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Discrete-time neural sliding-mode pinning control for synchronization of complex networks

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***Abstract*—A discrete-time neural sliding-mode pinning controller for synchronization of uncertain complex networks with nonidentical nodes is proposed. To achieve synchronization, a neural control scheme constituted by a Recurrent High-Order Neural Network identifier trained with an extended Kalman algorithm, and a discrete-time sliding-mode controller is used in a small fraction of nodes. The applicability of the proposed scheme is illustrated via simulations, using a network constituted by 8-nodes with four different types of chaotic systems.**

***Index Terms*—Complex networks, neural identification, pinning control, sliding-mode control.**

I. INTRODUCTION

Different models of complex networks have been proposed to analyze properties related to real systems, such as: Internet, the World Wide Web, gene regulatory networks, power grids, among others [1]–[3]. This fact has motivated research on fundamental properties and dynamical behavior of these networks in different disciplines [4]–[6]. Synchronization is an important dynamical behavior, which has been received overwhelming attention [7]–[9].

Pinning control is a control technique for complex networks, which guarantees the desired objectives applying local feedback controllers in a reduced quanty of nodes [10]–[12]. Different pinning control schemes have been developed in continuous-time framework to achieve synchronization [13]–[16]. It is well known that the discrete-time controllers are easier to implement in a digital processor, has lower cost, and better productivity, among others. To design discrete-time nonlinear control, there are different approaches as in [17], [18].

Artificial neural networks have been used to design intelligent control systems, due to their properties, such as learning, adaptation, classification, and function approximation [19]–[21]. Recurrent High-Order Neural Networks (RHONN) are a type of neural networks easy to implement, with simple structure, robustness, and on-line weights learning [22]–[24]. RHONN identifier schemes have been used to synthesize different discrete-time control techniques [25]–[27]. On the other hand, sliding-mode control is a well-known control technique, which is simple to design, is robust to uncertainties, has finite time convergence, and rejects of external bounded disturbances. This technique has been applied for synchronization of complex networks as in [28]–[31].

This paper proposes a neural sliding-mode pinning control to synchronize the whole complex network with non-identical nodes using discrete-time design method. An unknown discrete-time model of the pinned node is approximated by means of neural scheme, which consists of an RHONN identifier trained with an Extended Kalman Filter (EKF), the obtained neural model is used to synthesize discrete-time neural sliding-mode pinning control strategy, achieving the desired goals.

II. PROBLEM STATEMENT

Consider a network of N non-identical coupled nodes, given by

$$\dot{\mathbf{x}}_p = f_p(\mathbf{x}_p) + \sum_{j=1,\, j\neq p}^{N} c_{pj} a_{pj} \mathbf{\Gamma}(\mathbf{x}_j - \mathbf{x}_p), \qquad (1)$$

where each node is an n-dimensional dynamical system, $\mathbf{x}_p = [x_{p_1}, x_{p_2}, \ldots, x_{p_n}]^T \in \mathbb{R}^n$ is the state vector of node p for $p = 1, 2, \ldots, N$; $f_p : \mathbb{R}^n \mapsto \mathbb{R}^n$; $\mathbf{\Gamma} \in \mathbb{R}^{n\times n}$ is the inner coupling matrix; constant c_{pj} is the coupling strength between node p and node j; and the coupling matrix $\mathbf{A} = [a_{pj}] \in \mathbb{R}^{N\times N}$ denotes the coupling configuration of the network: $a_{pj} = a_{jp} = 1$ means of node p and j are connected; otherwise, $a_{pj} = a_{jp} = 0$. The node degree k_p is defined as

$$k_p = \sum_{j=1, j\neq p}^{N} a_{pj} = \sum_{j=1, j\neq p}^{N} a_{jp}, \quad p = 1, 2, \ldots, N,$$

and $a_{pp} = -k_p$, $p = 1, 2, \ldots, N$, which is defined as the diagonal element of matrix $\mathbf{A}$, with

$$c_{pp} = \frac{1}{k_p} \sum_{j=1, j\neq p}^{N} a_{pj} c_{pj},$$

for normalization.

The pinning control technique consists on controlling a reduced quantity of the network nodes, named as pinned ones.

Selecting the first l nodes as pinned, the controlled network can be written as

$$\dot{\mathbf{x}}_p = f_p(\mathbf{x}_p) + \sum_{j=1}^{N} c_{pj} a_{pj} \mathbf{\Gamma} \mathbf{x}_j + \mathbf{u}_p, \quad p = 1, \dots, l. \tag{2}$$

$$\dot{\mathbf{x}}_p = f_p(\mathbf{x}_p) + \sum_{j=1}^{N} c_{pj} a_{pj} \mathbf{\Gamma} \mathbf{x}_j, \quad p = l+1, \dots, N. \tag{3}$$

The control objective in this paper is to synchronize the whole network at a homogeneous stationary state $\bar{\mathbf{x}}$, satisfying

$$f(\bar{\mathbf{x}}(t)) = 0.$$

i.e.,

$$\lim_{t\to\infty} \|\mathbf{z}_p(t)\| = 0, \quad p = 1, \dots, N,$$

where $\mathbf{z}_p(t) = \mathbf{x}_p(t) - \bar{\mathbf{x}}(t) \in \mathbb{R}^n$. The control $\mathbf{u}_p$ is defined as a piecewise constant signal

$$\mathbf{u}_p(t) = \mathbf{u}_p(KT) := \mathbf{u}_p(k),\ KT < t < (K+1)T,\ K \in \mathbb{N},$$

where $p = 1, 2, \dots, l$ and $T > 0$ is the sampling time.

III. CONTROL SYNTHESIS

To synchronize the complex networks, a discrete-time neural sliding-mode pinning controller is presented. The proposed control scheme is displayed in Fig. 1, which is composed by a discrete-time RHONN structure, an EKF, a sampler, a Zero Order Hold (ZOH), and the discrete-time sliding-mode controller.

A. On-Line Neural Identification

Assume the unknown discrete-time model for pinned nodes $p = 1, 2, \dots, l$, defined as

$$\begin{aligned} \dot{\mathbf{x}}_p &\triangleq \mathbf{x}_p(k+1) \\ \mathbf{x}_p(k+1) &= \hat{f}_{pk}(\mathbf{x}_p(k)) + \hat{g}_{pk}(\mathbf{x}_p(k))\mathbf{d}_k + \mathbf{u}_p(k), \end{aligned} \tag{4}$$

where the vector functions $\hat{f}_{pk}(\cdot)$ and $\hat{g}_{pk}(\cdot)$, and disturbance $\mathbf{d}_k$ are unknown. Consider that the state measurements $\mathbf{x}_p(KT) := \mathbf{x}_p(k)$ are available at sampling instants KT, $K \in \mathbb{N}$, where T is a design parameter.

System (4) is represented by an RHONN scheme, given by

$$\mathbf{x}_p(k+1) = W_p^{*T}(k) \boldsymbol{\phi}_p(\mathbf{x}_p(k)) + \mathbf{u}_p(k) + \epsilon,$$

where $W_p^*(k) \in \mathbb{R}^{L\times n}$ are the ideal optimal constant weight matrices; $\boldsymbol{\phi}_p(\cdot) \in \mathbb{R}^L$ is defined as

$$\boldsymbol{\phi}_p(\mathbf{x}_p(k)) = \begin{bmatrix} \prod_{j\in I_1} \zeta_{ij}^{d_{ij}(1)} \\ \vdots \\ \prod_{j\in I_L} \zeta_{ij}^{d_{ij}(L)} \end{bmatrix},$$

where d_{ij} are non-negative integers; L is the respective number of high-order connections; $\{I_1, I_2, \dots, I_L\}$ is a collection of non-ordered subset of $\{1, 2, \dots, n\}$; and ζ_i is given by

$$\zeta_i = \begin{bmatrix} \zeta_{i_1} \\ \vdots \\ \zeta_{i_n} \end{bmatrix} = \begin{bmatrix} S(x_1) \\ \vdots \\ S(x_n) \end{bmatrix},$$

where $S(\cdot)$ is a hyperbolic tangent function. ϵ is the modeling error, so that

$$\|\epsilon\| = \epsilon_N,$$

where ϵ_N is a known bound. An RHONN in a series-parallel structure is implemented to identify (4), as

$$\boldsymbol{\chi}_p(k+1) = W_p(k)^T \boldsymbol{\phi}_p(\mathbf{x}_p(k)) + \mathbf{u}_p(k), \tag{5}$$

where $\boldsymbol{\chi}_p = [\chi_{p_1}, \chi_{p_2}, \dots, \chi_{p_n}]^T \in \mathbb{R}^n$ is the neuron state vector, $W_p \in \mathbb{R}^{L\times n}$ is the adjustable weights vector. The EKF learning algorithm is used to on-line adjust the neural weights, which determines the values W_p to minimize the identification error. This algorithm is given by [25]:

$$\begin{aligned} K_i(k) &= P_i(k) H_i(k) M_i(k), \\ w_i(k+1) &= w_i(k) + \eta_i K_i(k) e_i(k), \\ P_i(k+1) &= P_i(k) - K_i(k) H_i^T(k) P_i(k) + Q_i(k), \end{aligned} \tag{6}$$

with

$$\begin{aligned} M_i(k) &= \left[R_i(k) + H_i^T(k) P_i(k) H_i(k)\right]^{-1}, \\ e_i(k) &= x_i(k) - \chi_i(k), \quad i = 1, 2, \cdots, n, \end{aligned}$$

where L_i is the number of weights for each neuron; $e_i \in \mathbb{R}$ is the respective identification error; $P_i \in \mathbb{R}^{L_i\times L_i}$ is the prediction error associated covariance matrix; $w_i \in \mathbb{R}^{L_i}$ is the weights vector; η_i is a design parameter; $K_i \in \mathbb{R}^{L_i\times m}$ is the Kalman gain matrix; $Q_i \in \mathbb{R}^{L_i\times L_i}$ is the state noise associated covariance matrix; $R_i \in \mathbb{R}^{m\times m}$ is the measurement noise associated covariance matrix; and $H_i \in \mathbb{R}^{L_i\times m}$ is a matrix, in which each entry H_{ij} is defined as:

$$H_{ij}(k) = \left[\frac{\partial x_i(k)}{\partial w_{ij}(k)}\right]^T_{w_i(k)=w_i(k+1)},$$

where $\{i = 1, \cdots, n\}$ and $\{j = 1, \cdots, L_i\}$.

The EKF based algorithm (6) used to train (5) guarantees that $\mathbf{e}_p$ and W_p remain bounded. Details about stability analysis, see [21], [26].

B. Neural Sliding-Mode Control

System ((2), (3)) can be stabilized at the homogeneous stationary state employing a control law based on the RHONN identifier (5) trained with EKF based algorithm (6). To achieve this goal, discrete-time sliding mode control law is used, given by

$$\mathbf{u}_p(k) = \begin{cases} \mathbf{u}_{p_{eq}}(k) & \text{if } \|\mathbf{u}_{p_{eq}}(k)\| \le u_{max} \\ u_{max}\frac{\mathbf{u}_{p_{eq}}(k)}{\|\mathbf{u}_{p_{eq}}(k)\|} & \text{if } \|\mathbf{u}_{p_{eq}}(k)\| \ge u_{max} \end{cases}, \tag{7}$$

where $\mathbf{u}_{p_{eq}}(k) = -W_{p,k}^T \boldsymbol{\phi}_p(\mathbf{x}_p(k))$, u_{max} is a bound for the control $\mathbf{u}_p(k)$, i.e., $\|\mathbf{u}_p(k)\| \le u_{max}$.

For the discrete-time sliding-mode neural control law stability analysis, see [26]. The synchronization analysis is ongoing work.

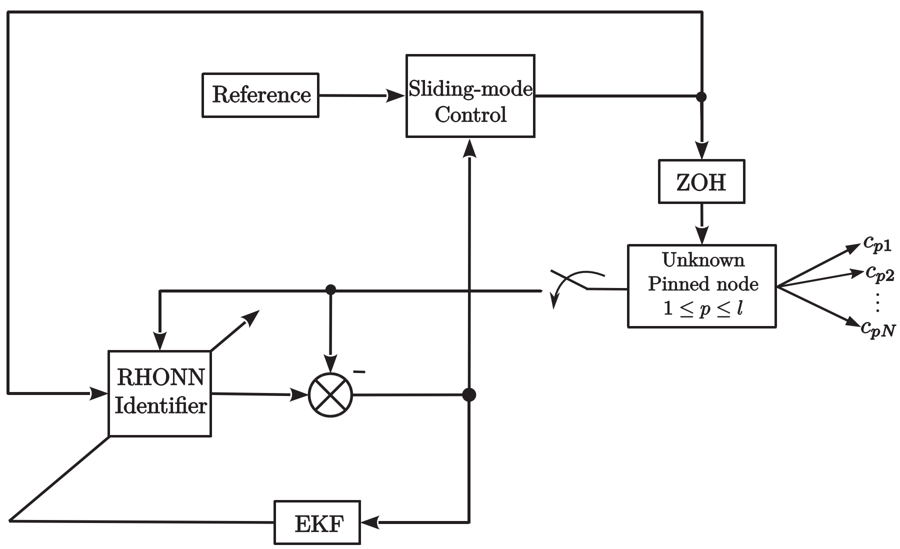

Fig. 1. Discrete-time neural sliding-mode pinning control scheme.

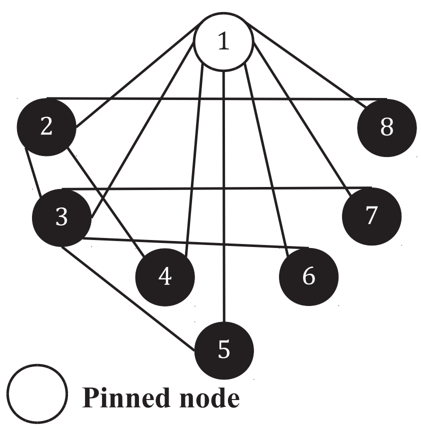

Fig. 2. Proposed network with 8 chaotic nodes, where Node 1 is the pinned node.

IV. SIMULATION RESULTS

To illustrate the effectiveness of discrete-time proposed control scheme to achieve synchronization, a complex network with 8-nodes is proposed, which is composed for four types of nodes, Chen's system (Node 1 and 2) [32], Rössler's system (Node 3 and 4) [33], Chua's system (Node 5 and 6) [34], and Lorenz's system (Node 7 and 8) [35] respectively. Fig. 2 presents this network, which has constants coupling strengths $c_{pj} = c = 20$, $\mathbf{\Gamma} = diag(1, 1, 1)$, and the desired synchronization point is given by $\bar{\mathbf{x}} = [0, 0, 0]^T$.

The proposed control law (7) is used to synchronize the whole network at $\bar{\mathbf{x}}$; Node 1 is selected as pinned one. The RHONN identifier used to model this pinned node $\mathbf{x}_1(k) = [x_{1_1}(k), x_{1_2}(k), x_{1_3}(k)]^T$, is given by

$$\begin{aligned}
\chi_{1_1}(k+1) &= w_1^T \phi_1(\mathbf{x}_1(k)) + u_1(k),\\
\chi_{1_2}(k+1) &= w_2^T \phi_2(\mathbf{x}_1(k)) + u_2(k),\\
\chi_{1_3}(k+1) &= w_3^T \phi_3(\mathbf{x}_1(k)) + u_3(k),
\end{aligned}$$

with

$$\begin{aligned}
\phi_1(\mathbf{x}_1(k)) =& [S(x_{1_1}(k)), S(x_{1_2}(k)), S^2(x_{1_1}(k))]^T,\\
\phi_2(\mathbf{x}_1(k)) =& [S(x_{1_2}(k)), S(x_{1_1}(k)), S(x_{1_1}(k))S(x_{1_3}(k)),\\
& S^2(x_{1_2}(k))]^T,\\
\phi_3(\mathbf{x}_1(k)) =& [S(x_{1_3}(k)), S(x_{1_1}(k))S(x_{1_2}(k)), S^2(x_{1_3}(k))]^T,
\end{aligned}$$

where $S(\cdot) = tanh(\cdot)$.

Simulations are done as follows. Until $t = 4s$, the proposed network evolves without control actions and interconnections ($\mathbf{u}_1 = 0, c_{pj} = 0$). After this lapse, the coupling strengths are selected as $c_{pj} = c = 20$, and the proposed discrete-time controller is applied at $t = 4s$; the entire network is synchronized at the desired point $\bar{\mathbf{x}}$. The control input signal applied to the pinned node is displayed in Fig. 3. The time evolution of neural network weights for node 1 and the identification error are presented in Fig. 4 and Fig. 5, respectively. Finally, Fig. 6 portrays the time evolution of network states.

Hence, for the complex network simulated with different chaotic nodes, the proposed control scheme achieves to minimize the identification error at an ultimate bound due to the effectiveness of the EFK-based training algorithm and simultaneously synchronizes the whole network to the desired point using the discrete-time neural sliding-mode pinning control applied in only one pinned node.

V. CONCLUSIONS

A discrete-time control strategy is proposed for the synchronization of complex networks. The proposed control neural scheme consists of an RHONN identifier trained with an EKF, which is used to design discrete-time neural sliding-mode pinning controller, guaranteeing that the control objectives are achieved for a complex network with non-identical chaotic nodes. A simulation example is

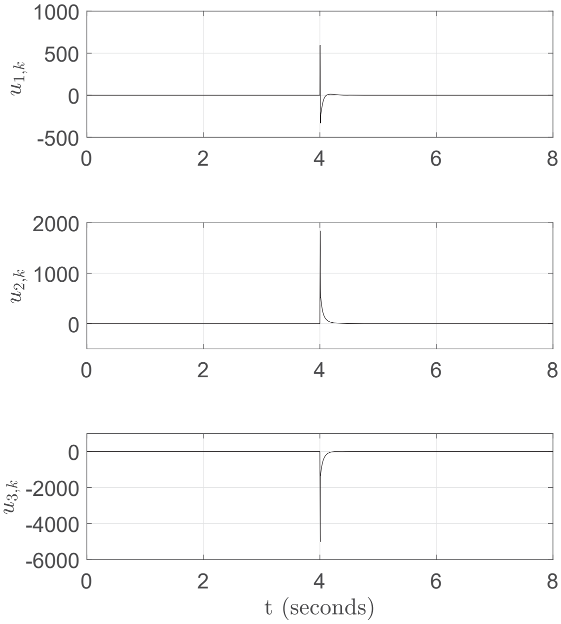

Fig. 3. Control input signal applied in the pinned node.

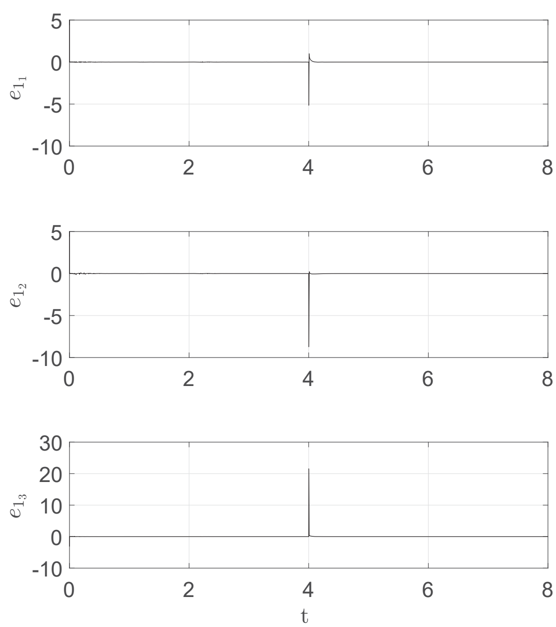

Fig. 4. Identification error evolution.

presented to verify the performance of the controller, which illustrates that whole network is effectively synchronized, even to different chaotic nodes using the proposed discrete-time control scheme.

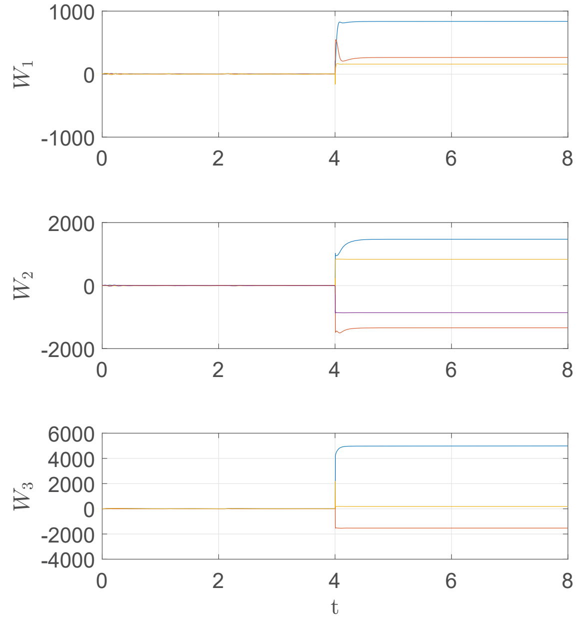

Fig. 5. Neural network weights evolution for Node 1.

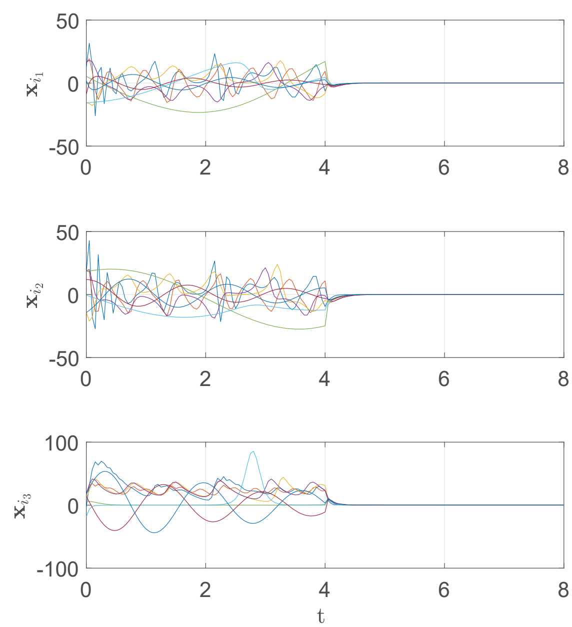

Fig. 6. Network states evolution.

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