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Noise Estimation in Measurements to Improve the State Estimation of Electric Power Systems

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***Abstract*— The accuracy in the estimation of the operating state of a power system depends on how biased the measurements used in the estimation are compared with their real values: perfect measurements. This work proposes a methodology, based on the double optimal regularization algorithm, for obtaining the most likely values of errors that are present in the set of measurements used to solve the problem of state estimation. Once these errors have been found, the measurements' database can be upgraded to perform a more accurate estimation of the operation status of a power system.**

Key words—State estimation, error estimation, sensitivity matrix, singular matrix inversion.

I. INTRODUCTION

In the context of power system analysis, the state estimation (SE) problem consists of computing the state variables that define its operating status, i.e. nodal voltage magnitudes and angles, from a redundant set of measurements and the system's topological configuration. The former is composed of measurements associated with power flows, power injections, complex voltages and complex currents [1]. The accuracy of estimates, which is directly associated with the values of measurements' errors, determines the quality and effectiveness of the control actions performed by the system's operators in maintaining the power system in a normal operation state.

In practice, the weighted least squares (WLS)-based estimators currently used in control centers have a post-processing technique for detecting the presence of measurements with gross errors. These kinds of measurements, referred to as bad data, are detected, identified and eliminated by using a bad data analysis approach. After this bad data analysis, the measurements' database is updated, and a new state estimation study is performed. This sequential solution process is carried out until no bad data are detected.

Different approaches for performing the bad data analysis mentioned above have been proposed in the literature: [2], [3], [4], [5], [6], [7] and [8]. All these proposals, however, have focused on measurements with gross error, thus neglecting small errors or noises contained in the measurements. Hence, the estimates are still biased against the real system's operation state. In this regard, if this estimation needs to be further improved, it is necessary to quantify the rate by which each measurement is biased from its perfect value.

A. Paper contributions

Based on the information mentioned above, this paper proposes an approach for quantifying the error associated with each measurement used to perform the state estimation of a power system. In this proposal, the double optimal regularization algorithm (DORA), a residual sensitivity matrix and measurements' residuals are used to estimate the most likely value of each measurement error. The information obtained from the probability density function (PDF) associated with each measurement error is also used for the estimation..

Unlike [5] and [6], the complete residual sensitivity matrix is considered for relating each measurement's residual to its corresponding unknown error such that not only the gross errors of measurements are quantified. Note that in [5] and [6] measurements without gross errors are assumed perfect: with no noise, which does not apply in practice. Lastly, a normalized residual sensitivity matrix is derived in the context of Hachtel's augmented matrix method for quantifying all the measurements' errors in a unit variance reference frame.

B. Paper organization

For the sake of completeness, the state estimation solved by Hachtel's augmented matrix method is described in Section II. The derivation of the normalized residuals' sensitivity matrix in the context of Hachtel's method is presented in Section III. The DORA and its proposed application are detailed in Section IV. Case studies are reported in Section V, followed by conclusions in Section VI.

II. STATE ESTIMATION USING HACHTEL'S METHOD

In electric power systems the operating state $\hat{\boldsymbol{x}} \in \Re^{n}$ is estimated from a set of measurements $\boldsymbol{z} \in \Re^{m}$ provided by a supervisory control and data acquisition (SCADA) system and a set of phasor measured units (PMUs). The SCADA system is capable of measuring branch power flows, nodal power injections and nodal voltage magnitudes: $\boldsymbol{z}_{\text{SCADA}} = \left[\boldsymbol{P}_{\text{flow}} \ \boldsymbol{Q}_{\text{flow}} \ \boldsymbol{P}_{\text{inj}} \ \boldsymbol{Q}_{\text{inj}} \ \boldsymbol{V} \right]^{\text{T}}$. On the other hand, the set of PMU measurements is composed of measurements associated with nodal voltages and branch current flows: $\boldsymbol{z}_{\text{PMU}} = \left[\boldsymbol{\theta}_{\text{PMU}} \ \boldsymbol{V}_{\text{PMU}} \ \boldsymbol{I}_{\text{PMU,r}} \ \boldsymbol{I}_{\text{PMU,i}} \right]^{\text{T}}$. The values of all these physical measurements are mathematically related to the true values of state variables $\boldsymbol{x}$ by $\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{\varepsilon}$ and to the

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estimates values of state variables $\hat{\boldsymbol{x}}$ by $\boldsymbol{z} = \boldsymbol{h}(\hat{\boldsymbol{x}}) + \boldsymbol{r}$. In these equations $\boldsymbol{h}(\hat{\boldsymbol{x}})$ is referred to as estimated measurements, and $\boldsymbol{\varepsilon} \in \Re^{m}$ denotes the vector of the measurements' errors with zero mean $E(\boldsymbol{\varepsilon}) = 0$. Lastly, $\boldsymbol{r} \in \Re^{m}$ is the vector of the measurements' residuals. Since the errors are independent, $\boldsymbol{R} \in \Re^{m \times m}$ is the error covariance matrix $\boldsymbol{R} = diag\left\{\sigma_{ii}^{\,2}\right\}$, where $\sigma_{ii} = \sigma_i$ is the standard deviation of the *i*-th measurement's error and reflects the expected accuracy of the corresponding measurement equipment.

The state estimation problem is solved by Hachtel's method by minimizing the sum of squared weighted residuals $J(\hat{\boldsymbol{r}}) = \left(\hat{\boldsymbol{r}}^{T}\boldsymbol{R}^{-1}\hat{\boldsymbol{r}}\right)/2$, while satisfying the two sets of equality constraints given by $\boldsymbol{c}(\hat{\boldsymbol{x}}) \in \Re^{c}$ and $\hat{\boldsymbol{r}} - \boldsymbol{z} + \boldsymbol{h}(\hat{\boldsymbol{x}}) = \boldsymbol{0} \in \Re^{m}$. These sets of constraints represent virtual measurements and estimated residuals $\hat{\boldsymbol{r}}$, respectively [9]. The Lagrangian method is applied to transform this constrained optimization problem into an unconstrained optimization problem by using two sets of Lagrange multipliers $\boldsymbol{\lambda} \in \Re^{c}$ and $\mu \in \Re^{m}$:

$$\mathcal{L}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{r}}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = J(\hat{\boldsymbol{r}}) - \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{c}(\hat{\boldsymbol{x}}) - \boldsymbol{\mu}^{\mathrm{T}}(\hat{\boldsymbol{r}} - \boldsymbol{z} + \boldsymbol{h}(\hat{\boldsymbol{x}})) \ . \quad (1)$$

The first-order optimality conditions of (1) are obtained with respect to (w.r.t.) $\hat{\boldsymbol{x}}, \hat{\boldsymbol{r}}, \boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. These conditions are then linearized w.r.t. $\hat{\boldsymbol{x}}$ by using Newton's method. Neglecting the Hessian terms, the following set of nonlinear algebraic equations must be solved for each *k* iteration to find the incremental vector $\Delta\hat{\boldsymbol{x}}^{k}$ [1]:

$$\begin{pmatrix} \boldsymbol{R} & \boldsymbol{H}(\hat{\boldsymbol{x}}^{k}) & \boldsymbol{0} \\ \boldsymbol{H}(\hat{\boldsymbol{x}}^{k})^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{C}(\hat{\boldsymbol{x}}^{k})^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{C}(\hat{\boldsymbol{x}}^{k}) & \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{\mu}^{k} \\ \Delta\hat{\boldsymbol{x}}^{k} \\ \boldsymbol{\lambda}^{k} \end{pmatrix} = \begin{pmatrix} \boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{x}}^{k}) \\ \boldsymbol{0} \\ -\boldsymbol{c}(\hat{\boldsymbol{x}}^{k}) \end{pmatrix} , \quad (2)$$

where $\boldsymbol{H}$ and $\boldsymbol{C}$ are the Jacobian matrices of $\boldsymbol{h}(\cdot)$ and $\boldsymbol{c}(\cdot)$, respectively, and $\boldsymbol{\mu}^{k} = \boldsymbol{R}^{-1}\hat{\boldsymbol{r}}^{k}$. Note that at the optimal solution $\boldsymbol{r} = \left(\boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{x}})\right) \cong \hat{\boldsymbol{r}}$.

III. Normalized Residual Sensitivity Matrix

The residual sensitivity matrix $\boldsymbol{S} \in \Re^{m \times m}$ relates the unknown errors of measurements to the measurements' residuals by

$$\boldsymbol{r} = \boldsymbol{S}\boldsymbol{\varepsilon} \ . \quad (3)$$

The residual sensitivity matrix is also related to the residual covariance matrix $\boldsymbol{\Omega}$ by

$$\boldsymbol{\Omega} = \boldsymbol{S}\boldsymbol{R} \ . \quad (4)$$

On the other hand, the residual covariance matrix $\boldsymbol{\Omega}$ given by (5) is derived from Hachtel's method, where the sub-matrix $\boldsymbol{A}_1 \in \Re^{m \times m}$ is given by (6) [9]:

$$\boldsymbol{\Omega} = \boldsymbol{R}\boldsymbol{A}_1\boldsymbol{R} \quad (5)$$

$$\begin{pmatrix} \boldsymbol{R} & \boldsymbol{H}(\hat{\boldsymbol{x}}) & \boldsymbol{0} \\ \boldsymbol{H}(\hat{\boldsymbol{x}})^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{C}(\hat{\boldsymbol{x}})^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{C}(\hat{\boldsymbol{x}}) & \boldsymbol{0} \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \boldsymbol{A}_3 \\ \boldsymbol{A}_2^{\mathrm{T}} & \boldsymbol{A}_4 & \boldsymbol{A}_5 \\ \boldsymbol{A}_3^{\mathrm{T}} & \boldsymbol{A}_5^{\mathrm{T}} & \boldsymbol{A}_6 \end{pmatrix} . \quad (6)$$

From (4) and (5), the residual sensitivity matrix is given by

$$\boldsymbol{S} = \boldsymbol{R}\boldsymbol{A}_1 \ . \quad (7)$$

The set of measurements used for the state estimation is obtained from different measurement equipment that has its own precision. This means that errors associated with measurements have values of a very different order of magnitude between each other. Hence, to represent all errors and residuals in a standardized reference frame, both errors and residuals are normalized w.r.t. their corresponding standard deviation:

$$\begin{aligned} \varepsilon_i^{Norm} &= \varepsilon_i \big/ \sigma_i \ , \forall i = 1,...,m \\ r_i^{Norm} &= r_i \big/ \sigma_i \ , \forall i = 1,...,m \end{aligned} \ . \quad (8)$$

By expressing (8) in vector form, the following set of equations is obtained:

$$\begin{aligned} \boldsymbol{\varepsilon} &= \sqrt{\boldsymbol{R}}\,\boldsymbol{\varepsilon}^{Norm} \\ \boldsymbol{r} &= \sqrt{\boldsymbol{R}}\,\boldsymbol{r}^{Norm} \end{aligned} \ . \quad (9)$$

Lastly, the normalized measurements' residuals $\boldsymbol{r}^{Norm}$ are obtained by substituting (9) and (7) into (3):

$$\begin{aligned} \sqrt{\boldsymbol{R}}\,\boldsymbol{r}^{Norm} &= \boldsymbol{R}\boldsymbol{A}_1\sqrt{\boldsymbol{R}}\,\boldsymbol{\varepsilon}^{Norm} \\ \sqrt{\boldsymbol{R}}\,\boldsymbol{r}^{Norm} &= \sqrt{\boldsymbol{R}}\sqrt{\boldsymbol{R}}\,\boldsymbol{A}_1\sqrt{\boldsymbol{R}}\,\boldsymbol{\varepsilon}^{Norm} \\ \boldsymbol{r}^{Norm} &= \sqrt{\boldsymbol{R}}\,\boldsymbol{A}_1\sqrt{\boldsymbol{R}}\,\boldsymbol{\varepsilon}^{Norm} . \end{aligned} \quad (10)$$

The normalized residual sensitivity matrix $\boldsymbol{S}^{Norm}$ is defined from (10) as the matrix that relates the normalized measurements' errors $\boldsymbol{\varepsilon}^{Norm}$ to the normalized measurements' residuals $\boldsymbol{r}^{Norm}$, i.e., $\boldsymbol{r}^{Norm} = \boldsymbol{S}^{Norm}\boldsymbol{\varepsilon}^{Norm}$ such that

$$\boldsymbol{S}^{Norm} = \sqrt{\boldsymbol{R}}\,\boldsymbol{A}_1\sqrt{\boldsymbol{R}} \ . \quad (11)$$

Since the vector $\boldsymbol{r}^{Norm}$ and the matrix $\boldsymbol{S}^{Norm}$ are obtained after solving (2), the values of unknown measurements' errors $\boldsymbol{\varepsilon}^{Norm}$ could be obtained from (12). Note, however, that $\boldsymbol{S}^{Norm}$ represents a set of ill-posed algebraic equations such that the matrix is not invertible:

$$\boldsymbol{\varepsilon}^{Norm} = \left(\boldsymbol{S}^{Norm}\right)^{-1}\boldsymbol{r}^{Norm} \ . \quad (12)$$

Based on the information mentioned above, the most likely solution of (12) can be obtained by using a numerical method that searches for the value of each error that is closer to its most frequent value in its corresponding probability density function. The numerical method adopted in this paper is detailed in the following section.

IV. Double Optimal Regularization Algorithm

The DORA method detailed in [10] iteratively searches for a double optimal solution of the ill-posed *n*-dimensional system of linear equations (12) in order to find the closest value of estimated normalized errors $\hat{\boldsymbol{\varepsilon}}^{Norm}$ that satisfies

$$\boldsymbol{r}_D = \boldsymbol{r}^{Norm} - \left(\boldsymbol{S}^{Norm}\hat{\boldsymbol{\varepsilon}}^{Norm}\right), \quad (13)$$

where $\boldsymbol{r}_D$ is the vector of DORA's residuals that satisfy a defined tolerance tol :

$$\left|r_{i,D}\right| < tol, \forall i = 1,...,\mathrm{m} \ . \quad (14)$$

In the context of this solution process, the importance of using normalized residuals and a normalized residual sensitivity matrix is that the specified tolerance has the same significance for all measurements' errors.

A. MATLAB DORA code

For completeness, the MATLAB code for solving the DORA is presented below, where the EeNorm is the vector of $\hat{\boldsymbol{\varepsilon}}^{Norm}$ with an initial value defined at the beginning of the iterative solution.

```
EeNorm=EeNorm0; %initial value (starting point)
rD=rNorm-SNorm*EeNorm;
iter=0;
while max(abs(rD)) > tol && iter < maxiter
    rD=rNorm-SNorm*EeNorm;
    u1=(SNorm*rD)/norm(SNorm*rD);
    [U,HA]=arnoldi(SNorm,u1,mm-1);
    J=SNorm*U(:,1:mm);
    C=J'*J;
    X=U*(C \ J');
    E=SNorm*X;
    In=diag(ones(size(SNorm,2),1));
    A=(rD'*(In-E)*SNorm*rD);
    B=(rD'*SNorm'*(In-E)*SNorm*rD);
    alfa0=A/B;
    z=X*rD+alfa0*(rD-X*SNorm*rD);
    gama=(beta*norm(z)^2*norm(SNorm*z)^2)^(-1/4);
    dx=gama*z;
    EeNorm=EeNorm+dx;
    iter=iter+1;
end
```

B. Selection of the initial value of estimated normalized errors

The Hachtel-based state estimation method is derived under the assumption that measurements' errors follow a Gaussian distribution with zero mean and that the standard deviation of each error is known [1]. Under these assumptions, the most frequent value of an error would be 0. Furthermore, if all these errors truly have a Gaussian distribution and are correctly represented by their corresponding standard deviation, the residuals obtained at the optimal solution of (2) should represent the set of errors that are closest to 0. Hence, in this case the initialization of $\hat{\boldsymbol{\varepsilon}}^{Norm}$ is $\hat{\boldsymbol{\varepsilon}}^{Norm} = \mathbf{0}$.

On the other hand, if the measurements' errors follow an asymmetric non-Gaussian distribution, e.g. a Gaussian mixture distribution, the most likely values of these errors are not null. To validate this statement, a Gaussian mixture of two components is shown in Fig. 1. The red line represents the Gaussian fit of the Gaussian mixture PDF and shows that the mean value is 0, while the most likely error is around 0.005. The effect of choosing the mean or the most frequent error as the initial value for the estimated errors will be tested in the case studies.

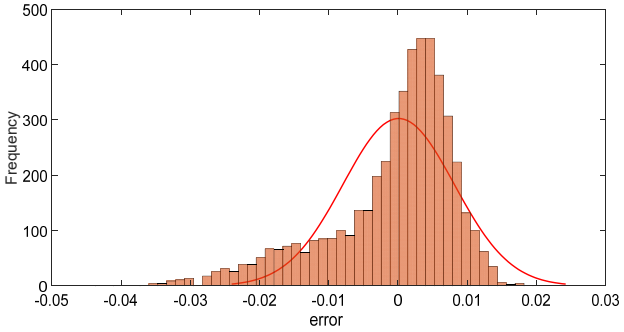

Fig. 1 Histogram of a Gaussian mixture of two components.

V. Case Studies

The proposed approach is used in this section to estimate the values of errors present in a set of measurements associated with the IEEE 14-bus test system. A first set of perfect measurements is directly generated from a power flow study by considering 37 branch power flows and three nodal voltage phasors. This set is then corrupted with errors $\boldsymbol{\varepsilon}^{Norm}$ generated from a set of random numbers that follow a specified probability density function. Lastly, the parameters used in the application of DORA correspond to a beta value of $1\text{x}10^5$, four Arnoldi terms, a convergence tolerance of 0.01, and a maximum number of iterations of 100.

The first case study consists of generating measurements' errors having a Gaussian PDF with zero mean and a specified standard deviation σ according to the type of measurement. These standard deviations are 0.01 for the branch power flows and 0.001 for the nodal voltage phasor measurements. The proposed approach is applied by considering an initial value of zero for all elements of $\hat{\boldsymbol{\varepsilon}}^{Norm}$. The estimated normalized errors $\hat{\boldsymbol{\varepsilon}}^{Norm}$ obtained by DORA are compared with the normalized residuals $\boldsymbol{r}^{Norm}$ obtained from the Hachtel-based state estimation study and with the normalized true errors $\boldsymbol{\varepsilon}^{Norm}$. These comparisons are schematically reported in Fig. 2. As expected, the solution of (12) is very similar to the residuals obtained in the optimal solution of (2).

The case study reported above is repeated, but the measurements' errors are now generated from a Gaussian mixture PDF. In this case, the following parameters are considered for the two components: means of 0.4σ and -1.4σ, standard deviations of 0.5σ and 1.3σ and weights of 0.8 and 0.2, respectively. The application of DORA results in the values of $\hat{\boldsymbol{\varepsilon}}^{Norm}$ reported in Fig. 3 when using the expected value of 0 as an initial value for all elements of $\hat{\boldsymbol{\varepsilon}}^{Norm}$. The values of $\boldsymbol{\varepsilon}^{Norm}$ and $\boldsymbol{r}^{Norm}$ are also reported in this figure. These results clearly show that the closest solution of (12) to the expected value is very similar to the residuals obtained from the estimation process, even though the errors do not follow a Gaussian PDF. This indicates that the

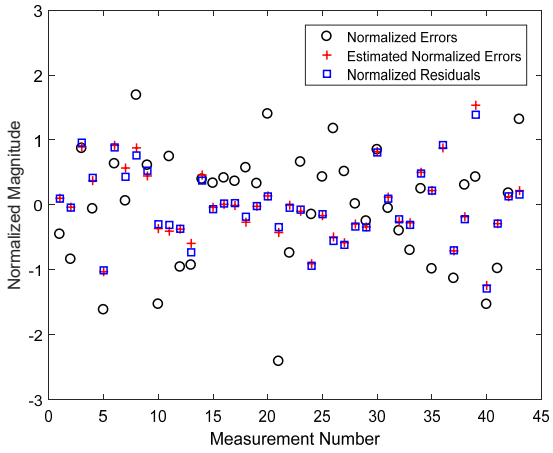

Fig. 2 Estimated errors, true errors and residuals of case study 1.

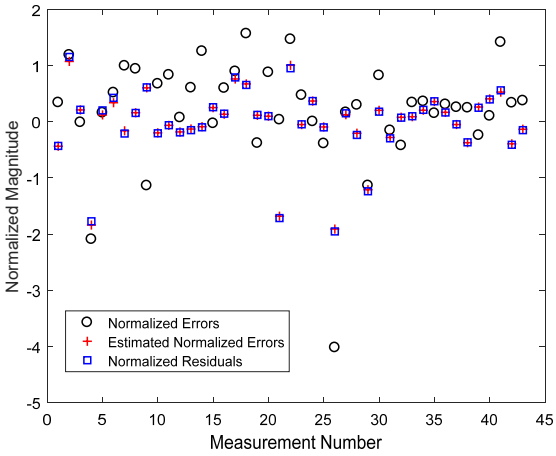

Fig. 3 Estimated errors, true errors and residuals of case study 2.

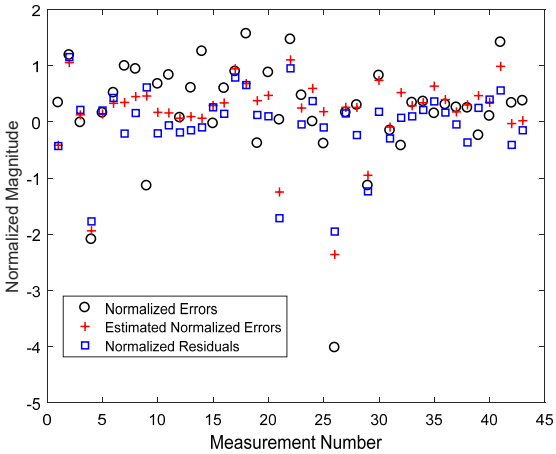

Fig. 4 Estimated errors, true errors and residuals of case study 3.

estimator still gives the best estimates under the consideration of the expected value.

Lastly, in order to quantify how the initialization of $\hat{\boldsymbol{\varepsilon}}^{Norm}$ affects the results obtained by the application of DORA, the values of $\boldsymbol{r}^{Norm}$ obtained in the last case study are used to estimate $\hat{\boldsymbol{\varepsilon}}^{Norm}$ when this vector is initialized at the most frequent value of the Gaussian mixture distribution. The results are presented in Fig. 4, which shows that the estimated errors are closer to the true errors most of the time. With this in mind, the measurement set can be upgraded by subtracting such an estimated error, and a new estimation process can be performed that yields better estimates.

In order to have a metric that allows comparing the solutions associated with the second and third test cases, the mean squared error (MSE) is calculated as the mean value of the squared differences between the true errors and the estimated errors such that

$$MSE = \frac{1}{m}\sum_{i=1}^{m}\left(\varepsilon_i^{Norm} - \hat{\varepsilon}_i^{Norm}\right)^2 \quad . \tag{15}$$

The MSEs obtained for the second and third test cases are 0.5414 and 0.3591, respectively. The smaller value of MSE obtained when using the most frequent value as the initial estimated error shows that this gives estimates closer to the true values.

A. Comparison with other methods

In this section, the approaches reported in [5] and [6] are applied to case study 2 to obtain the estimated errors from a selected measurement's subset. Once these errors have been computed, the corresponding MSE of each approach is computed by using (15) and compared with the MSE value associated with the proposed approach.

The selection of the subset of measurements to be used in the estimation of errors is determined by all measurement residues greater than a predefined threshold *Th*. Note that the authors in [5] and [6] define the normalized residuals in a different way than in our proposal. Their definition of normalized residuals $\boldsymbol{r}_N$ is given by

$$r_{Ni} = \frac{r_i}{\sqrt{\boldsymbol{\Omega}_{ii}}}, \forall i = 1,...,m \quad . \tag{16}$$

On the other hand, their definition of weighted residuals $\boldsymbol{r}_W$ corresponds to the definition of normalized residuals adopted in this paper: $\boldsymbol{r}_W = \boldsymbol{r}^{Norm}$.

Once the measurements' subset is determined, the subset of estimated errors are obtained by

$$\hat{\boldsymbol{e}}_S = \boldsymbol{S}_{SS}^{-1}\boldsymbol{r}_S \quad , \tag{17}$$

where $\boldsymbol{S}_{SS}$ is a sub-matrix of the residual sensitivity matrix $\boldsymbol{S}$ and $\boldsymbol{r}_S$ is a subvector of $\boldsymbol{r}$. In this context, the measurements' errors not contained in the selected subset are assumed to be 0.

Lastly, the decision of using either $|r_{Ni}| > Th$ or $|r_{Wi}| > Th$, as well as the selection of *Th*, has a direct impact on which measurements will be included in the selected subset. Hence, the normalized and weighted residuals were combined with different values of *Th* to obtain different subsets of measurements, which in turn were used to estimate the errors of measurements and to compute the MSEs reported in Table I.

Table I. MSEs obtained for different values of *Th*

	Values of Th					
	0.5	1	1.5	2	2.5	3
MSE w/ $\boldsymbol{r}_W$	1.0566	0.4268	0.4916	0.9493	0.9493	0.9493
MSE w/ $\boldsymbol{r}_N$	$5.7\text{x}10^{32}$	139.42	1.0286	0.4344	0.5880	0.9493

Table I shows that the measurements' subset obtained when using $\boldsymbol{r}_W$ instead of $\boldsymbol{r}_N$ gives better estimates of errors. Note also that a MSE value of 0.9493 is obtained when the subset of measurements is empty and all errors are assumed to be 0. This empty subset condition is found for thresholds greater than or equal to 2 (resp. 3) in the case of $\boldsymbol{r}_W$ (resp. $\boldsymbol{r}_N$). The MSE value of $5.7\text{x}10^{32}$ is obtained because of a singularity problem when inverting the sub-matrix $\boldsymbol{S}_{SS}$.

Finally, a comparison of the MSE values reported in Table I with the one obtained by the proposed method, i.e. 0.3591, demonstrates that our proposal provides a better estimate of errors. Moreover, the results obtained by using the methods proposed in [5] and [6] are very sensitive to the selected value of *Th* such that an erroneous selection can lead to singularity problems.

VI. Conclusions

In this proposal, the application of the double optimal regularization algorithm for obtaining the most likely measurement errors under Gaussian and Gaussian mixture distribution was presented.

It was shown that under Gaussian mixture errors the selection of the most frequent error as the initial guess of the recursive algorithm provides better results than the mean value.

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