2019 16th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)
Mexico City, Mexico. September 11-13, 2019

Servodrive chaotization: An MRAC approach using a nonlinear reference model

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***Abstract*—This paper introduces a Model Reference Adaptive Control (MRAC) algorithm where the reference model corresponds to a Duffing oscillator. The goal of the controller is to induce a chaotic behavior in a servodrive. An indirect approach is employed and a stability proof allows concluding that the model tracking error and the parameter estimates are bounded. Real-time experiments illustrate the performance of the proposed controller.**

***Index Terms*—Model reference adaptive control, Chaotic behavior, Servo systems.**

I. Introduction

Adaptive controllers are endowed with an estimation algorithm for updating its parameters [1]. One of the approaches for the design of this kind of algorithms is the so called Model Reference Adaptive Control (MRAC). A classic MRAC methodology employs a reference model with linear dynamics, and its goal is to design an adaptive controller such that the behavior of the controlled plant remains close to the behavior of the reference model despite parametric uncertainties [2]. An MRAC algorithm may be represented schematically by the block diagram depicted in Fig. 1 and it consists of four parts. A plant whose model contains unknown parameters, a reference model, which is used to specify the ideal behavior of the closed-loop control system, a control law depending on the parameter estimates $\hat{\theta}$, and an adaptation law to update the estimated parameters $\hat{\theta}$ [3].

The plant is assumed to have a known structure and its parameters are constant and unknown. The controller must have a perfect tracking capability, i.e. when the parameters of the plant are known the corresponding controller must make the outputs of the plant and the reference model identical. When the parameters of the plant are not known, the adaptation law must update the parameters of the controller so that it achieves asymptotically a proper follow-up in the absence of disturbances [3]. Besides, the adaptation law updates the parameters in the control law, and if it estimates the unknown controller parameters, then the adaptive controller is called a direct adaptive controller. On the other hand, if the adaptation law estimates the unknown parameters of the plant, hence the adaptive controller is called an indirect adaptive controller [2]. The main problem in MRAC is to design a control law that guarantees that the all the signals of the closed-loop system remain bounded and that in the absence of disturbances the tracking error, defined as the difference between the outputs of the reference model and the plant, converges to zero.

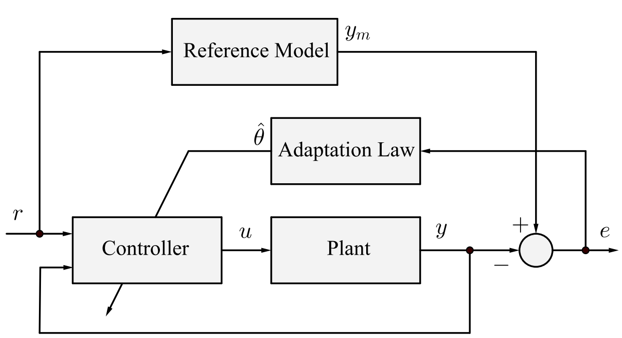

Fig. 1. Model Reference Adaptive Control.

A dynamic system is called chaotic when its evolution highly depends on its initial conditions. The above implies that two trajectories emerging from two nearby initial conditions separate exponentially over time [4], [5]. At this point it is worth mentioning some applications related to chaotic systems. Reference [6] describes an approximate model of a vibroformer working in a chaotic regime, which is used in the production of aluminum. In the chemical industry the *chaotic stirring*, especially of fluids and free-flowing materials including grains, sand or granular fertilizers, is an important area of research on the use of controlled chaos [7], [8]. Using chaos to generate high-speed and high-quality agitation would reduce the mass of reagents necessary in the reaction, which consequently decreases costs [9]. In [10] the authors propose and implement the chaotization of a Direct Current (DC) motor using time-delay feedback control to produce a desired chaotic motion in industrial mixing processes. In [11] the authors study a simple model that provides an idealization of a stirred tank. The fluid is assumed inviscid and incompressible, and its movement is entirely two-dimensional. The agitator is modeled as a point vortex that provides a source of unstable potential flux. By keeping the agitator in a fixed position, a marker movement occurs, and the agitator movement is not very efficient. If, on the other hand, the agitator is moved in such a way that the potential flow is chaotic, then the efficiency

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of the agitation operation improves.

Motivated by the applications of chaotic movement, the aim of this work is to present an indirect MRAC algorithm able to induce chaotic motion in a DC servodrive. A salient feature of the proposed approach is the use of a nonlinear model reference described by the dynamics of a Duffing oscillator working in a chaotic regime.

The work is composed of the following sections. Section II describes the proposed control scheme. Section III shows the stability proof for the closed-loop system. The laboratory prototype based on a DC servodrive and the experimental setup are described in Section IV. Section V shows the experiments, and the conclusions on the proposed approach are given at the end.

Notation: Let $A \in \mathbb{R}^n$ be a vector, where $A_{(i)}$ denotes the component i^{th} of the vector. Let $B \in \mathbb{R}^{n\times n}$ be a matrix, where $B_{(ij)}$ denotes the element in the i^{th} row and j^{th} column of the matrix. The term $\lambda_{\min}(B)$ represents the smallest eigenvalue of B and $\lambda_{\max}(B)$ represents the largest eigenvalue of B. The operator $||\cdot|| : \mathbb{R}^n \to \mathbb{R}$ denotes the Euclidean norm.

II. Non-linear Model Reference Adaptive Control

In this section, the servodrive and the nonlinear reference models are introduced. Subsequently, the control law is described and the dynamics of the closed loop system are obtained. The proposed control scheme is represented in the block diagram shown in Fig. 2.

A. Servodrive model

Consider a servodrive composed of a DC motor, a position sensor, and a power amplifier working in current mode. A model of this system is:

$$J\ddot{y} + F\dot{y} = ku + \eta \quad (1)$$

Variables y, $\dot{y}$ and $\ddot{y}$ are the position, velocity and acceleration respectively of the servodrive, u is the control input voltage, J is the servodrive inertia, F is the viscous friction coefficient, η is a bounded disturbance and k is a parameter related to the amplifier gain and the motor torque constant. Model (1) has the next alternative writing:

$$\ddot{y} = -a\dot{y} + bu + d \quad (2)$$

where $a = F/J$, $b = k/J$ and $d = \eta/J$. Note that a and b are positive constants and are assumed constant and unknown, and the disturbance d is assumed bounded, i.e. $|d| \leq D$.

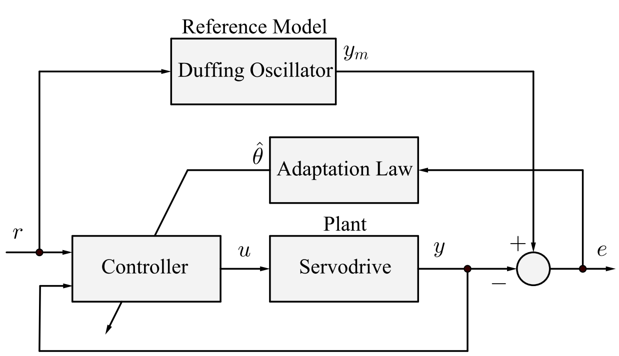

Fig. 2. Model Reference Adaptive Control.

B. Non-linear reference model

Consider the model of the Duffing Oscillator [12], which corresponds to the reference model within the proposed MRAC scheme, described by:

$$\begin{aligned}\dot{x}_1 &= x_2\omega\pi \\ \dot{x}_2 &= \left[-0.25x_2 + x_1 - 1.05x_1^3 + 0.3\sin(\omega\pi t)\right]\omega\pi\end{aligned} \quad (3)$$

where ω sets the frequency of the sinusoid input without affecting the shape of the signals x_1 and x_2 [13].

Define $y_m = Mx_1$, where y_m is the output of the reference model and $M > 0$ is a scaling factor. Using this definition allows writing (3) as:

$$\ddot{y}_m = \alpha\dot{y}_m + \beta y_m + \gamma y_m^3 + Fr \quad (4)$$

where $\alpha = -0.25\omega\pi$, $\beta = (\omega\pi)^2$, $\gamma = -1.05(\omega\pi/M)^2$, $F = 0.3M(\omega\pi)^2$ and the input signal corresponds to $r = \sin(\omega\pi t)$.

C. Control law

The model tracking error is defined as:

$$e = y_m - y \quad (5)$$

An expression for the second time derivative of (5) using (2) and (4) is:

$$\ddot{e} = \alpha\dot{y}_m + \beta y_m + \gamma y_m^3 + Fr + a\dot{y} - bu - d \quad (6)$$

Consider the case where the parameters of model (2) are known. It is desired to design a control law u producing the following closed loop dynamics:

$$\ddot{e} = -\sigma_1\dot{e} - \sigma_2 e, \quad \sigma_1, \sigma_2 > 0 \quad (7)$$

Equating (6) and (7) yields:

$$\alpha\dot{y}_m + \beta y_m + \gamma y_m^3 + Fr + a\dot{y} - bu - d = -\sigma_1\dot{e} - \sigma_2 e \quad (8)$$

from which the control law is obtained:

$$u = \frac{1}{b}\left[\alpha\dot{y}_m + \beta y_m + \gamma y_m^3 + Fr + a\dot{y} + \sigma_1\dot{e} + \sigma_2 e\right] \quad (9)$$

Define $z = \alpha\dot{y}_m + \beta y_m + \gamma y_m^3 + Fr + \sigma_1\dot{e} + \sigma_2 e$. Using this definition allows writing (9) as:

$$u = \frac{1}{b}\left[a\dot{y} + z\right] \quad (10)$$

If the exact values of a and b are unknown and only their estimates $\hat{a}$ and $\hat{b}$ are available, then, using them allows rewriting control law (10) as:

$$u = \frac{1}{\hat{b}}\left[\hat{a}\dot{y} + z\right] \quad (11)$$

D. Closed-loop dynamics

To obtain the dynamics of the system in closed loop, add and subtract the term $\hat{b}u$ in (2) thus leading to:

$$\ddot{y} = -a\dot{y} + bu - \hat{b}u + \hat{b}u + d \tag{12}$$

Define the parametric error vector $\tilde{\theta}$:

$$\tilde{\theta} = \hat{\theta} - \theta = \begin{bmatrix} \hat{a} - a \\ \hat{b} - b \end{bmatrix} = \begin{bmatrix} \tilde{a} \\ \tilde{b} \end{bmatrix} \tag{13}$$

Substituting (11) and (13) into (12) yields:

$$\ddot{y} = \tilde{a}\dot{y} - \tilde{b}u + z + d \tag{14}$$

Consequently:

$$\ddot{y} = \tilde{a}\dot{y} - \tilde{b}u + \alpha\dot{y}_m + \beta y_m + \gamma y_m^3 + Fr + \sigma_1\dot{e} + \sigma_2 e + d \tag{15}$$

from which, it is not difficult to obtain:

$$\ddot{e} + \sigma_1\dot{e} + \sigma_2 e = -\tilde{a}\dot{y} + \tilde{b}u - d \tag{16}$$

Define the error vector E and the regressor vector ϕ:

$$E = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \tag{17}$$

$$\phi = \begin{bmatrix} -\dot{y} \\ u \end{bmatrix} \tag{18}$$

Using these definitions allows writing (16) as:

$$\dot{E} = AE + \tilde{\theta}^\top \phi v - d_1 \tag{19}$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ -\sigma_2 & -\sigma_1 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad d_1 = \begin{bmatrix} 0 \\ d \end{bmatrix}$$

Here, A is a stable Hurwitz matrix, so $\forall Q \in \mathbb{R}^{2\times 2} > 0$, then the Lyapunov equation:

$$A^\top P + PA = -Q \tag{20}$$

has a unique solution $P \in \mathbb{R}^{2\times 2} > 0$ [14].

III. Stability Analysis

The analysis is performed using the next following Lyapunov function candidate:

$$V = E^\top PE + \tilde{\theta}^\top \Gamma^{-1}\tilde{\theta}, \quad \Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} > 0 \tag{21}$$

The time derivative of (21) is:

$$\dot{V} = \dot{E}^\top PE + E^\top P\dot{E} + 2\tilde{\theta}^\top \Gamma^{-1}\dot{\tilde{\theta}} \tag{22}$$

Substituting (19) and (20) into (22) yields:

$$\dot{V} = -E^\top QE - 2E^\top Pd_1 + 2E^\top P\tilde{\theta}^\top \phi v + 2\tilde{\theta}^\top \Gamma^{-1}\dot{\tilde{\theta}} \tag{23}$$

Consider the following algorithm for estimating θ:

$$\dot{\hat{\theta}} = -\Gamma\phi E^\top Pv + \beta\Gamma||E||\hat{\theta}, \quad \beta > 0 \tag{24}$$

Since θ is a constant, then $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$. Substituting $\dot{\hat{\theta}}$ into (23) gives:

$$\dot{V} = -E^\top QE - 2E^\top Pd_1 + 2\beta||E||\tilde{\theta}^\top\hat{\theta} \tag{25}$$

Under the assumption that $||\theta|| \leq K_\theta$, it follows that the upper bound for $\tilde{\theta}^\top\hat{\theta}$ corresponds to:

$$\tilde{\theta}^\top\hat{\theta} \leq K_\theta||\tilde{\theta}|| + ||\tilde{\theta}||^2 \tag{26}$$

Consider the upper bound of the following terms:

$$-E^\top QE \leq -\lambda_{\min}(Q)||E||^2 \tag{27}$$

$$-2E^\top Pd_1 \leq 2D\lambda_{\max}(P)||E|| \tag{28}$$

Substituting (26), (27) and (28) into (25) yields:

$$\begin{aligned} \dot{V} \leq & -\lambda_{\min}(Q)||E||^2 + 2D\lambda_{\max}(P)||E|| \\ & +2\beta||E||\left(K_\theta||\tilde{\theta}|| + ||\tilde{\theta}||^2\right) \end{aligned} \tag{29}$$

Substituting the following equality:

$$K_\theta||\tilde{\theta}|| + ||\tilde{\theta}||^2 = \left(||\tilde{\theta}|| + \frac{1}{2}K_\theta\right)^2 - \frac{1}{4}K_\theta^2$$

into (29) gives:

$$\begin{aligned} \dot{V} \leq & -\lambda_{\min}(Q)||E||^2 + 2D\lambda_{\max}(P)||E|| \\ & +2\beta||E||\left(||\tilde{\theta}|| + \tfrac{1}{2}K_\theta\right)^2 - \tfrac{1}{2}\beta||E||K_\theta^2 \end{aligned} \tag{30}$$

and factoring terms leads to:

$$\begin{aligned} \dot{V} \leq & -\tfrac{1}{2}\beta||E||K_\theta^2 \\ & -\psi\left[||E|| - \frac{2D\lambda_{\max}(P) + 2\beta\left(||\tilde{\theta}|| + \frac{1}{2}K_\theta\right)^2}{\lambda_{\min}(Q)}\right] \end{aligned} \tag{31}$$

where $\psi = \lambda_{\min}(Q)||E||$.

Hence, $\dot{V} \leq 0$ as long as:

$$||E|| \geq \frac{2D\lambda_{\max}(P) + 2\beta\left(||\tilde{\theta}|| + \frac{1}{2}K_\theta\right)^2}{\lambda_{\min}(Q)}$$

and the trajectories of the closed-loop system are uniformly ultimately bounded [15].

At this point, it is convenient to define:

$$\xi = -\phi E^\top Pv + \beta||E||\hat{\theta} \tag{32}$$

Using this definition allows writing (24) as:

$$\dot{\hat{\theta}} = \Gamma\xi \tag{33}$$

A critical problem in the control law (11) is the existence of a singularity, which happens when $\hat{b} = 0$. It can be solved by an algorithm for estimating θ endowed with a parameter projection. Consider that:

$$\theta = \begin{bmatrix} a \\ b \end{bmatrix}$$

belongs to Ω, which is a known compact convex subset of $\mathbb{R}^2$. Let $\hat{\Omega}$ be a convex subset of $\mathbb{R}^2$ containing Ω. Suppose that Ω is the convex hypercube:

$$\Omega = \{\theta \mid p_i \leq \theta_{(i)} \leq q_i\ ,\ 1 \leq i \leq 2\} \tag{34}$$

Let:

$$\Omega_\delta = \{\theta \mid p_i - \delta \le \theta_{(i)} \le q_i + \delta \, , \, 1 \le i \le 2\} \tag{35}$$

where $\delta > 0$ is chosen such that $\Omega_\delta \subset \hat{\Omega}$. Since Γ is a positive diagonal matrix, the projection $\Pr(\Gamma\xi)$ is taken as [15]:

$$\Pr(\Gamma\xi)_{(i)} = \begin{cases} \Gamma_{(ii)}\xi_{(i)}, & \text{if } p_i \le \hat{\theta}_{(i)} \le q_i \text{ or} \\ & \text{if } \hat{\theta}_{(i)} > q_i \text{ and } \xi_{(i)} \le 0 \text{ or} \\ & \text{if } \hat{\theta}_{(i)} < p_i \text{ and } \xi_{(i)} \ge 0 \\ \Gamma_{(ii)}\bar{\xi}_{(i)}, & \text{if } \hat{\theta}_{(i)} > q_i \text{ and } \xi_{(i)} > 0 \\ \Gamma_{(ii)}\check{\xi}_{(i)}, & \text{if } \hat{\theta}_{(i)} < p_i \text{ and } \xi_{(i)} < 0 \end{cases} \tag{36}$$

where:

$$\bar{\xi}_{(i)} = \left[1 + \frac{q_i - \hat{\theta}_{(i)}}{\delta}\right]\xi_{(i)}, \quad \check{\xi}_{(i)} = \left[1 + \frac{\hat{\theta}_{(i)} - p_i}{\delta}\right]\xi_{(i)}$$

The choice of δ such that $\Omega_\delta \subset \hat{\Omega}$, ensures that $\hat{b} \neq 0 \; \forall \, \theta \in \Omega_\delta$. The algorithm for estimating θ is taken as:

$$\dot{\hat{\theta}} = \Pr(\Gamma\xi) \tag{37}$$

Note that the algorithm for estimation θ with parameters projection does not affect the stability result previously obtained [2]. Therefore, denote by $(\Gamma\xi)_\perp$ the component of $\Gamma\xi$ perpendicular to the tangent plane at $\hat{\theta}$, so that:

$$\Gamma\xi = \Pr(\Gamma\xi) + (\Gamma\xi)_\perp \tag{38}$$

Since $\theta \in \Omega_\delta$ and Ω_δ is convex:

$$(\hat{\theta}^\top - \theta^\top) \cdot (\Gamma\xi)_\perp = \tilde{\theta}^\top (\Gamma\xi)_\perp \ge 0 \tag{39}$$

Define $\dot{V}_{Pr}$ as the time derivative of (21) evaluated using (37) for estimating θ. Substituting (37) into (22) yields to:

$$\dot{V}_{Pr} = \dot{E}^\top P E + E^\top P \dot{E} + 2\tilde{\theta}^\top \Gamma^{-1} \Pr(\Gamma\xi) \tag{40}$$

From the above equality and (38), it follows that:

$$\dot{V}_{Pr} = \dot{E}^\top P E + E^\top P \dot{E} + 2\tilde{\theta}^\top \xi - 2\tilde{\theta}^\top \Gamma^{-1} (\Gamma\xi)_\perp \tag{41}$$

Now, introducing equation (22) into (41) leads to:

$$\dot{V}_{Pr} = \dot{V} - 2\tilde{\theta}^\top \Gamma^{-1} (\Gamma\xi)_\perp \tag{42}$$

Finally, using the fact that $\tilde{\theta}^\top (\Gamma\xi)_\perp \ge 0$ yields $2\tilde{\theta}^\top \Gamma^{-1} (\Gamma\xi)_\perp \ge 0$, hence:

$$\dot{V}_{Pr} \le \dot{V} \tag{43}$$

and since $\dot{V} \le 0$ as long as:

$$||E|| \ge \frac{2D\lambda_{\max}(P) + 2\beta\left(||\tilde{\theta}|| + \frac{1}{2}K_\theta\right)^2}{\lambda_{\min}(Q)}$$

the trajectories of (19) are uniformly ultimately bounded [15]. The following proposition resumes the foregoing results.

Proposition 1: Consider the servodrive model (2) in closed loop with control law (11) and the reference model (4). If (37) updates the servodrive model parameters and:

$$||E|| \ge \frac{2D\lambda_{\max}(P) + 2\beta\left(||\tilde{\theta}|| + \frac{1}{2}K_\theta\right)^2}{\lambda_{\min}(Q)}$$

then, E and $\tilde{\theta}$ remain bounded and the closed-loop system is Ultimately Uniformly Bounded.

IV. Experimental Setup

Fig. 3 depicts the experimental setup. It consists of a brushed DC motor (Clifton Precision, model JDTH-2250-DQ-1C), an optical encoder (Servotek, model SA-7388-1), a power amplifier (Copley Controls, model 413) configured in current mode, and a box that galvanically isolates the data acquisition card from the power amplifier.

Servotogo II card performs the data acquisition. It has 13-bit digital-analog and analog-digital converters with a voltage range of $\pm 10V$. The algorithms are coded through real-time software (The MathWorks Matlab/Simulink, and Quanser Consulting WINCON). A personal computer (Intel Core 2 Quad processor) executes the software. The Simulink diagrams use a sampling period of $1ms$ and the Euler-ode1 integration method, which is selected for its simplicity.

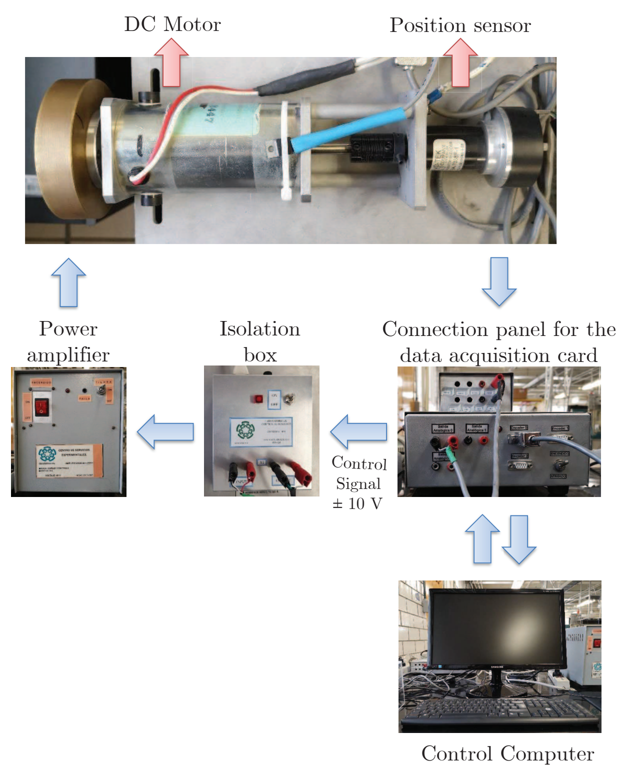

Fig. 3. Experimental setup.

The servomotor angular velocity $\dot{y}$ is estimated from position measurements through the next filter:

$$G_f(s) = \frac{300s}{s+300}\frac{300}{s+300} \tag{44}$$

V. Experimental Results

A. Experiments

In this subsection, the performance of the adaptive controller is evaluated experimentally. The experiments consist of applying the control law (11), the estimation algorithm (37), and the reference model (4) in the servodrive, using a reference signal $r = \sin(3\pi t)$. The parameters of the adaptive controller used in the experiments are $\omega = 3.0$, $M = 0.2$, $\sigma_1 = 40$, $\sigma_2 = 150$, $\beta = 0.01$, $\gamma_1 = 1$, $\gamma_2 = 20$, $\delta = 0.01$, $p_1 = 0.1$, $p_2 = 20$, $q_1 = 10$, $q_2 = 100$ and:

$$P = \begin{bmatrix} 4041.6667 & 6.6667 \\ 6.6667 & 25.1667 \end{bmatrix}$$

The initial values of the estimated parameters used in the experiments are $\hat{a}(0) = 0$ and $\hat{b}(0) = 1$. Note that matrix P is chosen large and matrix Γ small to obtain good convergence of the parameter estimates. Small values of the entries of P may produce large oscillations in the motor.

The closed-loop response is depicted in Fig. 4, the position error graph in Fig. 5, and the control signal in Fig. 6. For comparison purposes, the parameters of servodrive are estimated using the Least Square Method (LSM) [16], so the estimate parameters are $\hat{a} = 1.9527$ and $\hat{b} = 51.3030$. Fig. 7 and Fig. 8 show the evolution of the estimated parameters.

The Integral Squared Error (ISE) index allows the assessment of the model tracking performance. Further, performance is also assessed through the Integral of the Absolute value of the Control (IAC) and the Integral of the Absolute value of the Control Variation ($IACV$) indexes defined as:

$$ISE = \int_0^T [e(t)]^2\, dt, \quad IAC = \int_0^T |u(t)|\, dt,$$

$$IACV = \int_0^T \left| \frac{du(t)}{dt} \right| dt$$

which are evaluated at $T = \frac{2}{3}s$. Table I shows the values of these indexes obtained in the experiment.

TABLE I
PERFORMANCE INDEXES

ω	ISE	IAC	$IACV$
3.0	0.0237	0.1084	1.7143

B. Analysis of results

The results show that the adaptive controller using a chaotic reference model exhibits reasonable performance in the tracking of the output of the model y_m (see Fig. 4). It is experimentally verified that the model tracking error shown in Fig. 5, and the estimated parameters are bounded, see the Fig. 7 and 8. Based on Table I it is observed that the ISE

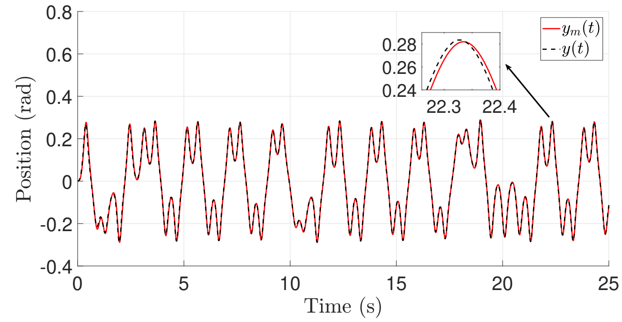

Fig. 4. Model tracking.

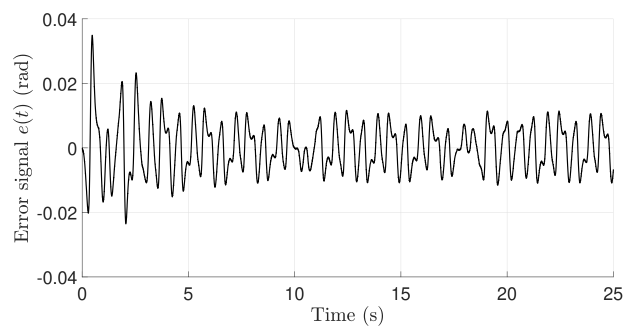

Fig. 5. Error signal $e(t)$.

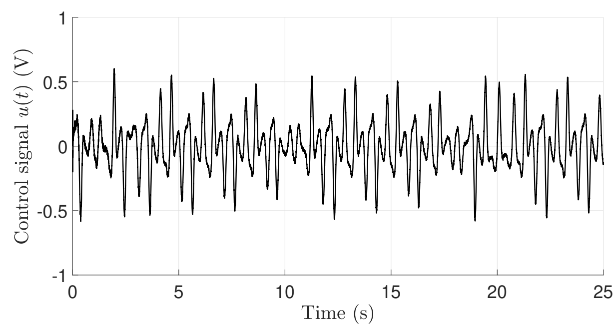

Fig. 6. Control signal $u(t)$.

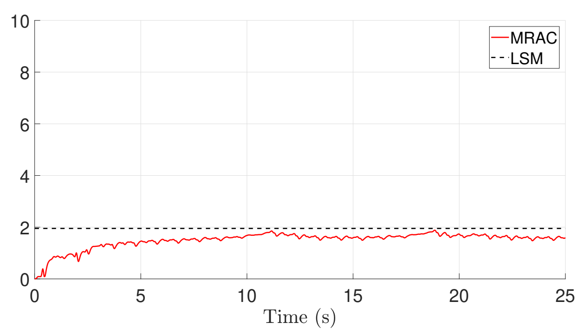

Fig. 7. Parameter estimate $\hat{a}$.

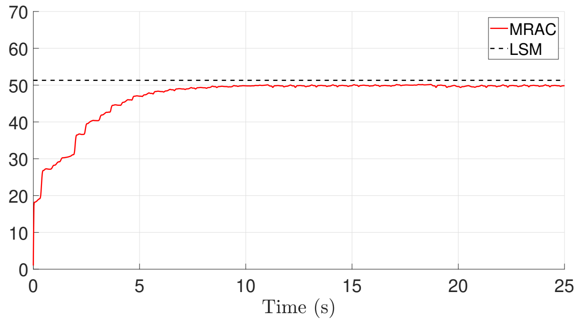

Fig. 8. Parameter estimate $\hat{b}$.

has a value close to zero, which indicates a proper model tracking. Further, the IAC index, which is related to the current consumption of the power amplifier is small. However, the $IACV$ index shows that the variation of the control signal has a higher value, this is due to the cutoff frequency used in the speed estimation filter (44), and its effects are shown in Fig. 6.

VI. Conclusions

In this work, an indirect adaptive controller was presented using the Model Reference Adaptive Control approach. The reference model is nonlinear and corresponds to the chaotic Duffing oscillator. The goal of this controller is to produce a chaotic behavior in a servodrive. A parameters projection was made to avoid a singularity in the control law. It is shown that the trajectories of the closed-loop system states are uniformly ultimately bounded. The real-time experiments carried out in a laboratory prototype allow verifying the performance of the proposed controller. One application of the chaotization of a DC motor is in the chemical industry, where it has been proven that chaotic movement improves mixing results. The proposed approach may use other chaotic systems provided that the degree of the controlled plant be the same as the one of the chaotic system.

Acknowledgment

The assistance of Gerardo Castro Zavala and Jesús Meza Serrano in preparing the laboratory testbed is recognized. The second author appreciates the support of CONACyT-MEXICO through a Ph.D. scholarship.

References

[1] K. J. Åström and B. Wittenmark, *Adaptive Control*. Mineola, NY: Dover Publications, 2008.

[2] S. Sastry and M. Bodson, *Adaptive control, stability, convergence and robustness*. Prentice Hall, 1989.

[3] W. L. Jean Jaques Slotine, *Applied Nonlinear Control*. Prentice Hall, 1991.

[4] S. Boccaletti, J. Kurths, G. Osipov, D. Valladares, and C. Zhou, "The synchronization of chaotic systems," *Physics Reports*, vol. 366, no. 1, pp. 1 – 101, 2002.

[5] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, pp. 821–824, Feb 1990.

[6] M. Paskota, "On modelling and the control of vibroformers in aluminium production," *Chaos, Solitons & Fractals*, vol. 9, no. 1, pp. 323 – 335, 1998.

[7] J. M. Ottino, *The kinematics of mixing: stretching, chaos, and transport*. Cambridge University Press, 1989, vol. 3.

[8] A. Sharma and N. Gupte, "Control methods for problems of mixing and coherence in chaotic maps and flows," *Pramana*, vol. 48, no. 1, pp. 231–248, Jan 1997.

[9] B. R. Andrievskii and A. L. Fradkov, "Control of chaos: Methods and applications. ii. applications," *Automation and Remote Control*, vol. 65, no. 4, pp. 505–533, Apr 2004.

[10] K. T. Chau, S. Ye, Y. Gao, and J. H. Chen, "Application of chaotic-motion motors to industrial mixing processes," in *Conference Record of the 2004 IEEE Industry Applications Conference, 2004. 39th IAS Annual Meeting.*, vol. 3, Oct 2004, pp. 1874–1880 vol.3.

[11] H. Aref, "Stirring by chaotic advection," *Journal of Fluid Mechanics*, vol. 143, p. 121, 1984.

[12] R. Garrido and R. Miranda, "Dc servomechanism parameter identification: A closed loop input error approach," *ISA Transactions*, vol. 51, no. 1, pp. 42 – 49, 2012.

[13] C.-C. Fuh and H.-H. Tsai, "Adaptive parameter identification of servo control systems with noise and high-frequency uncertainties," *Mechanical Systems and Signal Processing*, vol. 21, no. 3, pp. 1437 – 1451, 2007.

[14] C.-T. Chen, *Linear System Theory and Design*, 3rd ed. New York, NY, USA: Oxford University Press, Inc., 1998.

[15] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 1996.

[16] L. Ljung, Ed., *System Identification (2Nd Ed.): Theory for the User*. Upper Saddle River, NJ, USA: Prentice Hall PTR, 1999.