [q_1, q_2, \dot{q_1}, \dot{q_2}]^T = [x_1, x_2, x_3, x_4]^T \tag{9}$$

the dynamical Pendulum is given by

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ f_1(t,x(t))+\bar{M}_{12}u(t) \\ f_2(t,x(t))+\bar{M}_{22}u(t)\end{bmatrix} \tag{10}$$

A. Robust Controller Design

The control proposal for the pendulum in a non-inertial framework is based on the Second Adaptive Sliding Mode Control, providing adequate adjustment of the magnitude of a discontinuous control as in [18], within the "reaching phase" when the state trajectories are out of a sliding surface.

The control strategy consist in stabilize the attitude states of the Acrobot in the inverted position while rejecting the undesired non-inertial effects induces by the cart in motion. In our work will not take into consideration the impulse phase as it will be already set as initial condition at the desired position:

$$x(0) = (x_1(0), x_2(0), x_3(0), x_4(0)) = \left(\frac{\pi}{2}, 0, 0, 0\right)$$

In order to design the ASMC with surface-adaptation (11) method is necessary to define a sliding variable. As in [8], this variable will take into consideration the position and velocity states of the pendulum. This particular sliding variable is chosen as in [22]. The sliding variable chosen is

$$s := c_1x_1 + c_2x_2 + x_3 + x_4, \tag{11}$$

with $c_1, c_2 > 0$. The sliding variable dynamic is derived:

$$\begin{aligned}\dot{s} &= f_1(x,t)+f_2(x,t)+c_1x_3+c_2x_4+\\ &(\bar{M}_{12}+\bar{M}_{22})u(t)\\ &= \varphi(x,t)+c_1x_3+c_2x_4+\bar{M}\tau\end{aligned}$$

where $\bar{M} = \bar{M}_{12}+\bar{M}_{22}$, and the undesired effects $f_1(x,t)+f_2(x,t)$ were grouped in

$$\varphi(x,t) := f_1(x,t)+f_2(x,t)$$

The control $\tau(t)$ must drives the sliding variable (11) to the stability in finite time,

$$\tau(t) = -(1/\bar{M})(u + c_1x_3 + c_2x_4) \quad c_1, c_2 > 0 \tag{12}$$

and it can be obtained with the Second ASMC considering the control as

$$u(s,t) = -K(t)sign(s(x(t),t)) \tag{13}$$

the gain $K(t)$ can vary in the range $[\eta, k^+]$, $\eta > 0$ is a minimal value of K. The coefficient $K(t)$ satisfies:

$$\dot{K} := \begin{cases}\bar{K}|s(x(t),t)|sign(|(s(x(t),t)|-\epsilon) & if \quad K > \eta\\ 0 & if \quad K \leq \eta\end{cases} \tag{14}$$

with $\bar{K} > 0$, $\epsilon > 0$ and a small enough positive η. The parameter ϵ is introduced in order to get only positive values for K. Then the controller is described as:

$$\begin{aligned}\tau(t) = -(1/\bar{M})(K(t)sign(s(x(t),t)+c_3x_3+c_4x_4)\\ c_3, c_4 > 0\end{aligned}$$

B. Analysis of the stability for the Adaptive version

The theorem in [18] describes the main stability property of the SMC with the gain adaptation. With the Acrobot (10), presented in the form:

$$\dot{x}(t) = f(t,x(t)) + b(t,x(t))u(t,x(t)) \tag{15}$$

$$\begin{gathered}x(t) \in \mathbb{R}^n, f: \mathbb{R}^+ \text{x} \mathbb{R}^n \to \mathbb{R}^n\\ u: \mathbb{R}^+ \text{x} \mathbb{R}^n \to \mathbb{R}, b: \mathbb{R}^+ \text{x} \mathbb{R}^n \to \mathbb{R}^n\end{gathered}$$

for which we assume that $s(x) = 0$ $(s \in C^1)$ and is in the form (13) with the gain adaptation law (14), it means that the sliding surface $s(x) = 0$ is achieved in a finite time t_f as [18] based in the theorem where the SM control is established for all $t \geq t_f$

$$|s(x(t),t) < \delta| \quad with \quad \delta = \sqrt{\epsilon^2 + \Psi_p^2/\bar{K}\Gamma_p} \tag{16}$$

where Ψ and Γ are functions whit proof based on the Lyapunov approach and for $\alpha = 1 - \epsilon_0$ where ϵ_0 is a small enough positive number, it follows [18], in ASMC, the suggested adaptation procedure provides $k(t)$ tending to a vicinity of the minimum possible value k_{min}, that is

$$k(t) := \begin{cases}\frac{1}{1-\epsilon_0}k_{min}(t) & if \quad k_{min}(t) \geq \epsilon\\ \epsilon & if \quad k_{min}(t) < \epsilon\end{cases} \tag{17}$$

$$k_{min}(t) := \frac{|\Phi(t,x(t))|}{1+\lambda\sqrt{||x(t)||^2+\epsilon}} \tag{18}$$

satisfying $|\Phi(t,x(t))| := \dfrac{\nabla^T s(x)f(t,x)}{\nabla^T s(x)b(t,x)}$

III. SIMULATION RESULTS

A numerical simulations was created for the Pendulum Acrobot. It was test in the Acrobot on a cart with the Second ASMC using the physical parameter in the simulation, although, an Sliding mode controller was compared shown in TableII, considering the scalar discontinuous control action $u = u(s, t)$ as $u(s, t) = -K(t)sign(s)$ and s from (11). The initial conditions, in both cases, are chosen for the Acrobot in a unstable equilibrium point, in the upward position:

$$(x_1(0), x_2(0), x_3(0), x_4(0)) = (\frac{\pi}{2}, 0, 0, 0)$$

TABLE II: Simulation parameters of SMC and ASMC for the Acrobot on a Cart.

Parameter	**Numerical value**	**Parameter**	**Numerical value**
m_1	$0.265kg$	c_1	1
m_2	$0.265kg$	c_2	3
L_1	$0.206m$	g	$9.81m/s^2$
L_2	$0.206m$	$\bar{w}$	$0.5m/s^2$
l_{c1}	$0.103m$	$I_{1,zz}$	$2.3428x10^{-4}kgm^2$
l_{c2}	$0.103m$	$I_{2,zz}$	$2.3428x10^{-4}kgm^2$
η	0.001	ϵ	0.001
k^+	9	$\bar{K}$	1
K	0.939	c_3, c_4	1

The Second ASMC stabilizes the states x_1 and x_2. The Fig.2 shows the angular position of the Link1 from the initial condition $x_1(0)$ who stabilizes close to $\frac{3\pi}{2}$, it is, 4.7124 radians and the Fig.4 shows to be close to 0 radians, physically is the angle between both links. Also, the pendulum is affected by non inertial effects induced by the cart, however the controller presents robustness. Fig.3 and Fig.5 are shown the errors and its zoom.

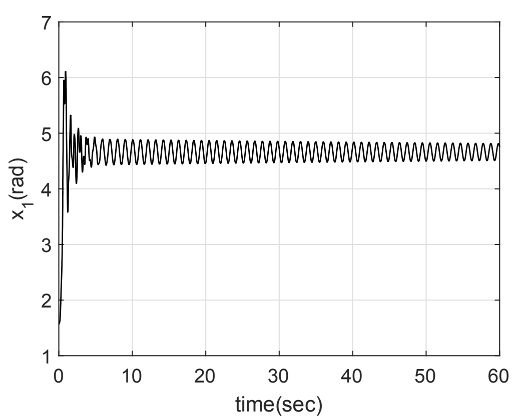

Fig. 2: Angular position of state x_1: Link 1 .

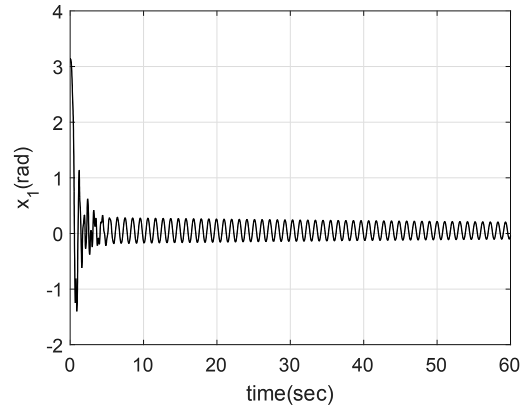

Fig. 3: Error position of state x_1 of Acrobot on cart: Link 1.

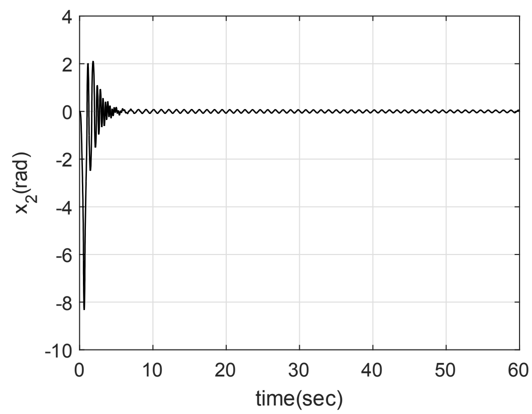

Fig. 4: Angular position of state x_2: Link 2

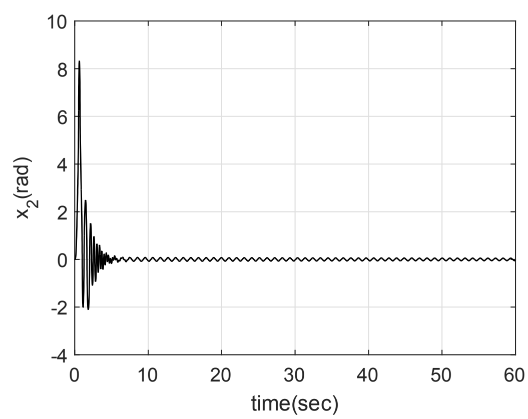

Fig. 5: Error position of state x_2 of Acrobot on cart: Link 2.

Fig.6 and Fig.7 shows the response of the states x_3 and x_4 with ASMC, corresponding to the angular velocities:

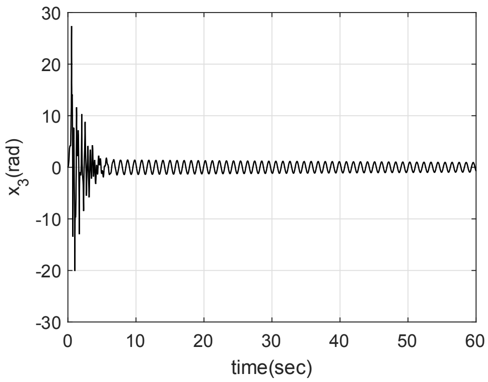

Fig. 6: State x_3: Angular velocity of the Link 1.

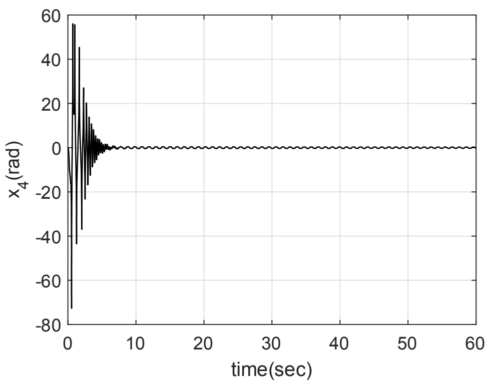

Fig. 7: State x_4: Angular velocity of the Link 2.

In Fig.8 and Fig.9 is appreciate the robustness of the adaptive method compared with SMC:

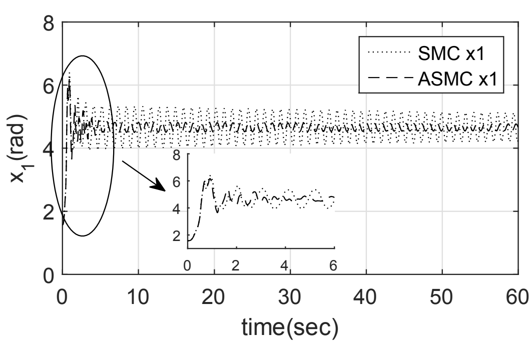

Fig. 8: Comparison for state x_1: SMC vs ASMC.

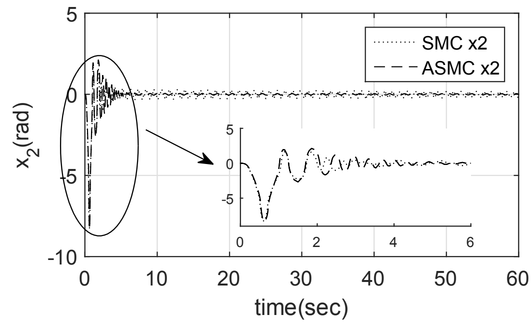

Fig. 9: Comparison for state x_2: SMC vs ASMC.

The states x_1,x_2 and its error are presented in Fig.10 and Fig.11. It shows the Second ASMC carry out to stabilization with more robustness in comparison with SMC:

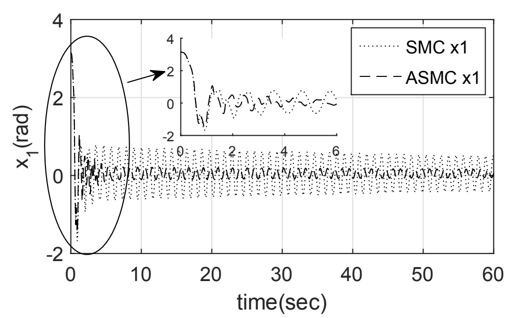

Fig. 10: Error position of the state x_1.

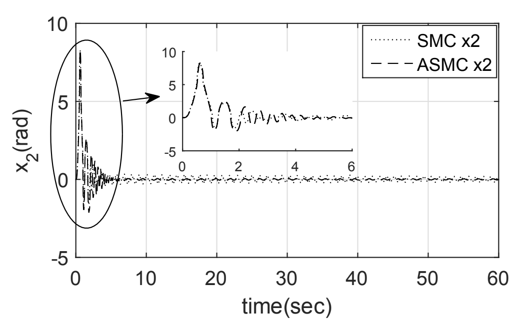

Fig. 11: Error position of the state x_2.

And in the Figure is shown the performance of $K(t)$ respect to time as in (14), it is, when $K(t)$ satisfies $\dot{K}$ and force to the control to slide to the surface s:

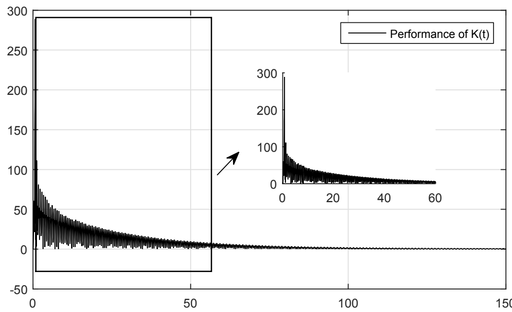

Fig. 12: Performance of $K(t)$.

IV. CONCLUSION

It work presented the mathematical model for Acrobot on a cart in a non-inertial framework with an accelerated framework. In addition, the Second Adaptive Sliding Mode Control design was used to the stabilize the Acrobot on a cart in the upward position. In the simulation results, the performance of the non-inertial system with the adaptive methodology was compared with the response of an Sliding Mode Control, and the results showed robustness with the Second ASMC to reject the non-inertial undesired effects and its errors signals.

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