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Nested Stabilization for Connected Cruise Control via the Delay Lyapunov Matrix*

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***Abstract*—In this paper, we study the stability of a vehicle platoon following model called Connected Cruise Control (CCC) [4]. The analysis is carried out in the time-domain via the delay Lyapunov matrix. We propose a nested stabilization methodology based on the construction of stability charts for the leader up to the tail vehicle. Two examples illustrate our results.**

***Index Terms*—Time delay, Connected cruise control, Lyapunov matrix, Linear system**

I. Introduction

In the last years, the ongoing development on wireless communication [6] vehicle to vehicle (V2V) or vehicle to infrastructure (V2I) has allowed useful applications in traffic systems. Typical traffic problems include gasoline consumption [13] or collision avoidance [12]. Vehicles equipped with sensors monitoring physical variables as headway (distance), position, velocity and acceleration have been a key piece in the implementation of a variety of control algorithms in platoon of vehicles. One of the first control strategies is the Adaptive Cruise Control (ACC) [2] where the equipped vehicle monitors the vehicle ahead with the objective of maintaining a regulated inter-vehicular distance. Another control strategy is the Cooperative Adaptive Cruise Control (CACC) [8] which considers a group of vehicles that are led by a prescribed leader vehicle. A more recent control strategy is the Connected Cruise Control (CCC) [14], in contrast with the CACC strategy, the participation of human-driven vehicles that are not equipped with range sensors or wireless transceivers is considered. A frequency-domain approach of the CCC strategy is presented in [14] and a time-domain approach is given in [5].

The stability analysis of a platoon of vehicles is classified as plant stability and string stability. Plant stability indicates whether or not a vehicle is able to approach to the steady state when no disturbances occur. If disturbances are applied to the leader vehicle, the string stability indicates the ability of the platoon to attenuate them. The stability chart is a practical tool for tunning parameters and ensuring stability in platoons of vehicles. This graph shows regions in the space of selected parameters where the platoon approaches a steady state. Some results in the frequency domain offer stability charts for problems as: stochastic communication delays [11] or connectivity structures [14], to name a few.

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In this work, we obtain stability charts for tunning parameters of platoon of vehicles by using a time domain approach via the delay Lyapunov matrix. We propose a methodology that consists in tunning stable parameters with the help of the stability chart. This tunning approach is repeated from the leader up to the tail vehicle.

The work is organized as follows. In Section II, we present the linear system models with multiple concentrated delays, namely, commensurate delays. The main concepts and results to test their stability are reminded. In Section III, the CCC strategy is explained and the nested stabilization design is described. In Section IV, we apply this strategy in two illustrative examples. Finally, some concluding comments are given in Section V.

Notation: We express the time derivative of a function as $\dot{y}(t)$, while $y'(\tau)$ denotes the derivative with respect to τ. The notation $A > (\ngeq)\ 0$ means that the matrix is positive definite (not positive semidefinite). PC $([-h_m, 0], \mathbb{R}^n)$ denotes the space of piecewise continuous functions defined on $[-h_m, 0]$.

II. Preliminaries

In this section, we present a multiple delay linear system. The stability analysis of this system is carried out in the time domain via the delay Lyapunov matrix. To achieve this, necessary and sufficient stability conditions are reminded in terms of the delay Lyapunov matrix.

A. Linear system with multiple concentrated delays.

Consider a linear system with multiple concentrated delays of the form

$$\dot{x}(t) = \sum_{j=0}^{m} A_j x(t - h_j), \quad t \geq 0, \tag{1}$$

where $A_0, \ldots, A_m$ are constant real $(n \times n)$ matrices, and $0 = h_0 < h_1 < \ldots < h_m$ are the delays. The initial functions φ are taken from $PC([-h_m, 0], \mathbb{R}^n)$. The restriction of the solution $x(t, \varphi)$ of system (1) on the interval $[t - h_m, t]$ is denoted by

$$x_t(\varphi) : \theta \to x(t + \theta, \varphi), \quad \theta \in [-h_m, 0].$$

As explained in [7], system (1) is related to a delay Lyapunov matrix $U(\tau)$, $\tau \in [-h_1, 0]$ via a Lyapunov-Krasovskii functional of complete type. This matrix is a continuous

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function and is associated to a positive definite matrix W. The delay Lyapunov matrix is solution of the boundary value problem:

$$\begin{aligned} U'(\tau) &= \sum_{j=0}^{m} U(\tau - h_j)A_j, \quad \tau \geq 0, \\ U(\tau) &= U^T(-\tau), \quad \tau \geq 0, \\ \sum_{j=0}^{m} \left[U(-h_j)A_j + A_j^T U(h_j)\right] &= -W. \end{aligned} \tag{2}$$

This set of equations (2) called dynamic, symmetry and algebraic properties, respectively, admits a unique solution when system (1) satisfies the Lyapunov condition (the characteristic equation has no eigenvalues that are symmetric with respect to the imaginary axis). In the case of commensurate delays where $h_j = jh$, $j = 0, ..., m$, the delay h is the basic delay and the semi-analytic method provides a solution to the boundary value problem (2) that is exact, up to the computation of a matrix exponential. For more details of the construction of the matrix $U(\tau)$, see [7].

B. Necessary and sufficient stability conditions in terms of the delay Lyapunov matrix.

In [3], necessary and sufficient stability conditions are formulated exclusively in terms of the delay Lyapunov matrix. It is worth noting that the sufficiency of these conditions was established for a special choice of the delays and a large enough parameter r.

Theorem 1: [3] System (1) is exponentially stable if and only if the Lyapunov condition holds and for every natural number $r \geq 2$,

$$\left\{ U\left(\frac{j-i}{r-1}H\right)\right\}_{i,j=1}^{r} > 0. \tag{3}$$

Moreover, if the Lyapunov condition holds and system (1) is unstable, there exists r such that

$$\left\{ U\left(\frac{j-i}{r-1}H\right)\right\}_{i,j=1}^{r} \ngtr 0,$$

here, the scalar variable H represents the maximum delay h_m. A novel methodology for avoiding computational burden at the time of constructing the delay Lyapunov matrix is given in [1].

III. Modeling framework

In this section, the model for vehicle platooning called Connected Cruise Control (CCC) [14] is presented. In this model, all vehicles follow a leader in a straight line without changing lanes. With regard to the information signal transmission, this is carried out downstream in only one direction as shown in Figure 1. It is important to point out that air-drag and rolling resistance are neglected. Without further ado, the CCC model [14] is

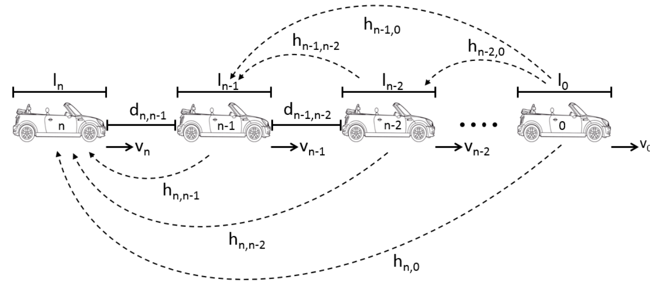

Fig. 1. Connected vehicle systems. The variables l and d are the length of the vehicle and the inter-vehicular distance, respectively. The variable h represent the time delay generated by signal transmission. Finally, the variable v is the velocity of the vehicle.

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= \sum_{j=0}^{i-1} \gamma_{i,j} \left[\alpha_{i,j}(V(d_{i,j}(t-h_{i,j}) - v_i(t-h_{i,j}))\right. \\ &\quad \left. + \beta_{i,j}(v_j(t-h_{i,j}) - v_i(t-h_{i,j}))\right], \end{aligned} \tag{4}$$

where s_i, and v_i are position and velocity of the i^{st} vehicle, respectively. The variables $\alpha_{i,j}$ and $\beta_{i,j}$ are control gains. The time delay is represented by $h_{i,j}$. The variable $\gamma_{i,j}$ is used to include human vehicles into the group. Here, we consider that all vehicles are equipped with monitoring sensors, hence, the value of $\gamma_{i,j}$ is always defined as 1.
The inter-vehicular distance between two vehicles is defined as

$$d_{i,j}(t) = \frac{1}{i-j}(s_j(t) - s_i(t) - \sum_{k=j}^{i-1} l_k). \tag{5}$$

The CCC strategy is based on a non-linear model, this can be seen in the range policy function $V(d)$ which gives a desired velocity according to the inter-vehicular distance d. An advantage of this model is the soft response, that is introduced through the nonlinear function $V(d)$.

$$V(d) = \begin{cases} 0, & d \leq d_{st}, \\ \frac{V_{max}}{2}[1 - cos(\pi(\frac{d-d_{st}}{d_{go}-d_{st}}))], & d_{st} < d < d_{go}, \\ V_{max}, & d \geq d_{go}. \end{cases} \tag{6}$$

According to [10], common parameters in (6) are set to $d_{st} = 5[m]$, $d_{go} = 35[m]$, $V_{max} = 30[m/s]$, .

A. Linearized model

The linearization of the CCC model (4) is realized in the vicinity of the uniform flow equilibrium when the perturbations are

$$\tilde{s}_i(t) = s_i(t) - s_i^*(t), \quad \tilde{v}_i(t) = v_i(t) - v^*, \quad v^* = V(d^*),$$

for more details, see [4] and [14].
In order to test stability, a head-to-tail transfer function in the frequency domain which has the objective of establishing a relation between the velocity of the vehicle i and j is defined in [4] as follows,

$$V_i(s) = \sum_{j=0}^{i-1} T_{i,j}(s) V_j(s)$$

$$T_{i,j}(s) = \frac{\gamma_{i,j}(s\beta_{i,j} + \varphi_{i,j})e^{sh_{i,j}}}{s^2 + \sum_{k=0}^{i-1} \gamma_{i,k}(sk_{i,k} + \varphi_{i,k})e^{-sh_{i,k}}} \tag{7}$$

where,

$$\varphi_{i,j} = \frac{\alpha_{i,j} V'(d^*)}{i-j}, \quad k_{i,j} = \alpha_{i,j} + \beta_{i,j}$$

for $j = 0, \ldots, i-1$. These transfer functions are called *link transfer functions* and can be seen as the dynamic gains between vehicles.

B. Methodology of the nested stabilization design

In this work, our aim is to test plant stability by using the head-to-tail transfer functions. Our method consists in the evaluation of the quasi-polynomials on the denominator of each transfer function by using the delay Lyapunov matrix in the time-domain framework. The methodology is explained as follows

1) Once the number of members in the platoon is defined, we calculate the required transfer functions, according to (7).
2) We sketch the stability chart of the quasi-polynomial of the denominator of the transfer function between the vehicles 1 and 0.
3) Next, we select stabilizing parameters for gains and delay values.
4) We continue with constructing the stability charts of all transfer functions between vehicles 2, 1 and 0. Once again, stabilizing parameters are chosen for gains and delays.
5) We repeat the process increasing the number of vehicles until the tail vehicle is reached.

Although in this method the complete stability region is obtained for certain parameters, we do not know which ones are the best. One way to select the parameters can be done by adding a σ-stability analysis to the proposed strategy, in order to obtain the maximum exponential decay.

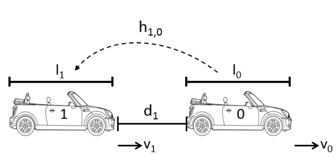

Fig. 2. Two connected vehicle systems under the CCC strategy.

IV. Illustrative examples

In this section, we present two scenarios. The first scenario includes two vehicle following each other. The second scenario addresses the case of three vehicles.

Example 1: we realize a stability test of two interconnected vehicles under the CCC approach, the interconnection is depicted in Figure 2. According to (7), the link transfer function between these vehicles is

$$T_{1,0}(s) = \frac{(s\beta_{1,0} + \alpha_{1,0}V'(d^*))e^{-sh_{1,0}}}{s^2 + (s(\alpha_{1,0} + \beta_{1,0}) + \alpha_{1,0}V'(d^*))\, e^{-sh_{1,0}}}$$

We can represent the quasi-polynomial of the transfer function $T_{1,0}(s)$ in the time-domain. Let $x(t) = [\tilde{v}_1(t) \quad \dot{\tilde{v}}_1(t)]^T$, then

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - h_{1,0}) \tag{8}$$

where,

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; A_1 = \begin{bmatrix} 0 & 0 \\ -\alpha_{1,0}V'(d^*) & -(\alpha_{1,0} + \beta_{1,0}) \end{bmatrix}$$

According to (6), we chose $d^* = 20m$ and calculate $V'(d^*) = \pi/2$. Note that system (8) only has three free parameters, the gains $\alpha_{1,0}$ and $\beta_{1,0}$ and the delay $h_{1,0}$. The procedure to set a stable point is based on the delay Lyapunov matrix. Observe that system (8) is of the form (1), namely, a linear system with multiple concentrated delays. We follow the next steps to obtain a stability chart in the space of parameters $(\alpha_{1,0}, \beta_{1,0}, h_{1,0})$ and to find a stable point

1) We construct the delay Lyapunov matrix of system (8). (For a detailed explanation, see [7]).
2) We apply the necessary and sufficient stability conditions [3] on a desired interval of selected parameter values.

We study the parameter behavior of system (8) in three cases. In the first case, we set the delay to $h_{1,0} = 0.2$, so that the design parameters are $\alpha_{1,0}$ and $\beta_{1,0}$. In Figure 3, the stability region for the parameters $(\beta_{1,0}, \alpha_{1,0})$ is shown. Hereinafter the continuous lines of all figures are obtained by using the *D*-partition method proposed in [9].
In the second case, the gain $\beta_{1,0} = 0.7$ is set in the dotted

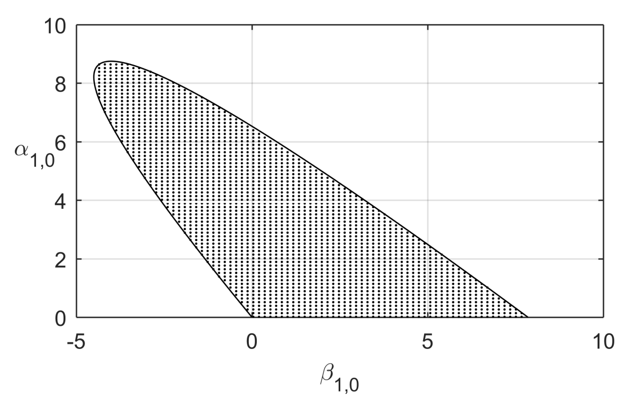

Fig. 3. Example 1, stability chart $(\beta_{1,0}, \alpha_{1,0})$ when $h_{1,0} = 0.2$.

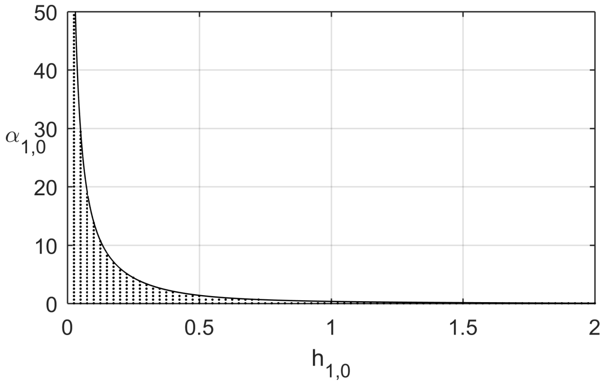

Fig. 4. Example 1, stability chart $(h_{1,0}, \alpha_{1,0})$ when $\beta_1 = 0.7$.

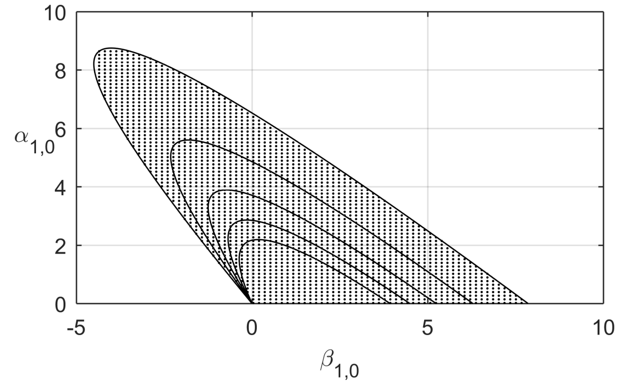

Fig. 6. Example 1, level curves for the stability chart in Figure 5. The stability region shrinks when the delay $h_{1,0}$ is increased.

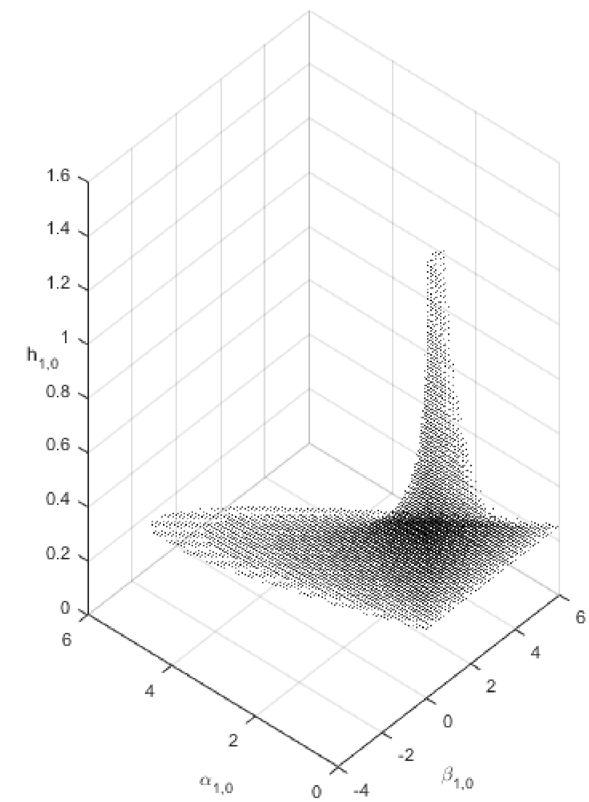

Fig. 5. Example 1, stability chart $(\alpha_{1,0}, \beta_{1,0}, h_{1,0})$.

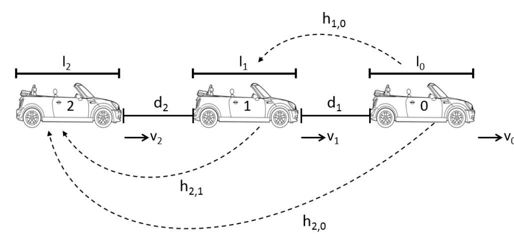

Fig. 7. Three connected vehicle systems under the CCC strategy.

stable region in Figure 3. Next, we obtain the stability region for the delay $h_{1,0}$ and gain $\alpha_{1,0}$. This region is depicted in Figure 4. Note that when the parameter $h_{1,0}$ increases, the stability region shrinks.

In the third case, we consider that all parameters are free. A 3D graph is shown in Figure 5, as expected the stability region reduces when the delay $h_{1,0}$ increases. Notice that, the graph in Figure 5 shows the complete parameter behavior for the case of two vehicles following in caravan. Finally, level curves representing a top view of Figure 5 are depicted in Figure 6. We conclude that increasing the delay parameter $h_{1,0}$ significantly reduces the stability region of the system for the gain parameters.

Example 2: A scenario of three vehicles is studied in this example. The interconnection structure is shown in Figure 7. According to (7), we need to analyze the three transfer functions

$$T_{1,0}(s) = \frac{V_1(s)}{V_0(s)}, \quad T_{2,1}(s) = \frac{V_2(s)}{V_1(s)}, \quad T_{2,0}(s) = \frac{V_2(s)}{V_0(s)},$$

where

$$T_{1,0} = \frac{(s\beta_{1,0} + \varphi_{1,0})e^{-sh_{1,0}}}{s^2 + (sk_{1,0} + \varphi_{1,0})e^{-sh_{1,0}}}, \tag{9}$$

$$T_{2,1} = \frac{(s\beta_{2,1} + \varphi_{2,1})e^{-sh_{2,1}}}{s^2 + (sk_{2,0} + \varphi_{2,0})e^{-sh_{2,0}} + (sk_{2,1} + \varphi_{2,1})e^{-sh_{2,1}}}, \tag{10}$$

$$T_{2,0} = \frac{(s\beta_{2,0} + \varphi_{2,0})e^{-sh_{2,0}}}{s^2 + (sk_{2,0} + \varphi_{2,0})e^{-sh_{2,0}} + (sk_{2,1} + \varphi_{2,1})e^{-sh_{2,1}}}. \tag{11}$$

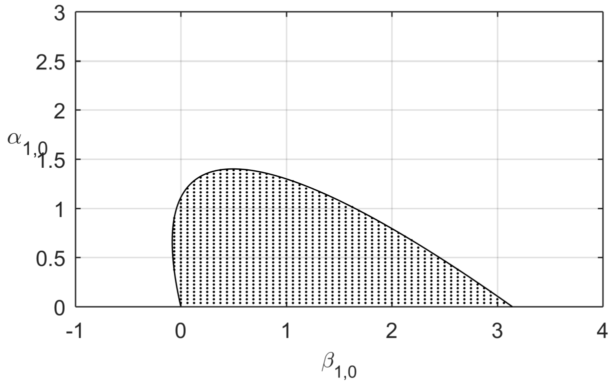

Fig. 8. Example 2, stability chart $(\alpha_{1,0}, \beta_{1,0})$ when the delay $h_{1,0}$ is assumed to be 0.5.

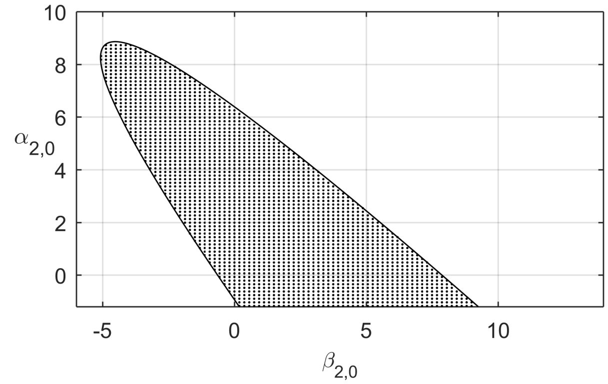

Fig. 9. Example 2, stability chart $(\beta_{2,0}, \alpha_{2,0})$.

In Example 1, the link transfer function $T_{1,0}$ was completely studied. Note that, the quasi-polynomials of the denominator of the link transfer functions (10) and (11) are the same. Hence, we only need to analyze the stability of two quasi-polynomials, the denominator of the transfer function (9) and the one repeated in (10) or (11). As a result, we ensure plant stability for all the vehicle platoon.

Once we define the quasi-polynomials to be analyzed, we apply the nested design methodology as follows.

For the transfer function $T_{1,0}$, we set up the delay parameter $h_{1,0} = 0.5$. Next, we show in Figure 8 the stability chart for the free parameters $\alpha_{1,0}$ and $\beta_{1,0}$. Then, we choose the ordered pair $(\beta_{1,0}, \alpha_{1,0}) = (0.7, 0.6)$ which falls inside the dotted stability region in Figure 8. Thus, we ensure plant stability between vehicles one and zero.

For the transfer functions $T_{2,1}$ and $T_{2,0}$, we have the quasi-polynomial

$$s^2 + (sk_{2,0} + \varphi_{2,0})e^{-sh_{2,0}} + (sk_{2,1} + \varphi_{2,1})e^{-sh_{2,1}} = 0 \tag{12}$$

where

$$k_{2,0} = \alpha_{2,0} + \beta_{2,0},\ k_{2,1} = \alpha_{2,1} + \beta_{2,1},$$

$$\varphi_{2,0} = \frac{1}{2}V'(d^*),\ \varphi_{2,1} = V'(d^*).$$

A representation of the quasi-polynomial (12) in the time-domain is

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - h_{2,0}) + A_2 x(t - h_{2,1}) \tag{13}$$

where,

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ -\varphi_{2,0} & -k_{2,0} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 0 \\ -\varphi_{2,1} & -k_{2,1} \end{bmatrix},$$
$$k_{2,1} = \alpha_{2,1} + \beta_{2,1}, \quad k_{2,0} = \alpha_{2,0} + \beta_{2,0},$$
$$\varphi_{2,1} = \alpha_{2,1} V'(d^*), \quad \varphi_{2,0} = \frac{\alpha_{2,0} V'(d^*)}{2},$$

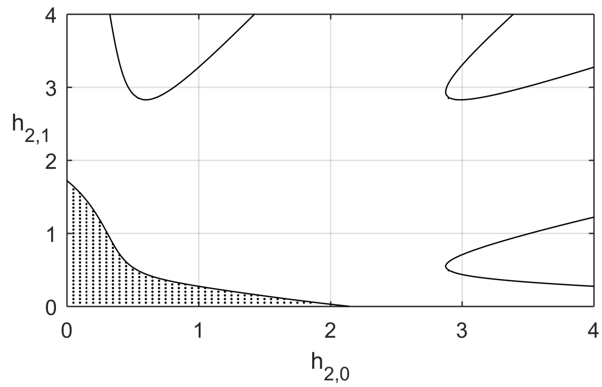

Fig. 10. Example 2, stability chart $(h_{2,0}, h_{2,1})$.

We observe that there are six free parameters corresponding to gains and delays between vehicles 2, 1 and 0. In addition to the point selected in the Figure 8, we consider the ordered pair $(\beta_{2,1}, \alpha_{2,1}) = (0.7, 0.6)$ and the delays $h_{2,1} = 0.5$, $h_{2,0} = 0.2$. Here, the gains $\alpha_{2,0}$ and $\beta_{2,0}$ are free parameters. The stability chart of system (13) is depicted in Figure 9 for $(\beta_{2,0}, \alpha_{2,0})$. The dotted area represents the region where all parameters ensure stability for the link transfer functions $T_{2,0}$ and $T_{2,1}$. Moreover, a σ-stability analysis can be done to system (13) in order to find the ordered pair in Figure 9 that ensures a maximum exponential decay as shown in [5].

In another case, we realize a delay parameter design, in other words, we set the parameters $(\beta_{2,0}, \alpha_{2,0}) = (\beta_{2,1}, \alpha_{2,1}) = (0.7, 0.6)$. Our aim is to find the space of parameters for the delays $(h_{2,0}, h_{2,1})$. The corresponding stability region of the delay space parameters is depicted in Figure 10. It shows us that the selection of delays has to be done carefully.

V. Conclusion

The obtained results show that the nested stabilization design can be used as a tunning tool when we search for stabilizing parameters. An advantage is that the time-domain approach via the delay Lyapunov matrix not only allows the construction of stability charts in the space of gains, but also in the space of delays, so that stabilizing delays can be also selected. Future work includes a σ-stability analysis at each step of the nested stabilization design in order to improve the exponential decay of the platoon response.

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